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SAFE BUILDING

A TREATISE

GIVING IN THE SIMPLEST FORMS POSSIBLE

THE PRACTICAL AND THEORETICAL RULES AND FORMULÆ USED IN THE CONSTRUCTION OF BUILDINGS

BY

LOUIS DE COPPET BERG, F. A. I. A.

MEMBER AMERICAN SOCIETY OF CIVIL ENGINEERS

VOLUME TWO

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TABLE OF CONTENTS OF VOL. II.

PAGE

LIST OF TABLES ix

LIST OF FORMULAE xiii

PREFACE TO VOL. II. xv

CHAPTER VIII.

THE NATURE AND USES OF IRON AND STEEL 1–55

CHAPTER IX.

RIVETS, RIVETING AND PINS 56–105

CHAPTER X.

PLATE AND BOX GIRDERS.. 106–135

CHAPTER XI.

GRAPHICAL ANALYSIS OF STRAINS IN TRUSSES.. .. 136–178

CHAPTER XII.

WOODEN AND IRON TRUSSES 179–199

CHAPTER XIII.

COLUMNS.. 200–212

TABLES 213–262

INDEX 263–279

LIST OF TABLES IN VOL. II.

[*See Index*, "*How to use Tables.*"]

	PAGE
TABLE XXVI. — Analysis of cast-irons	8
TABLE XXVII. — Densities and weights of cast-irons . .	8
TABLE XXVIII. — Classification of irons and steels . . .	26
TABLE XXIX. — Classification of structural irons and steels	*at end of text.*
TABLE XXX. — Amount of extension and contraction, in inches, of cast and wrought iron bars, 100 feet long, under different strains.	"
TABLE XXXI. — Length of cast or wrought iron bars, in feet, that will stretch or contract exactly one inch under different strains.	"
TABLE XXXII. — Ultimate breaking strength of materials under different kinds of strains (dead, rolling, intermittent, etc.).	48
TABLE XXXIII. — Proportion for screw-threads, nuts and bolt heads	*at end of text.*
TABLE XXXIV. — Recent tests of irons and steels . .	"
TABLE XXXV. — Bearing values for iron and steel rivets and pins	"
TABLE XXXVI. — Bearing values for iron and steel pins and bolts, 1″ to 3″ diameter . .	"

TABLE XXXVII. — Bearing value for iron and steel pins and bolts, 3″ to 6″ diameter . . *at end of text.*

TABLE XXXVIII. — Shearing, bending and tensional values for iron and steel rivets, pins and rods of one-fourth to one inch in diameter. . . . . "

TABLE XXXIX. — Shearing, bending and tensional values for iron and steel pins and rods 1″ to 3″ in diameter. . . "

TABLE XL. — Shearing, bending and tensional values for iron and steel pins and rods 3″ to 6″ in diameter. . . "

TABLE XLI. — Wrought-iron riveted girders: strength of webs. . . . .

TABLE XLII. — Wrought-iron riveted girders: strength of the angle bars. . . "

TABLE XLIII. — Wrought-iron riveted girders: strength of flanges . . . . "

TABLE XLIV. — Table of wind-pressures on roofs . . . 140

TABLE XLV. — Strength of hollow cylindrical columns of cast-iron (3″ to 7″ diameter). . . . 213

TABLE XLVI. — Strength of hollow cylindrical columns of cast-iron (8″ to 10″ diameter.) . . . 214

TABLE XLVII. — Strength of hollow cylindrical columns of cast-iron (11″ to 13″ diameter) . . . 215

TABLE XLVIII. — Strength of hollow cylindrical columns of cast-iron (14″ to 16″ diameter) . . . 216

TABLE XLIX. — Safe compressive loads per square inch on wrought-iron columns. 217

TABLE L. — Properties of Phoenix columns . . . 218

TABLE LI. — Natural sines, etc. 219, 220

TABLE LII. — Natural sines, tangents and secants . . 221–224

TABLE LIII. — Logarithms of numbers 225–227
TABLE LIV. — Squares, cubes, square and cube roots and reciprocals of numbers 228–242
TABLE LV. — Multiplication table in inches for rectangular sections 243–245
TABLE LVI. — Circumferences and areas of circles from 0 to 100, also squares up to 100 246–256
TABLE LVII. — Decimal parts of an inch for each 1-64 of an inch 257
TABLE LVIII. — Decimal parts of a foot for each 1-64 of an inch, from 0″ to 12″ 258, 259
TABLE LIX. — Reduction of English feet and inches to metres 260
TABLE LX. — Reduction of English inches to millimetres . 261
TABLE LXI. — Reduction of English weights to kilograms. 261
TABLE LXII. — English measures and weights 262

LIST OF FORMULÆ IN VOL. II.

NO. OF FORMULA.		PAGE
100.	— Effect of thickness on tensional strength of cast-iron.	34
101.	— Effect of thickness on cross-breaking strength of cast-iron	34
102.	— To find modulus of rupture of a material from cross-breaking test	38
103.	— Same as 102, if specimen 1″ x 1″ and 12″ long, and pressure applied in centre,	38
104.	— Extension of cast-iron under tension	44
105.	— Contraction of cast-iron under compression	44
106.	— Effect of variable strains	49
107.	— Greatest pitch for rivets	65
108.	— Least pitch for rivets	65
109.	— Number of rivets required at lap and butt joints to resist bearing	68
110.	— Number of rivets required at lap and butt joints to resist single shearing,	68
111.	— Number of rivets required at lap and butt joints to resist double shearing.	68
112.	— Number of rivets required at butt joints to resist bending or cross-breaking,	68
113.	— Number of rivets required at butt joints to resist bending or cross-breaking,	68

NO. OF FORMULA. PAGE

114. — Thickness of plate required to resist rivets compressing or tearing same, 69

115. — Length of plate needed between rivet and edge of plate . . 69

116. — Load on cantilever only, or on overhanging pin end, nearer reaction, 87

117. — Same as 116, farther reaction 87

118. — Load on both beam and cantilever, or on pin and overhanging end, nearer reaction, 88

119. — Same as 118, farther reaction. 88

120. — Required thickness of eye-bar at pin 89

121. — Horizontal flange strain on rivets, in plate girders . . . 108

122. — Number of rivets required in ends of each layer of flange plates, 111

123. — Horizontal slide tendency in plates of riveted girders . . 112

124. — Required thickness of web in plate girders 115

125. — Total compression strain on end stiffeners of plate girder . 116

126. — Total compression strain on intermediate stiffeners of plate girders, 116

127. — Where stiffeners are needed in plate girders 116

128. — Average fibre stress in flanges of plate girders . . . 134

129. — Increase in stress for curved compression (struts) or tension (tie) members, 160

130. — Amount of reaction (left side) due to wind. 167

131. — Amount of reaction (right side) due to wind 167

132. — Safe load on each roller under end of truss. 198

Formulæ for calculating natural sines, cosines, tangents, etc. . . 220

PREFACE.

IN presenting the second volume of "*Safe Building*" to the public, a few words of explanation seem in place.

In the preliminary announcement it was said that there might "possibly" be added, chapters on Drainage, Plumbing, Heating, and Ventilating; the work, however, is of sufficient length without these subjects, and as they really form separate studies from those of construction, it has been thought best to omit them.

While the first volume of this work, which has been before the public for several years, has been favorably received, criticisms have been made on the absence of an index and on the meagre treatment of iron constructions. These wants are supplied by Volume II. The writer believes that no fuller or clearer treatment of riveted girders, or of truss work generally has been published than will be found in this volume; while he has *personally* prepared the index with the greatest care, in order to adapt the work for reference, as well as for study. The reference tables at the end of Volume II have all been newly made up and carefully corrected; they are so numerous that it is believed that every table needed by an architect in his practice will be found there, avoiding the necessity of other hand-books. Throughout both volumes the writer has carefully read all proofs to insure correctness.

The work has extended over several years; this has not been due to any lack of enterprise or liberality on the part of the publishers; but to the fact that the study, writing, and proof-reading had to be done in such time as the writer could secure outside the duties of an exacting professional practice.

SAFE BUILDING.

CHAPTER VIII.

THE NATURE AND USES OF IRON AND STEEL.

THE introduction of the use of iron into the construction of buildings has practically revolutionized modern architecture; the introduction of steel promises to make equally great changes. The cost of these materials is comparatively so much greater than the ordinary materials used, such as brick and wood, and, again, the uniformity of their composition and strength is such, that in their use the smallest factors-of-safety are used; that is, the size of material used is very much more nearly equal to its ultimate strength than is the case when using cheaper or less uniform materials. Where, therefore, we "run so closely to the wind," it is essential that the nature and use of the material be thoroughly understood by the architect. Iron is used in three different kinds in building; namely, wrought-iron, steel, and cast-iron. Each has its uses and merits, and its disadvantages. All are really but iron in different combinations. Their differences depend mainly on the amount of carbon they contain. The more carbon, the more brittle, but harder is the iron. The less carbon, the more flexible and elastic, but softer is the iron.

Three Kinds of Iron.

Wrought or rolled iron is the softest, that is iron in its purest form. As it combines with itself a small amount of carbon, it becomes soft steel. The absorption of more carbon makes harder steel, until finally it becomes cast-iron. Pure or real metallic iron does not occur in nature, in commercial quantities, if at all. It is extracted

Iron Ores. from the various ores of iron, the chief of which are known as magnetite, red and brown hematite, limanite, siderite, etc., being various combinations of iron, with oxygen, forming oxides; of carbon and oxygen forming carbonates; and of hydrogen and oxygen forming hydrates. Other minerals, rich in iron ore, are found, but cannot be used in the manufacture of irons, on account of the large percentages of sulphur, copper, phosphorus and other substances they contain, which, if present in the finished product even to the smallest extent, render it unfit for most uses.

In the manufacture of pig-iron, the ore — or preferable an intelligent mixture of ores — is smelted in a blast furnace with the addi-

Manufacture of Pig-Iron. tion of coal or coke (or a mixture of both), as fuel, and limestone, or some substitute, as a "flux." The blast furnace itself is roughly of the form of an upright hollow cylinder, sometimes 100 feet high, but usually from 50 to 80 feet high and from 20 feet to 25 feet in diameter. The structure

Blast Furnace. has a strong masonry foundation on which rest about eight cast or wrought iron columns, some 10 to 20 feet in height. These sustain a plate-iron casing enclosing the whole furnace from bottom to top. Inside, the furnace is of the shape of two truncated cones, placed base to base over each other, with a short cylinder at

Hearth and Crucible. the bottom, being thus somewhat narrower at the top and bottom. The bottom of the furnace is called the "hearth;" about 5 feet to 7 feet above the hearth is the "crucible;" from hearth to crucible the furnace is cylindrical and from 6 feet to 12 feet diameter. From the crucible to the "bosh" which is some 20 feet to 30 feet above the hearth, the furnace enlarges to some 14 feet to 20 feet diameter, sometimes even 25 feet diameter. From here to the "throat" which is the extreme top, the furnace narrows down again, being some 10 feet to 15 feet diameter at the top. The furnace is lined inside with an infusible lining of fire-brick, and the charging of ore, flux and fuel is kept up constantly, and of course the fire and smelting process kept going, without stop, barring accidents, for many months at a time, and until this lining gives out; as a rule, the fire is continuous for from two to four years. The lower end of the furnace is closed save for an orifice at the bottom pierced through the walls about horizontally and known as the "hearth." In this pit the melted iron as it is reduced, being heavier than the flux, impurities or fuel, settles down

and collects, until sufficient is obtained to justify the tapping or withdrawal of the plug from the orifice, when, of course, the pressure from above forces out the molten iron, which being thus withdrawn flows off through dikes and furrows in the sand of the casting-house floor. This tapping is done from three to four times every twenty-four hours. The main or feed channels through which the metal flows off directly from the furnace are known as the "sows"; at right angles to these, at frequent intervals, are the short furrows known as the "pigs." These are of convenient size for handling, and when cooled, are broken from the "sows" and form what is known in commerce as "pig-iron."

Casting Pig-Iron.

Just under the crucible, that is, above the level to which the melted iron is allowed to rise in the hearth, or some 3½ feet to 6 feet above the hearth, there are from five to eight radial openings in the walls of the furnace admitting the "tuyères" which are blast nozzles, cooled by the circulation of water in them, and through which hot or cold air is forced horizontally into the blast furnace. The product is known accordingly as "hot blast" or "cold blast." The pressure under which this air has to be forced in, varies, according to circumstances, between 3 and 13 pounds per square inch.

Hot and Cold Blast.

As already remarked, the walls of the furnace widen out above this forming what is known as the "bosh" or the main body of the furnace. Above this the walls usually narrow down, the narrowing, however, depending upon the ore used or the product desired. This part is called the "stack." In the bosh and in the stack the distinctive phenomena of the blast-furnace mainly take place. The top where the walls always narrow down considerably, forming the "throat" of the furnace, is usually closed nowadays by a cone drawn up against a conical hopper, and only opened to allow of the charging of ore, flux and fuel, which is done in alternate layers, after lowering the cone a little. The cone, of course, being again drawn up tightly into place, after the charging. The furnace is kept constantly full, to the throat, being charged as often as the material settles or is withdrawn. The charge usually consists, first, of from 1 to 3 tons of fuel — (coal, or coke, or both) — and then a mixture of ores in proportion of 1½ ton of ore to each ton of fuel. After this the limestone or flux is put in, being in weight from 40 per cent to 60 per cent of the ore. The materials are

Charging Furnace.

hoisted to the top in iron barrows by proper machinery. Where the tops of furnaces are kept closed, the blast — (or heated gases at the top) — is conducted off through flues immediately underneath and around the top. Part of these heated but otherwise waste gases are then passed through iron flues to the brick chambers, called "hot blast stoves," or around iron pipes, and serve to heat the "blast" or feed draught of the furnace, where a "hot blast" is used, thus saving fuel, increasing the output and effecting a considerable change in the nature of the pig-iron.

Hot Blast.

The air for the hot blast is heated by passing it through the above mentioned iron pipes around which the gases play; or, where the brick chambers are used, the gases are turned into the chambers alternately, that is, one after another until the brick linings of one set of chambers are highly heated; the gases are then turned into the alternate set of chambers and the air to be used in the hot blast is admitted to the first set and becomes quickly heated to a temperature of from 900° to 1500° F. by contact with the hot bricks. As these cool the process is reversed, the alternate chambers being now used. These chambers are about 17 feet diameter, 60 feet high cylinders of plate-iron, made air-tight and lined with fire-brick. The interior being lined with a mass of intersecting flues of fire-brick. The balance of the hot gases which do not pass to these chambers are used to heat the boilers, which supply the necessary steam-power for the hoisting machinery, forced blasts, etc.

The process of smelting ores into pig-irons is, then, roughly this: The ore, flux and fuel are charged into the furnace from the top, in alternate layers at stated periods. A fierce fire is kept going and supplied with the necessary air (either hot or cold) for combustion at the bottom by means of a forced draught. As the above layers descend in the furnace they change their nature. The fuel gives out carbonic oxide which reduces the iron. The latter gradually separates from its impurities and combines with more or less carbon from the fuel. The ashes of the fuel and impurities of the ores combine with the flux (the melted limestone), and when all reaches the bottom we have the pure melted iron (with more or less carbon) at the very bottom or hearth; over this, in the crucible, float the melted flux and combined impurities; above come layers of less perfect iron, flux, partly-consumed fuel, and so on to the top. Before drawing off the melted iron at the

Description of Process.

bottom, the impurities and flux, known as the "slag," immediately above the melted iron, are first drawn off.[1]

Chemical Process.

To be more technical in the above description, we should say that the ores of iron, whether oxides, carbonates or hydrates, are reduced either by their preparatory roasting or during their early passage down the top of the furnace shaft to the state of oxide of iron (ferric oxide).

The ensuing reaction in the furnace is, therefore, for all practical purposes, the reduction of this ferric oxide ($Fe_2 O_3$) when red hot by the action of carbonic oxide (C O) produced by the incomplete combustion of the fuel farther down the furnace. The iron gives up its oxygen to the carbonic oxide leaving metallic iron (which then takes up with some carbon) and carbonic dioxide ($C O_2$) which passes away in the waste gases. It should be noted here that pure metallic iron is infusible at the temperature obtainable in the blast-furnace. Its combination with carbon, however, to the extent of from 2 per cent to 5 per cent renders it easily fusible, and constitutes the pig or cast iron. Were it not for this fact the blast-furnace would be impracticable, as can be readily imagined.

Use of Flux.

The mission of the limestone or other fluxes, is mainly, when melted, to effect a more ready fusion or separation of the earthy impurities or "gangue" in the ore and to take up the ashy remnants of the fuel. It is found that the earthy bases are more fusible to an extraordinary degree when they are present together in numbers. Further, the addition of lime takes care of the silica present in the ores, which otherwise would unite with the iron, forming silicates of iron, which, though fusible, are difficult of reduction, and further prevent to a certain extent the taking up of carbon by the reduced iron, thus entailing a waste in two ways. The ordinary gangue or matrix of iron ore itself is clayey (argillaceous) or quartzose (silicious). The addition of lime or limestone (or dolomite) results in the formation of a "slag" which is readily fusible at the existing temperature. This slag, which when cold somewhat resembles bottle-glass, is much lighter than the molten iron, and as it collects above it, is drawn off just before casting from the surface of the melted iron in the hearth through openings placed at the proper level, just below the crucible.

[1] This slag forms the basis of the "mineral wool," largely used for various purposes.

Brands of Pig-iron.

To undertake to enumerate all of the brands of pig-iron used in casting would be an endless task. A few, however, may be here mentioned.

Amongst those principally used in the New York market are:

Coltness, Summerlee, Shotts, Langloan, Dalmellington, Clyde, Eglinton, Glengarnock, Gartsherrie.	All Scotch irons: used as *softeners*, in connection with scrap-iron or lower grades of American pig-iron for cheap and inferior castings.
Cleator, Lowther, Lonsdale.	English Bessemers: are soft and strong and are used in place of best Scotch iron.
Manhattan, (New York), Low Moor, (Virginia), Thomas, (Penn.), Crane, (Penn.), Musconetcong, (Penn.), Sloss, (Alabama), Woodward, Spearman, Carbon, Granger, South Pittsburgh, Alice, Citico, Chattanooga, Hudson, Cold Spring, Sheridan, Leesport, Coleraine, Brier Hill, Secaucus, Castle, Poughkeepsie, Copley, Glendon, Andover, Taylor, Cornwall, Bethlehem, Stanhope, Allentown, Harry Clay, Harrisburg, Mill Creek.	All American brands.[1] The Manhattan is very fine; Secaucus and Castle are very strong; Thomas and Glendon are very popular and their Nos. 1 and 2 largely used for strong and good castings. The last three on the list are weak and soft cinder irons and are unfit for architectural uses, being used principally for stove-plates and pipe making.

Grading of Pig-iron.

All pig-irons are graded in three kinds, namely, Mill iron, Foundry iron and Bessemer[2] iron. Each of these is again subdivided into the following six grades:

No. 1, No. 2, No. 3, Grey Forge, Mottled, White.

[1] For makers' names and addresses, see "The Directory of the Iron and Steel Works of the United States," published by the American Iron and Steel Association, 261 S. 4th Street, Philadelphia.

[2] Any foundry iron which is sufficiently low in phosphorus (not over 0,1 per cent) and silicon can be used in the Bessemer process.

"No. 1" is the best and strongest, "No. 2" the next best, and so on to the "White," which is the poorest quality. Grey irons contain more graphitic carbon and are softer and more fusible than white irons, which contain more combined carbon, and are much harder and more brittle.

If the pig-iron on fracture is dark grey with spots it is soft and will run freely into the mould, making a good casting but not a strong one. Black specks, if present, mean carbon. If the carbon in the iron is chemically combined, it will show white metal, with no specks, on fracture, in which case the iron is very hard and brittle and will not flow easily into the mould, but will make a very strong casting.

For rolling or mill work the most used are the Nos. 2 and 3, Grey Forge and Mottled of the mill irons. For castings the most used are the Nos. 1, 2 and 3 and Grey Forge of foundry irons; the Mottled and White being usually sold for cheap mill-work.

For steel the iron should be as free as possible from phosphorus and sulphur, and the same, so far as possible, for rolled-iron. The presence of these makes iron fluid and soft and good for fine castings, but unfits it for rolling or forging.

Irons for mill and steel work are usually much stronger than for foundry work.

Scotch Softeners.

Scotch irons are used in castings to make the melted iron more fluid, to soften it; but they greatly weaken the casting. For very fine castings, Coltness is the best and softest. For ordinary architectural castings, such as columns, lintels, etc., either Glengarnock or Eglinton (both Scotch irons) can be used; using one-third Scotch to two-thirds of some good American iron; using Nos. 1 and 2 of the latter in equal proportions.

Sloss (American) iron is now frequently used by good manufacturers as a softener in place of Scotch iron.

Strong Pig-Irons.

For good and yet strong castings, use Thomas, Crane, Copley, Manhattan, Low Moor, Glendon or Coleraine. Add Sloss or Scotch for extra fine castings. Or add Glendon, Secausus or Castle for extra strong castings, using the No. 1 mill irons for the strongest work.

For rolled iron-work use Glendon, Andover, Taylor, Thomas, Stanhope, Allentown, Cornwall or Bethlehem. The latter two being used for steel.

In every case the better qualities (Nos. 1 and 2) will, of course, give the best results.

Unwin compiles (from a paper published by Mr. Turner in the *Transactions* of the Iron and Steel Institute of 1885) the following Tables of percentages, density and weight, for cast-iron:

TABLE XXVI.

ANALYSIS OF CAST IRONS.

	Combined Carbon.	Graphitic Carbon.	Silicon.
Greatest softness	0,15	3,1	2,5
" hardness	—	—	under 0,8
" general strength	0,50	2,8	1,42
" stiffness	—	—	1,0
" tensile strength	—	—	1,8
" crushing strength	over 1,0	under 2,6	about 0,8

TABLE XXVII.

DENSITIES AND WEIGHTS OF CAST IRONS.

Material.	Density.	Weight per cubic foot in lbs.
Dark-grey foundry-iron	6,80	425
Grey foundry-iron	7,20	450
Mottled foundry-iron	7,35	458
White iron	7,50	474

For wrought-iron Unwin gives this analysis:

Carbon 0,02 to 0,25 per cent; Manganese 0,0 to 0,3 per cent; Silicon 0,0 to 0,2 per cent; Sulphur 0,0 to 0,015 per cent; Phosphorus the same, and Pure Iron 99 to 99,5 per cent.

For steel, of course, the proportions vary greatly with the amount of carbon it contains.

From pig-iron is produced cast-iron, steel and wrought-iron. Castings are generally made from pig-iron. In some rare cases cheap castings are made the same as the pig-iron, from the "first melting," that is, directly from the molten ore. Most castings, however, are made from the second melting, that is, the pig-iron is re-melted in a cupola or reverbatory furnace with more flux to take up any remaining impurities, and the molten mass run out into the moulds. The second melting makes of course very much better work. The moulds generally consist of a wooden flask or box, made in parts which are secured together by clamps, the parts generally being halves, top (cope) and bottom. This flask is filled all around the *inside* with a lining of a special dampened black or green (fascia) sand; into this sand the mould or pattern of the outside of the casting is pressed and, after ramming the sand, removed, leaving, of course, the impression of the outside of casting. The core is then secured into position in the flask. The core is the reverse of the mould being sometimes a sand pattern covered on the *outside* with a similar rammed sand made to fit the hollow spaces on the inside of the castings, and stiffened with an iron piece called the "arbor," but usually made of a baked mixture of sand and clay or flour, which forms a friable, infusible mass, which can be easily broken up and removed after the casting has cooled. The top flask is then secured over the whole, and the molten metal is run in through convenient holes left through the flask, until all the space between the core and mould is filled with metal. The gases generated escape through holes left in the cope, and from the core by putting iron rods along the arbor which are withdrawn before casting. The whole is then covered with sand and allowed to cool slowly if of varying thickness; if the casting is long and thin it is "stripped" of the moulding sand to prevent warping. The inside of the mould is frequently painted with coal dust or charred oak, or dusted with flour of plumbago, which generates hot gases that prevent the too quick chilling of the external surfaces of the casting by the dampened sand and makes a clearer surface finish. When cold the top flask is lifted off, the arbor withdrawn and the finished casting removed from the bottom flask. The casting is then chipped off and finished. Small and cheap castings, such as separators, are revolved in a rumble (a sort of barrel) for the finishing process. It will be seen, therefore,

Manufacture of Cast-iron.

Moulds, flask and core.

that the most economical castings will be those which are so designed that the pattern can be easily withdrawn and the flask readily made in simple parts; in which case it is only necessary to line the flask with a fresh layer of sand for each casting. Whenever possible, castings should be made in an upright position (the metal being, of course, run in from the top), as the long core, if horizontal, being unsupported except at its ends is apt to sag in the centre, or float with the metal, making uneven thicknesses in the casting above and below the core. To avoid this the core should be supported by small iron "chaplets," which mix and combine with the casting. There is also danger of slag and other impurities that may be carried into the mould floating to one side. In upright castings or where there are large vents they will float off on top.[1] To discover whether the casting is or is not uneven, the architect should have every column or long hollow casting "tapped" at about midway of its length. Tapping consists of drilling a hole, about three-eighths of an inch diameter through the shell. A small wire with the end bent at right angles is inserted in the hole; the bent part is drawn closely to the inside of shell and the outside of shell marked on the wire, which on being withdrawn gives the exact thickness of shell. In case of rectangular or round castings there should be four such holes bored, two opposite each other and the other two the same, but at right angles to the line of the first two. All of course in the same plane, and about midway of the length of casting.

Economical Castings.

Inspection by Tapping.

The amount of metal in the shell is then readily ascertained, also whether the casting is of even thickness.

For a good strong casting it is very essential that all parts should be designed (and cast) of even thickness. Castings of uneven thickness vary greatly in strength for two reasons. For some reason not thoroughly understood, though generally attributed to the influence of the gases produced in the flask, the outside layers or "skin" of castings is supposed to be very much stronger than any other part. The metal on the inside seems to diminish in strength the farther it lies from the outside. (Experi-

Even thickness best.

[1] On the muzzle end of cannons are cast heavy "sink heads," which contain the greater part of all impurities, leaving the good solid material for the guns proper. In heavy castings a flow or pathway is provided, more metal is poured than needed, and as it flows off it carries with it the dirt and impurities.

ments as to whether the skin does or does not add strength, however, are contradictory, the latest seeming to disprove the theory of its extra strength.)

Thin castings strongest.

It is found, however, by actual experiment that castings from the same metal, made at the same time, vary according to their thickness in strength, the thinnest being the strongest in proportion to their metal, and the thickest the weakest.

Another serious objection to uneven thickness in castings is the uneven cooling; that is, the thin parts cool before the thicker ones, and the consequence is the production of internal strains in the metal, the parts which have cooled rapidly, contracting and pulling away from the other parts. This means tension, and a constant tension right in the metal and before any external strains come upon it; this frequently is the cause of the sudden and otherwise inexplicable failure of the casting.

Danger of Sudden Cooling.

It is essential, therefore, that not only should cast-iron cool evenly, but also as slowly as possible, to allow the contracting parts to relax.[1] It is well-known that if a casting — column or lintel for instance — be heated in a fire and then suddenly be cooled by a stream of water played against it, that it is liable and likely to snap in two from the sudden tension put on the metal in rapid cooling. For this reason all such castings should be surrounded with non-conducting, fireproof materials wherever placed in inflammable buildings, or surrounded by inflammable goods.

Big castings bad.

Another essential in good castings is, not to have them too big and heavy; otherwise it is impossible to keep the entire mass of same fluidity; that is, to prevent parts from cooling before other parts have yet been filled with molten metal. The result of such castings is apt to be weak from internal strains, due to unequal casting and cooling, to be weak on account of the thickness of the metal, and to be spongy or filled with air bubbles, due to the gases escaping at one end and mixing with the molten metal running in at the other end. Sand-holes in castings are, of course, due to the use of a poor quality of sand or improper ramming or to the presence of dirt, scum or other impurities.

[1] When castings of unequal thicknesses are required, as for instance in fly-wheels and pulleys, the strains from unequal shrinkage are prevented by so removing the sand from the hot casting as to have all parts cool together. The thin parts or arms being kept covered until the heavier rim and hub have been hurried in cooling by uncovering them.

Avoid all sharp angles.

Another essential in good castings is to round all angles, that is, not to go abruptly from one surface to another at a right or sharp angle to the first: this, for the same reason that uneven thickness is avoided, namely, the danger of internal tensional strains.

In casting plates it is desirable to have a large circular hole, if possible, in the centre of the plate, or else several holes, which will largely prevent the great strain, in cooling, on the centre of the mass.

Shrinkage in Castings.

The contraction of cast-iron in cooling, from its bulk when fluid depends on its thickness and its mixture. It is about $\frac{1}{8}''$ per foot for thin castings, for castings $1\frac{1}{2}''$ thick about $\frac{1}{12}''$ per foot, and for castings $2''$ thick about $\frac{1}{16}''$ per foot. The moulds and patterns are, therefore, made correspondingly larger than required by the architect's drawings.

Steel-castings.

All that has been said of iron-castings applies equally to steel-castings. But as these are made at a temperature some 700° to 800° F. higher than for cast-iron, the contraction is, of course, much greater, and also the danger from unequal shrinkage. To avoid this, steel-castings are usually annealed, that is, left for some 24 hours in a furnace at 1700° F. — (the melting point being about 3500° F.) — and are then allowed to cool slowly.

Manufacture of Wrought-iron.

In the manufacture of rolled or wrought iron the pig-iron forms the basis, but is sometimes first refined, though in many of the grades the refining is omitted. This refining consists in the conversion of the carbon contained in the pig-iron from the graphitic to the combined state. This is done by breaking the pig-iron into convenient lengths for handling and melting the iron and adding to it different substances to attract and remove the earthy impurities. To reduce the carbon the most usual process is the forcing of strong currents of air through tuyères downwards onto the melted mass, which stirs it up and uniting with the carbon, forms a carbonic oxide gas and ignites and burns part of the carbon away. The melted mass is run out into shallow metal troughs, and cooled suddenly by streams of cold water. The object of this sudden cooling, is to make white iron, that is, to prevent the remaining carbon from separating itself and resuming the graphitic form.

The refined (or pig) iron is next puddled; this consists sometimes,

Refining and Puddling. but rarely, in remelting the iron, being the third melting, adding flux to attract the impurities, they and the flux being run off as iron slag; but usually in puddling the iron while it is still in pasty form, immediately after the removal of the carbon in the second melting or refining process. The pure iron forming a pasty mass is collected into separate lumps called puddle balls or loups, by the puddler, who stirs the mass around with his puddling bar, thus gradually forming the balls. When the puddle ball is of sufficient size, usually weighing from one to six hundred pounds, the puddler draws it out of the fire, and throws it on a wheelbarrow or truck; it is now run as quickly as possible to a steam hammer, where it is thoroughly pounded and thus welded into wrought-iron; or in more modern mills it is put into a rotary (Burden) squeezer which consists of a corrugated pinion and iron collar, situated one within the other, but eccentrically; the pinion revolving inside the collar, which is stationary, the puddle ball going in at the wide end, is kept constantly revolving by the corrugations, and at the same time is squeezed and drawn through and reduced between the pinion and collar, finally emerging at the other (narrow) end in cylindrical shape, and in nature partially rolled-iron, the slag and gases being squeezed out. From here it is taken while still heated, and several times run through between a pair of rollers which revolve in opposite directions under great pressure; these rollers have grooves along their surfaces, the largest at one end and diminishing gradually towards the other, thus gradually shaping the mass as desired into "muck bars," which are pieces of partially rolled-iron of convenient size and shape for handling, usually from 3 inches to 6 inches wide and $\frac{1}{2}$ inch to $\frac{3}{4}$ inch thick in section. The length of each bar, of course, depends upon the weight of puddle ball.

From puddle ball to muck bar.

When beams or other rolled-iron shapes are to be rolled the cold muck bars are sheared off to convenient lengths and sufficient of them tied together with wire into bundles to make the desired length of beam or other shape. This is called piling. Usually from five to nine muck bars are piled on top of each other in each bundle.

System of Piling.

The details of piling and size of muck bars, of course, vary in different shops.

The Phœnix Iron Company form their pile for beams over nine

inches deep very ingeniously, by rolling bars of different shapes, which are keyed and fitted into each other and are driven and held firmly together by wedges, as shown in Figure 160.

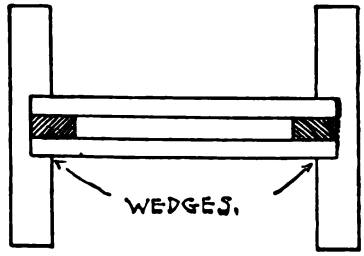


Fig. 160.

The writer believes that there are only a few mills using this method, which is to be regretted, as the advantage of having the grain or joints of the rolled-section longitudinal to each part is very great over the method where the seams run at right angles to the web, as is the case in the ordinary piling.

Rolling out the Piles.

These bundles or piles are put into the fire and heated to an almost white heat. They are then put on barrows or trucks and run as fast as possible to the rolls. These are arranged in pairs or in threes in a long line. The rollers are about eight feet long, the ends supported in cast-iron stanchions (called housings) of tremendous strength, the pressure against them being very great when the iron is forcing its way between two rollers. In "two-high mills" there are two rows of rollers over each other, in "three-high mills" there are three rows of rollers over each other.

Description of Rollers.

Each row of rollers revolves in opposite direction to the next row over, the result being that the iron is drawn in and squeezed between them. The roller has cut along its surface grooves which gradually shape the heated mass to its final section. The first set of teeth shape the iron roughly to the shape desired, the next pair come a little nearer to the shape, and so on till the last pair which give it the exact shape; for large sections it sometimes requires two sets of three-high rollers for each piece, but usually one set answers to roughen and finish the piece. Of course the length of the piece increases as its cross-section is diminished by the squeezing process of the rollers. Where the rollers are three-high, the heated bundle or pile is run through the first set of grooves in the lower pair of rollers in one direction; as it comes out on the other side, it is picked up by a pair of tongs (usually manipulated by hand, but by steam when large pieces are being rolled), and carried to the upper pair of rollers,

Process of Rolling.

where it runs back through their first pair of grooves but in the opposite direction. It is then carried to the second pair of grooves in the lower rollers where it runs through in the same direction as through the first, then back through the second pair of grooves in the upper rollers and so on to the end. Where the mill is only two high the rolling is all done in the same direction, the iron being run back over the upper roll, its friction on and the turning of the roll facilitating greatly the work of getting it back quickly. The operation of rolling continues during the one heating of the bundle or pile of refined iron, as, of course, the furnaces would not be large enough to re-heat long pieces. The rolling must, therefore, be very rapid, and no delay should take place. The pressure of the rolls is so great that if the mass meets with any obstacle, instead of stopping and wedging in the roll the balance will run on out and curl the piece all up, sometimes even pushing the end up through the roof. When the iron has made its final passage through the rollers and is properly shaped in section, it usually is all curled up lengthways in both directions. It is still at a cherry red heat and is, therefore (after having the defective parts at the ends cut off by a circular saw), either run onto or quickly carried by a gang of men and placed lengthwise across a long series of skids or rollers. At the other ends of these skids are iron upright stops. A gang of men armed with heavy wooden mauls or mallets belabor the flat side of the section its entire length till it touches the skids at every point. They now move it over against the stops and belabor the short edge or flat side of flange, till the flat side of the other flange touches the stops at every point. This, of course, straightens the beam or other section lengthwise in both directions. It is now allowed to cool and is ready to sell for use. Plate-iron is straightened as above. Beams and channels are straightened cold by being run through a gag press. Angles are straightened cold by running through straightening rolls.

Straightening the Pieces.

Single Refined-iron.

Iron rolled as above is the ordinary rolled-iron of commerce and is known as merchantable or "single refined-iron."

For better qualities the muck bars are heated and rolled into plates of single refined-iron. When forming the bundle or pile the top and bottom layers consist of the single refined plates, the intermediate layers being muck bars. Or the top, bottom and centre

layers are of single refined-iron plates, the other layers being muck bars. This is generally known as "extra refined-iron" and a small advance is charged for it over single refined-iron.

Extra Refined-iron.

For the best work *all* the layers in the pile or bundle are made of single refined plates, no muck bars at all being used. This is known as "double refined-iron" and a considerable extra price is charged for it over the cost of single refined-iron.

Double Refined-iron.

For very important tension-members in trusses or arches, or for important hangers it is very desirable to use the double refined-iron. But, as a rule, the best results will be obtained by being satisfied with whatever kind of iron the mill is accustomed to roll. It stands to reason that a mill will not upset its regular shop practice and line of work for a single order unless well paid for it, so that, as a rule, it is more speedy and more economical to take their regular iron and use a larger size of cross-section. Then, too, where double refined-iron is ordered, the most constant supervision will be needed to get it. The men will probably prefer to relapse into their regular ways even if the principals do not. Then, too, in a report to the Government of a large series of experiments made at many mills and tested at the Watertown Arsenal, it was found that the result of the double refining was harmful at times, and that its value depended entirely upon the nature of the muck bar.

Where the muck bar was of a decidedly poor quality, showed air holes, blisters, and a general crystalline appearance on fracture, or spluttered greatly while being run through the rotary squeezer or the steam-hammer, denoting the presence of much slag and impurities, that is, when the quality of the muck bar resembled that of a dirty and poor cast-iron, the double refining was of great service in adding strength to the material. Where, however, the muck bar was of a good quality, the puddle ball spluttering but little in the rotary squeezer or in the steam-hammer, and the muck bar itself showing on fracture a goodly part of fibrous section and the balance with but little impurities, the double refining was found to positively injure and weaken the iron, to dry it up, as it were, leaving the successive layers distinctly visible, when the iron is tested by eating away its edge with muriatic acid, or in circular pieces directly showing and opening the successive layers on fracture. This in the case of double refining of poor-quality muck bars was not nearly so marked.

Result of Several Rollings.

It concludes, therefore, that some impurities help to amalgamate the final layers, and that where the muck bar is already of a partly wrought-iron nature — partly fibrous on fracture — that double refining works an injury. For important work, therefore, there should be a constant mill supervision by an inspector stationed there for that purpose, and the question of double refining or not should be left for decision to tests made on the premises.

Extra refined-iron, however, can safely be used at all times, the only drawback being its additional cost, and the doubts whether it will be furnished as specified unless the work is closely watched and inspected.

Use of Old Iron. Instead of using muck bars to make up the bundles for the final heating, the mills sometimes use the cut off and otherwise waste ends of beams and other rolled-iron; there is no objection to this if the iron is not too dry, but, as a rule, it is a very good practice, as the iron thus gets an extra rolling or double refining. Nor is there any other objection than above to the use of old rails, beams, or scrap iron, provided always that all such material is free from all rust and is thoroughly cleaned; where mixed with muck bars, and thoroughly cleaned, and perfectly welded, it is greatly to be recommended.

The frequent rolling of English iron within a reasonable limit, according to Sir W. Fairbairn, improves its quality and strength; if, however, it is rolled too often, re-heated more than five times, he says it quickly loses its strength.

Effect of Rolling. Pure iron, that is, iron [*practically*] free from carbon, consists when heated to a white heat of a mass of crystals of cubical form. The effect of rolling is to elongate these cubical crystals into long silky fibres which interlace and bind themselves together and form the mass of all good wrought-iron. A repeated sudden shock or blow, when cold, or continuous heavy vibrations, tends to restore the fibres to their original form, that is, crystallize them, the iron becoming very brittle and weak. It is for this reason, in testing wrought-iron, that if it shows crystalline specks on fracture, it is considered to be of a poor quality, provided of course the fracture has not been made by sudden blows; if, however, the broken section is entirely fibrous the nature of the wrought-iron is correct and considered good. If wrought-iron contains sulphur it is "red short," or cold short and difficult to roll.

Wrought-iron is usually sold by the pound, and each rolled section is known by its weight per yard. The reason of this is that the weight in pounds of a section of wrought-iron one yard long is exactly ten times its area of cross-section in square inches. One-tenth of the weight per yard in pounds is then the area of cross-section in square inches. Steel is weighed up the same as iron and then two per cent added to what would be the weight of iron.

Why weight per yard.

In the above the writer has purposely used the words "wrought" and "rolled" iron interchangeably. They are really the same thing. Their nature is supposed to be exactly identical. If any distinction can be drawn, it would be that in rolled-iron the elongated fibres are formed by rolling, while in wrought-iron they are formed by hammering, either by hand or machinery.

Nature of Wrought and Rolled Iron Similar.

Where a large number of pieces of equal sections are needed, rolling is to be preferred; but for small orders (unless the order is for some regular mill section), hammering by hand, or by steam hammer and die, is generally preferable; these latter are frequently also known as forged-iron. Welding is the process of heating two separate pieces of iron and uniting them together, while hot, by hammering or forging, the fibres of the two parts interlacing and forming a joint nearly as strong as the material. Any iron that can be forged, rolled or welded is said to be malleable.

There is a quality of iron known as malleable cast-iron, which combines, to a certain extent, the qualities of both wrought and cast iron, being cast-iron of a semi-wrought-iron character, which can be forged. The forging, however, has to be done *cold*, not hot, as with wrought-iron. The process is described as embedding cast-iron in powdered hæmatite ore or in scales of oxide of iron, or other oxides, and raising it to a bright red heat in an annealing oven. Pieces of cast-iron not over one-half inch thick can be made entirely malleable by this process; with thicker pieces, however, it is only practicable to render the outside skin malleable to a depth of about one-quarter inch, the depth, of course, depending on the length of treatment, the interior retaining its unaltered cast-iron character.

Malleable Cast-iron.

The manufacture of steel is still in its early development and as a result great uncertainty attends the results obtained,

Manufacture of Steel.

unless proper tests are made at every stage. Nor have the different methods of manufacture been tested sufficiently long to definitely establish the superiority of either.

It is partly for this reason, and also on account of the many different kinds of steel produced — dependent on the varying amount of carbon it contains — that there are so many radically different processes of manufacture.

As already remarked, steel in its chemical composition is the intermediate between wrought and cast iron — wrought-iron being theoretically free from carbon, while cast-iron has ordinarily a good deal more carbon than steel and *holds it in a different form*. Steel with but little carbon is called "mild steel," with much carbon "hard steel," the former being the more reliable of the two for structural work. It should be noted, however, that whereas in the earlier stages of the manufacture of steel its chemical synthesis was made the criterion, yet to-day the oft-mooted question as to whether a certain product is iron or steel is more often determined by reference to its physical characteristics than to its analysis. The physical characteristic of steel is that it may be *cast* into wholly malleable ingots, that is, the *cast* ingot or lump can be rolled or forged in its heated state without any intermediate process. This is not true as regards cast or pig iron which is entirely non-malleable, while, on the other hand, wrought-iron cannot be cast, not being fusible at temperatures obtaining in the arts.

Characteristic of Steel.

The great importance and future of steel lies in the above property as also in its large tensile strength, its hardness, and its further advantages of being capable of "hardening" and of taking a "temper." It combines within one metal the ductility and weldability of wrought-iron with the hardness and fusibility of cast-iron, and this with a tensile strength greater than that of wrought-iron.

Hardening consists in raising the steel to a red heat and suddenly cooling it in water; this greatly hardens steel but does not affect wrought-iron, while cast-iron would fly to pieces under such treatment. Hardened steel can be softened by tempering, this is done by re-heating the hardened steel, but to a *lower* temperature than before, and again cooling it, but slowly, when it will be tempered or softened.

Hardening Steel.

Steel retains magnetism greatly. When struck, steel rings out a sharper metallic sound than iron, but in physical appearance it often

resembles wrought-iron to such a degree that it is difficult to distinguish between them by merely looking at their fractured parts. Steel is malleable at a lower heat than wrought-iron, and for this, (and other reasons, but principally on account of the danger of "burning" it,) — it is usually rolled at a much lower temperature than wrought-iron; being, as a result, much more solid, that is, further from the melting point, than is iron when passing through the rolls, it requires very much greater power and heavier machinery than the latter.

Steel rolled colder than Iron.

The production of steel direct from the ore is perfectly feasible, but has not yet attained to any relative commercial importance.

Direct Process.

All the different processes of manufacturing steel really narrow down to the two principal methods of either the addition of a certain percentage of carbon to pure or malleable iron, or the removal of part of the carbon from pig-iron. To give all the variations practised in these two principal methods would be impossible here, but under the first method we can cite the cementation process or the manufacture of the commercial "cast" steel; as an intermediate process the "Open Hearth" processes; and under the second method the well-known "Bessemer" process.

Two Principal Methods.

Of these, in its main features the Cementation process is the oldest. Briefly, bars of wrought-iron are packed in charcoal in air-tight retorts or "pots" and subjected to a white heat for over a week's time. The wrought-iron absorbs the carbon or alloys with it, showing a blistered appearance — whence the name of "blister" steel — and becomes readily fusible.

Cast-steel.

These blister steel bars when piled and rolled once constitute the "single" shear steel of commerce, or if re-piled and re-rolled the "double" shear steel. If, however, they are broken up and melted in a crucible and cast into an ingot which is re-rolled, the quality and uniformity of the product are much improved and it is known as "cast" steel; the principal use of which is for cutlery and edge tools. Cast-steel is the hardest, strongest and densest steel made, but when raised beyond a red heat becomes so brittle that it cannot be forged. It cannot be welded and flies to pieces under a powerful blow.

Open Hearth Process.

The *Open Hearth* process may be justly regarded as the intermediate process because it is either the carbonization of or adding

of carbon to wrought-iron, or the decarbonization of or removal of carbon from pig-iron, both reactions usually existing in one and the same heat, as in the best known *Siemens-Martin* process, which is a typical and probably one of the best open-hearth methods to-day.

It consists of melting a combination of irons in a basin-shaped hearth under a reverberatory roof by playing on them what is really a blowpipe with a flame oxidizing (removing carbon) from the mass; or reducing it (adding carbon) at will (but usually oxidizing). The combination consists of pig-iron and wrought scrap-iron or old steel rails, or else of pig-iron and iron ore, or sometimes a combination of all the above; when melted down, the one with or in the other, the mass is stirred about to render the mixture homogeneous, and after preliminary testing of samples, is duly poured into ingot moulds. Thus the cast-iron gives up its excess of carbon to the rest of the charge having less or none, and the resultant product, which can be controlled with a nicety that is surprising, is a steel of high grade, with carbon between limits that are well governed from the start to the close of the conversion. The ingots are allowed to cool and are then sold to mills not owning steel furnaces, who, of course, have to re-heat the ingots before rolling them. Where a mill manufactures its own ingots, however, the hot ingot is run directly to the blooming mill and rolled into "blooms" while still hot.

Condition known by tests.

In the pneumatic or Bessemer process the carbon is entirely removed from the pig-iron by oxidation and then the necessary amount of carbon to produce the desired quality of steel is afterwards re-introduced. This seemingly roundabout method is adopted, because it would be impossible otherwise to determine promptly or exactly enough when the decarbonization should cease, the entire removal of all the carbon taking only some twenty minutes. It is easy to determine, however, when the carbon, silicon, etc., are completely oxidized, and then to introduce carbon in a known proportion which is done by adding "spiegeleisen" as hereafter mentioned. It would not be so easy, however, otherwise to know at just what point sufficient carbon was removed.

Bessemer Process.

The above operations are conducted in an egg-shaped vessel open on top, known as the "converter," hung like a cannon on "trunnions," into which melted pig-iron is poured from a cupola furnace,

blast or wind furnace. This converter has a perforated lining in the bottom through which a forced vertical blast, at a pressure of 20 to 25 pounds per square inch, enters and bubbles up through the molten metal. The action of this blast is distinctly oxidizing, the oxygen in the air seeming to first combine with the silicon in the pig, forming silica or quartz, which rises as slag to the top. The manganese and carbon in the pig are next consumed, the carbon passing off in gaseous combination with the oxygen and being thus eliminated. By the changing appearance of the "flame," which at first is of a bright yellow appearance but very turbulent and full of flying sparks, then settles to a more steady and clearer flame of pink or amethyst color, and finally settles down and disappears almost completely from the mouth of the converter, the various stages and final total elimination of the carbon are known, and at the end the blast is shut off. The metal in the converter is at this moment approximately pure iron, slightly oxidized or "burnt," and is in a spongy friable mass and non-malleable. Molten "spiegeleisen" is then poured in and its addition is accompanied by a violent boiling reaction, accompanied by a reappearance of the flame at the mouth of the converter. This Spiegeleisen is simply a German pig-iron very high in carbon and manganese of known proportions, and is so-called from its mirror-like, shiny, crystallized formation.

Condition known by flame.

The object of the carbon in this addition is to turn the pure iron to steel; the object of the manganese is to take up any excess of oxygen in the melted iron by forming oxides of manganese, thus preventing the steel from being what is known as "red short" or burned.

This mixture of decarbonized iron and spiegel is allowed to rest in the converter for a short time or is sometimes mechanically stirred up and then, after pouring off the slag, the contents of the converter are tipped into a ladle swinging radially on a crane-arm over a semi-circular row of ingot moulds into which the melted steel is poured from an opening in the bottom of the ladle controlled by a plug. These ingots weigh 4 to 5 tons each: they are allowed to cool as they are in the mould, or else in a "soaking pit," which is simply a device for allowing them to cool slowly on the annealing principle; this slower cooling improves the quality of the product by more thoroughly eliminating the included gases and avoid-

Casting of Ingots.

ing the internal strains due to otherwise too-rapid cooling. But, as already remarked, where the blooming mill is close at hand, the ingots are not allowed to cool, but are run hot to the rollers.

The lining of the converter is usually "ganister" (silicious sand) which is of an acid (silicle) reaction and gives the name of "Acid process" to that usually adopted in this country. In England and on the Continent and presently in Alabama, where ores run too high in phosphorous and sulphur to permit of their successful use in the Bessemer process, the acid lining is sometimes replaced with a basic or alkaline lining made from dolomite, etc. The lime and magnesia in this lining unite with the sulphur and phosphorous in the pig forming a fusible slag, thus rendering their elimination possible. This expense is not required, however, with most pig made from American ores as may be inferred from the fact that it does not represent the ordinary practice in this country.

Clapp-Griffiths Process.

It is not necessary within these limits to explain the "Clapp-Griffiths" process, about the merits of which so much controversy has been had lately, further than to state that it is an intended modification of the Bessemer process, by which it is hoped to modify and cheapen it to the point of rendering it adaptable to plants of small capacity, and also to secure a better control over the quality and *homogeneity* of the product desired.

Taking all things together, however, the superiority to-day for structural iron seems to lie with steel metal made under the Open Hearth process, (first introduced into this country by the New Jersey Steel and Iron Co.), because the results desired are under more perfect and intimate control. Ample opportunity is afforded at various stages of the process to test and duly correct the quality of the product, and a greater *uniformity* of the product itself seems to be obtained than with the other processes. For railway bars, Bessemer steel has at present largely the greater production being also cheaper and is undoubtedly well enough adapted to it, but for structural purposes, such as ships, bridges, roofs and girders, the best opinion to-day seems to incline to the preferment of the Open Hearth product on account of its superior uniformity.

But, as already emphasized several times, very little is really known to-day of the properties of steel and new facts are coming to light every day. Its manufacture and determination are

undoubtedly still in their development, and though events point strongly to steel becoming the metal of the near future, there exists among many reasonably conservative men, a wide and well-grounded distrust of its use in the higher engineering or architectural structures, on account of its mysterious behavior, and frequent erratic and inexplicable failures. It should never be used, except after the most rigid and frequent tests, which of course add greatly to the expense attendant on its use.

Use of Steel not yet commendable.

Except that being cast in large masses requiring very heavy preliminary "breaking-down" or "blooming" mills to adapt it to the finishing strains, and that usually it is not piled, its subsequent manipulation to produce the ordinary structural forms is very similar to that already described for wrought-iron.

The ingot, while hot, is run to the blooming-mill, and there run through rollers to form the "blooms," which are each of the necessary weight in one piece, to make a whole beam or other shape. The cross-section of the bloom is about 3 inches by 9 inches, the length depending on the weight and length of beam or other shape desired, usually from 5 feet to 30 feet, the latter weighing a little over a ton each. These blooms occupy in the rolling of steel the corresponding place to the piled muck-bar or other iron in the rolling of iron.

Rolling of Steel Bloom.

The bloom is heated in a furnace and run through rollers, which gradually shape it to its final section, the same as with wrought-iron; the only difference being that the bloom is not heated to the same high temperature as iron, and, therefore, its resistance to squeezing when passing through the rolls is very much greater than with iron. Steel plants are, therefore, not only very expensive, but the amount of breakage in housings and rolls is very great.

The Phœnix Co. are preparing to make steel stock of similar size as from the iron muck-bars, and to pile them similarly. This will probably be done in time by all the mills, and may lead to a considerable saving in the expense of replacing broken machinery.

The writer understands that at present the Bessemer process is employed at the Homestead Works in Pittsburgh (Carnegie, Phipps & Co.); and by the Columbia Iron and Steel Co., of Uniontown, Pa.; and by the North Chicago Rolling-Mill Co., of Chicago; and that ingots made by the Open Hearth pro-

Processes used by Mills.

cess are bought and used by the Passaic Rolling Mills, of Paterson, N. J., and by the New Jersey Steel and Iron Works, of Trenton, N. J.; and that the latter also buy and use ingots made by the Bessemer process.

That the Clapp-Griffiths (modified Bessemer) process is used by the Pottsville Iron & Steel Co., of Pottsville, Pa.

The Siemens-Martin (Open Hearth) process is used by the Phœnix Iron Co., at Phœnixville, Pa., who roll their ingots; and that the works now building for the Pencoyd Iron Works at Pencoyd, Pa., are also for the Open Hearth process, but that they intend to hammer their ingots.

These eight mills embrace all the leading mills in the United States where structural iron and steel of I-beam, channel, angle, deck-beam or tee-shapes are rolled.

A very cheap and inferior grade of steel, known as "puddled steel," is made by stopping the puddling of pig-iron before all the carbon is removed.

Puddled Steel.

Many attempts have been made to establish exact lines of demarcation between wrought-iron, steel and cast-iron, but none are very satisfactory, though a few are here given.

Classification of Irons and Steel.

Dr. Percy says if the iron be pure, contains *no* carbon, it is wrought-iron (though practically wrought-iron contains some 0,25 per cent of carbon). In this condition the iron is soft, tenacious, weldable, but not fusible.

If the iron contains *some* carbon, from 0,15 per cent to 1,8 per cent, it is steel, the different kinds of steel varying according to the quantity of carbon; the different steels being accordingly more or less elastic, malleable, forgeable, fusible and capable of hardening and tempering.

If the iron contains *much* carbon, from 2 per cent to 6 per cent, it is cast-iron and is hard, brittle and fusible, but it is neither weldable nor forgeable.

Table XXVIII (on page 26), does not, however, agree with what is called steel in American practice. Steel with 0,12 per cent of carbon is considered "mild steel," and with 0,36 per cent of carbon "hard steel." The former is used for structural purposes. Steel with more than 0,12 per cent of carbon will not stand the tests for the Government cruisers.

The following (Table XXVIII) appeared originally in "*Bauermann's Metallurgy.*"[1]

TABLE XXVIII.

CLASSIFICATION OF IRONS AND STEELS.

Name.	Percentage of Carbon.	Properties.
1. Malleable iron.	0,25	Is not sensibly hardened by sudden cooling.
2. Steely iron.	0,35	Can be slightly hardened by quenching.
3. Steel.	0,50	Gives sparks with a flint when hardened.
4. Steel.	1,00 to 1,50	Limits for steel of maximum hardness and tenacity.
5. Steel.	1,75	Superior limit of welding steel.
6. Steel.	1,80	Very hard cast steel, forging with great difficulty.
7. Steel.	1,90	Not malleable hot.
8. Cast-iron.	2,00	Lower limits of cast-iron, cannot be hammered.
9. Cast-iron.	6,00	Highest carburetted compound obtainable.

Classification of Structural Iron and Steel.

In Table XXIX the writer has attempted to classify systematically the various kinds of iron and steel used in buildings. The table has been largely based on a similar table prepared for the *Stevens Indicator* by Mr. William Kent.

Manganese is frequently used instead of carbon to harden steel, it hardens it considerably (though not so much so as carbon) and does not reduce its ductility as much as carbon.

Chromium and tungsten are added to and produce very hard but yet ductile steels, but add considerably to the expense. Chrome steel being used largely for fine tools and vault-works. While cast-iron can be melted at 2732° F. and cast-steel at 3500° F., wrought-iron is practically not fusible. By adding, however, some ½ to 1 per cent of aluminium to wrought-iron its point of fusion is reduced some 300° to 400° F. and it becomes practicable to melt it. This is done in crucibles of fire-clay or plumbago and in petro-

Mitis.

[1] Copied from "*Notes on Building Construction.*"

leum furnaces. The product is a comparatively new metal and known as *mitis*. Its resistance to tension is about one-fifth greater than that of wrought-iron, while its ductility is about the same.

Resume of Qualities.

In "*Notes on Building Construction*" is given such a very able short résumé of the difference between cast and wrought iron and steel, that it will be given here with but slight modifications.

"The great differences in the characteristics of cast-iron, and wrought-iron and steel, are briefly recapitulated below, and these determine the uses to which they are respectively applied.

Cast-iron.

"Cast-iron has little tensile strength, but affords great resistance to compression.

"It is hard, brittle, wanting in toughness and elasticity, and gives way without warning, especially under sudden shocks or changes of temperature. It is easily melted and run into various shapes.

"The castings thus produced are liable to air-holes and other flaws, which reduce their strength. Small castings are stronger in proportion to their size than large ones." Castings should be of even thickness throughout and all sharp angles should be avoided.

"Cast-iron can be cut or turned with edge tools, but is not malleable either when cold or hot, nor is it weldable.

"It is not so easily oxidized in moist air as wrought-iron. In salt water, however, it is gradually softened and converted into plumbago.[1]

"Cast-iron is peculiarly adapted for columns, bedding-plates, struts, chairs, shoes, heads, and all parts of a structure which have to bear none but steady compressive strains; also for gutters, water-pipes, railings, grate-fronts, and ornamental work of nearly every description.

"It has been much employed for girders, but is an untrustworthy material for those of large size, or in important positions. It is liable to crack and give way without warning under sudden shocks, and also under extreme changes of temperature, such as occur in the case of buildings on fire, where the girders may become highly heated, and then suddenly cooled by water being poured on them." For lintels of short span it is used on account of the ease of adapting it to any condition. The main objection to the use of cast-iron is its brittle and unreliable nature, danger of flaws, and danger of snapping in case of fire and sudden cooling by a stream of water.

[1] Or dissolved.

"Malleable cast-iron possesses originally the fusibility of cast-iron, and eventually acquires some of the strength and toughness of wrought-iron. It may be used for heads, shoes and other joints in roofs, and for all articles in which intricacy of form has to be combined with a certain amount of toughness.

Malleable Cast-iron.

"Wrought-iron has many most valuable qualities, though these differ considerably as to degree in different varieties of the material.

Wrought-iron.

"Its tensile strength is three or four times as great as that of cast-iron, but it offers not half the resistance to compression.

"It is, however, very tough and ductile, and therefore gives way gradually instead of suddenly snapping.

"Its elastic limit is equal to about half its ultimate strength, and it will bear repeated loads below that limit without injury.

"Wrought-iron is practically infusible, is malleable hot and cold, is weldable at high temperatures, and can be forged into various shapes.

"It is subject to 'hot and cold shortness' produced by impurities, and to other defects. Large sections are more likely to contain flaws than small ones. Bars are, as a rule, stronger than plates, and plates are stronger with the grain than across it.

"Malleable iron rusts quickly in moist air, but stands salt water better than cast-iron.

"The great tensile strength of wrought-iron leads to its employment for tie-rods, bolts, straps, and all members of any structure which are exposed to tensile stress; it is also much used for members which undergo compression," particularly if very long compared to their diameters, or if exposed to fire. "It should be employed for all important iron beams and girders, especially those exposed to sudden shocks. In its various forms it comes into play in a variety of ways in roofs, braced girders, and iron structures of all kinds. Corrugated sheets are much used for roof coverings.

"Steel differs even more than wrought iron in the characteristics of its several varieties.

Hard and Soft Steel.

"It has a high tensile strength, much greater than that of wrought-iron. Its resistance to compression is also much greater. Moreover, it has a harder surface, and is better able to resist wear and tear.

"Hard steels, containing a large proportion of carbon, are fusible,

easily tempered, have a high tenacity and elastic limit. Their resistance to compression is enormous, especially when they are tempered, but they cannot be easily welded or forged, are brittle, and very uncertain in quality.

"Soft mild steels have a tenacity and resistance to compression, and an elastic limit somewhat" (proportionately much) "higher than wrought-iron. They can be hardened and tempered, but not easily. They are weldable and easily forged, and afford a very (?) reliable and ductile material adapted for structures subject to sudden shocks.

"Steel is more easily oxidized than wrought-iron, and far more easily than cast-iron.

"Steel is at present hardly used at all by the builder. Sometimes bolts," pins, "and cotters are made of steel for large roofs.

"It is not adopted for engineering structures to anything like the fullest extent of which it is capable, but is required by the engineer for tools, rails, boilers, machinery, wheels, etc., and is coming into use for some of the larger roofs and bridges."

Forging and Welding.

The process of welding different pieces of iron together is a very important one and as it occurs frequently should be well understood and carefully watched. All iron to be forged to any shape should be thoroughly heated to an almost white heat, but not too much for fear of burning it; the heating should be as uniform as possible and the iron should be worked as quickly as possible. The heat destroys the fibrous nature of the iron, and crystallizes it, but the fibres are again restored by hammering. Care must therefore be taken to thoroughly hammer all parts. All angles should be avoided in forgings. Welding is the same process as forging, except that two separate parts are joined together. The metal in each is heated and the two are thoroughly hammered together. It is very easy to weld two pieces of wrought-iron together, as the iron becomes sticky at a comparatively low temperature. Pieces of steel can be welded together if both are low in carbon. Those high in carbon cannot be welded. Iron and steel can be welded together if the steel does not contain too much carbon. Where the amount of carbon contained in the two pieces to be welded varies, the one with least carbon should be heated to a greater temperature than the other. With cast-iron no welding is possible. Welded joints in tension are considerably weaker than the original metal.

Welded Steel and Iron.

Sometimes layers of steel and iron are piled together, heated and rolled out into one mass, called three-ply steel and iron, five-ply, etc., according to the number of plates in the pile. These are used for safes and vault-work; after being rolled the steel is tempered or softened, the iron not being affected by the process. The plates are now fitted, drilled, etc., and then are hardened, when it becomes impossible to drill them on account of the extreme hardness of the steel layers, nor can they be broken by sledge hammers or hydraulic rams, on account of the elasticity of the iron layers, which are not affected by the hardening process.

Hardening and Tempering Steel.

In hardening steel the colder the water and the more suddenly the heated metal is plunged into it, the harder will be the quality. By using oil for the cooling plunge instead of water the steel is less likely to crack or warp during the process. To remove the brittleness from hardened steel it is tempered as already explained, by reheating it to a much lower temperature, however, than before, and again plunging it in a water bath to cool or allowing it to cool slowly. Hardening steel adds greatly to its tensile strength and increases the limit of elasticity, but on the other hand greatly reduces its ductility.

Softening or Annealing Steel.

To soften very hard steel it is annealed, that is, heated to a red heat and then allowed to cool very slowly in an annealing oven. The latter is a brick air-tight chamber in which several pieces of heated steel are placed to cool. The process of cooling is, of course, very slow, as the heat is retained by the brick walls and the radiations of heat from the different pieces greatly retard each other in cooling. In this way steel can be made to regain its original softness.

Case Hardening.

"Case hardening" is a process by which wrought-iron retains its tough and elastic nature but changes only its external skin to a depth of from $\frac{1}{16}$" to $\frac{3}{8}$" into hard steel so as to better resist wear and tear from external causes. This is accomplished by placing the iron in a box with bone dust and heating it to a red heat; it is kept thus for from half an hour to eight hours, according to the depth of skin desired and then quenched in cold water.

If the whole is to remain malleable it is not quenched in water but allowed to cool very slowly which leaves a skin of soft or mild steel. This is worked as desired and on completion the whole is re-heated

and quenched suddenly, which hardens the steel skin, but does not affect the iron interior.

Protection from Rust. The protection of iron after erection is a very important subject to the architect. If iron is exposed to the air, particularly alternating damp and dry air, or to wet mortar, it will surely rust unless protected. Rust means the gradual scaling, corrosion, and crumbling of the whole. Cast-iron is the least liable to rust; wrought-iron, however, is very liable to it, and steel even more so. In cast-iron the skin formed during the cooling of the metal by fusion with the mould sand and absorption of gases, is not only supposed to give great strength to the metal, but offers its best protection against the weather. When this is removed or damaged, the iron is not only supposed to be greatly weakened in strength but it is certainly very much more liable to rust.

Rust is like a disease, and must be entirely cleaned off the metal before covering it, or it will continue to eat its way in. Where sand-paper is not sufficient for its removal, a bath of diluted sulphuric, muriatic, nitric or other strong acid should be resorted to.

All parts which are planed and are to make exact joints, and cannot, therefore, be painted, should be kept carefully covered at all times until erection with a heavy coating of lard and white lead mixed.

Metallic Paints. All iron and steel pieces should be so designed that all parts are readily accessible for painting. For this purpose it is usual to use any of the so-called metallic paints, which are made mainly from earths rich in iron ore (red oxide of iron). A heavy coat of this should be put on immediately after finishing, first removing all loose scales. In practice, however, this is rarely done, as it is found cheaper by the iron contractor to do his own painting, rather than to pay the mill price. The architect should insist, however, that every speck of rust and all loose scales be cleaned off before any painting be done, and should not be talked out of it by the so-called practical man (who always has plenty of "pocket judgment").

In place of metallic paints, a mixture of red lead and linseed oil makes an excellent protector, but is rather more expensive than most metallic paints, as it does not cover so much surface as the latter. Some authorities claim that lead paints set up galvanic action with the iron, and so injure it. On the other hand, if the

metallic paint is made from a protoxide of iron, instead of red oxide, it is said it will rust within itself.

Finishing Paints. There should never be less than two coats of metallic or red-lead paint on iron before attempting to finish same in colors, bronzing or gilding. It will take at least two additional coats of white lead and oil paint in colors to hide the color of the metallic paint. These two additional coats should also be put on under and before bronzing or gilding. Bronzing is done on interior work only, and is done by painting with a mixture of bronze powder and varnish. In gilding, the paint is covered with a coat of the usual oil gold-size, and then the hammered gold-leaf is put on.

Galvanizing Process. Iron is frequently galvanized, which gives it an effectual protecting skin so long as this remains intact, and is not cracked or broken by bending or blows. This skin consists of a thin coating of zinc. The iron is first cleaned by being soaked in a weak solution of sulphuric acid, is then sand-papered and washed. After this it is dipped, while hot, into a bath of chloride of zinc and then "plunged into molten zinc, the surface of which is protected by a layer of sal ammoniac."

Tarring Process. In many cases, particularly with cast-iron pipes, the iron is tarred. This is a very good protection for the iron, but is apt to cover up and hide defects, such as sand-holes, etc.; for this reason the use of tarred pipes is sometimes prohibited, notably by the New York City Board of Health. The iron is heated to 700° F., and dipped into a mixture of coal-tar, pitch and five per cent to six per cent of linseed oil, heated to 300° F. The iron is left in it till it acquires the temperature of the mixture, when it is removed. In practice, the iron is usually dipped into the mixture without the preparatory heating.

Japanning Process. Sometimes iron, notably registers and hardware, is japanned. This consists in painting the iron with a mixture of lead-paint, oil and copal varnish, with successive coats of copal varnish, all dried at a very high temperature.

Electro-plating Process. Sometimes iron is protected and beautified by electro-plating. This consists of depositing on the iron in an electric-bath successive, but very thin, layers of brass, copper, bronze, etc. These effects are very beautiful, but expensive.

The Bower-Barff process turns the outside skin of the iron into a magnetic oxide of iron, the color being of a very beautiful dark

blue-black, and susceptible of a high polish. The iron is thoroughly cleaned and put into an air-tight chamber, and kept at a very high temperature. Superheated steam, or air heated to a very high temperature, sometimes as high as 1600° F., is then passed over the iron for from five to seven hours.

Bower-Barff Process.

The effectiveness of this coating to resist rust depends upon the length of the exposure in the oven and the height of the temperature.

A strong cement for filling poor joints between ironwork under compression is made of sal ammoniac, iron filings and sulphur. The more iron used in proportion to the other ingredients, the slower will the cement set.

Iron Cement.

It is frequently attempted to get extra-strong irons or steels by re-working them. In cast-iron, as a rule, re-melting, within reasonable limits, increases the strength of the iron. Box gives one case where the second melting added some forty per cent to the original tensile strength of the pig-iron; the third melting added some fifty per cent to this, or more than doubled the original strength; and the fourth melting added another twenty per cent to this, or the metal became about two and one-half times its original strength (tensile). On the other hand, Mr. Fairbairn found that the transverse, tensile and compressive strength of cast-iron was reduced gradually to the third or fourth melting, then increased with each melting till they reached their maximum at about the twelfth melting, and after that again decreased very rapidly. Accordingly, if the strength of the pig-iron were = 1,0 the third melting (second after pig) would give a minimum transverse strength of 0,82 and a minimum tensional strength of 0,77; while the maximum transverse strength 1,41 and the maximum tensile strength 1,32 would be reached at the twelfth melting.

Re-melting Cast-iron.

With compression, the minimum strength 0,92 would be reached at the fourth melting, and the maximum 2,18 at the fourteenth re-melting. According to Gauthier's analysis, re-melting turns gray irons to white, decreases the graphitic carbon and the silicon, and increases the combined carbon, thus rendering the iron weaker in resisting tensile or transverse strains, but stronger in resisting compression.

However, the whole subject is very uncertain, the increase or decrease of strength by re-meltings depending evidently on the par-

ticular mixture of iron used. A test in each case would be the only reliable criterion. It must be remembered, too, that re-meltings mean much additional expense.

If cast-iron be kept melted in the furnace for a long time — from one to three hours — it can be greatly increased in tensile strength, some experiments showing nearly doubled strength.

Cold-blast iron seems to be stronger than hot-blast (in tension), varying with the temperature of the test piece, the cold-blast iron being nearly twenty-five per cent stronger than iron made with a blast at 250° F.

Effect of Thickness.

Mr. Thomas Box has collected all the data obtainable as to the effect of thickness on cast-iron, and from them is obtained the formula:

$$t_1 = \frac{t}{\sqrt[2.3]{b}} \qquad (100)$$

Where b = the least thickness of a casting, in inches.

Where t = the ultimate tensional stress, per square inch, of a similar casting, whose least thickness is one inch.

Where t_1 = the ultimate tensional stress, per square inch, of a casting, whose least thickness is b inches,

and

$$k_1 = \frac{k}{\sqrt[2.3]{k}} \qquad (101)$$

Where b = the least thickness of a casting, in inches.

Where k = the ultimate modulus of rupture, per square inch, of a casting whose least thickness is one inch.

Where k_1 = the ultimate modulus of rupture, per square inch, of a casting whose least thickness is b inches.

Proportionate Strength of Castings.

Accordingly if the transverse or tensional strength of a casting one inch thick be = 1,0 we should have:

For $\frac{1}{2}$ inch thick = 1,350
For 1 inch thick = 1,000
For $1\frac{1}{4}$ inch thick = 0,907
For $1\frac{1}{2}$ inch thick = 0,838
For $1\frac{3}{4}$ inch thick = 0,783
For 2 inch thick = 0,739
For $2\frac{1}{2}$ inch thick = 0,671
For 3 inch thick = 0,620

For 4 inch thick = 0,547
For 5 inch thick = 0,497
For 6 inch thick = 0,459
For 7 inch thick = 0,429
For 8 inch thick = 0,405
For 9 inch thick = 0,385
For 10 inch thick = 0,367

According to Hodgkinson the tensile strengths of cast-iron bars 1″, 2″ and 3″ thick would be as 1,0 is to 0,8 and to 0,77 or slightly more than the above.

With wrought-iron the strength, according to Clay, increases with each rolling to the sixth; and then again decreases; or if the strength (tensional) of the muck-bar be = 1,0 then that of a piece piled and rolled three or four times = 1,36; and if six times = 1,41; decreasing again to = 1,0 at the twelfth re-working. This, however, as already stated is shown to be very doubtful with American irons, and largely dependent on the nature of the muck-bar. Kirkaldy has shown that welding does not make as strong a joint as the original iron. He made some eighteen experiments on welded bars of wrought-iron from 1¼″ to ¾″ diameter, the result being that none were equal to the original strength. The maximum strength attained was 97,4 per cent; the minimum 56,2 per cent and the average was 80,62 per cent of the original strength. Welded joints should, therefore, be taken at only about four-fifths of the strength of iron in calculating for tension or transverse strains. It is usual, however, to call for sufficient extra metal in all welded joints, so that the pieces must invariably break elsewhere than at the joint, when tested.

Effect of Re-rolling Wrought-iron.

In wrought-iron the skin seems to add some strength to the metal and for this reason, partly also on account of the more intricate interlacing of fibres in bar-iron, which is worked on all sides, bar-iron is generally stronger than plate-iron; plate-iron being about 84 per cent of bar-iron. The plate-iron itself is always stronger with the grain than across it, resembling wood in this quality. Experiments have shown the tensile strength to be nearly 10 per cent greater along the grain, than across it. Shearing would show even greater difference, being some 25 per cent easier with the grain than across it.

Bar-iron Strongest.

Wrought-iron should always be heated before hammering or working.

Its tensile stress is greatly reduced by cold hammering; experiments (according to Box) showing the reduction to be

a loss of 32 per cent if 2″ thick
a loss of 36 per cent if 1½″ thick
a loss of 13 per cent if 1″ thick

or an average loss of 27 per cent by cold hammering.

Effect of Cold Rolling.

By annealing the iron the strength can be partially restored but not entirely, the loss in the above experiment still being after the annealing

some 14 per cent in the 2″ thick iron
some 4 per cent in the 1½″ thick iron
some 8,6 per cent in the 1″ thick iron

or an average loss of about 9 per cent.

Unwin, however, claims the exact opposite. According to tests quoted by him, wrought-iron and mild steel increase in tensional strength and have a greater elastic limit, if worked cold, but suffer a large loss in ductility. The working however must be uniform on all parts and not local otherwise the material is weakened by uneven resistance. Annealing, according to Unwin, removes the effects of cold rolling or cold working and should be used where this is only local or on certain parts, as in the case of punching of rivet holes, where the part around the hole is hardened by cold hammering and the rest left unhardened. He cites the process of wire-drawing, which is similar to cold rolling and certainly adds greatly to the tensional strength of the material.

Avoid Blue-heat.

He finds one very curious fact, however, and that is that while neither cold nor hot working hurts wrought-iron or mild steel, working them at an intermediate or "blue-heat," that is at about 470° to 600° F., is positively harmful. In many tests it was found that samples of either, about ⅜ inches thick, could be bent back and forth on an average some twenty times, without breaking, if they were either cold or red hot. But if heated to an intermediate blue-heat they all broke readily after being bent back and forth only two or three times. Annealing ordinary wrought-iron means a loss of from 5 per cent to 10 per cent in tensile strength. The strength is not affected, however, by low temperatures nor ordinary variations in temperature, nor very much at higher temperatures. At high temperatures up to from 600° to 900° F. the tensile strength of wrought-iron

increases somewhat. Beyond this it decreases rapidly. Pure wrought-iron will increase up to a higher temperature than impure iron. Neither cast-iron nor steel are affected much by low temperature nor the ordinary variations in same.

Steel varies with carbon in strength.

With steel the tensional, transverse and compressive stresses vary greatly with the composition of the material. As already stated the more carbon it contains, the less elastic, but the harder is the steel, while the less carbon it contains, the more elastic, but less hard will it be. The ultimate limits for tension per square inch are given in Table IV from 42000 pounds to 108000 pounds and for compression from 90000 pounds to 150000 pounds, all per square inch.

Steel like wrought-iron is stronger in bars than in plates (plate-steel being about 80 per cent of bar-steel), and is stronger with the grain than across it. The proportions for tension and shearing across or along the grain being about the same in steel as in the wrought-iron. In regard to re-working, according to Mr. Clay, the strength of steel is increased to the fourth piling, and then declines, until at the seventh re-working it is weaker than after the first working. Or, if the tensional strength of the first working is $=1{,}0$ the fourth is $=1{,}253$ and the seventh $=0{,}94$. This, however, is quite as dubious as the advantage of re-working wrought-iron very often. Welded joints in steel are very much weaker than the original metal, having only some 40 per cent to 55 per cent of the strength of the unwelded parts.

If steel plates are worked cold they lose strength greatly. (Unwin, however, denies this for mild steel if working is uniform.)

Do not punch steel cold.

Punching them cold decreases their tensional strength some 33 per cent, if very hard steel; the milder the steel the less will the loss be. But this loss can be restored again by annealing after punching. Annealing instead of damaging steel plates greatly increases their tensional strength, particularly if the steel is very hard, adding some 50 per cent to same; but is of slight, if any, advantage in mild steel. But after all said, the only sure method for the architect to pursue is to make careful tests to see whether the material possesses the necessary qualities and strengths he desires. Chemical tests should therefore give way to practical tests.

The tensional or compressive ultimate strength can be readily

ascertained by testing small pieces in testing-machines, and making proper allowances for the differences in sizes of test-pieces and full size members. These machines are of various sorts, but generally by hydraulic power or weights, greatly increased by leverage or screw-action, they tear small specimens apart, or else crush them, the effects on the pieces being carefully noted and recorded. Some of the machines make automatic records. The kind of machine used is not so essential to the architect so long as he has ascertained its reliability. From the tensional and compressive ultimate strength the cross-breaking strength of the specimen can be calculated; or the modulus of rupture can be obtained by breaking specimens across with centre loads and calculating the modulus of rupture by the following Formula:

Tests for strength.

Modulus of rupture from tests.

$$k = \frac{3}{2} \cdot \frac{w.l}{b.d^2} \qquad (102)$$

Where k = the ultimate modulus of rupture of a material, in pounds, per square inch.

Where w = the load or amount of pressure, in pounds, applied at the centre, required to break a specimen of rectangular cross-section, lying on two supports.

Where l = the clear length, in inches, between supports.

Where b = the width of specimen, in inches, measured (across the specimen) at right angles to the line of pressure.

Where d = the depth of specimen, in inches, measured along the line of pressure.

Where the test specimen is exactly square in cross-section and of one square inch area, that is one inch by one inch, and where the supports are exactly twelve inches apart, and the load or pressure applied in the centre, the modulus of rupture will always be eighteen times the load or pressure, or:

$$k = 18.w \qquad (103)$$

Where k = the ultimate modulus of rupture of a material, in pounds, per square inch.

Where w = the load or pressure, in pounds, applied at the centre, required to break a test specimen, of one inch by one inch (square) cross-section, lying on supports exactly twelve inches apart. The shearing strength is generally found by direct tests.

Plane off test specimens.

Test specimens of steel, wrought or cast iron, should never be broken off suddenly, or by blows; nor should they be jarred, as otherwise the fibres crystallize more or less and this greatly affects the result. They should be carefully planed off in the machine-shop.

Another important point is to be sure to have all test specimens, when not cut off the actual pieces being used, of the same material and general thickness as the pieces.

In cast-iron, frequently, test-pieces are cast on to each piece, these are broken off and tested. They should not be broken off, however, except in the presence of the architect or his inspector. If they are of the same thickness as the piece, they offer a fair test; if, however, they are much smaller and thinner than the piece no reliance can be placed on the result.

In wrought-iron and steel it is best to roll the pieces a little too long and plane off the superfluous ends for testing.

Other Tests.

Besides the tests as to strength there are many tests resorted to to ascertain approximately the quality of the material. A few will be given here. If cast-iron be struck on its edge a sharp blow with a chisel or hammer, it is of soft and good quality if it can be indented. If it breaks it is very hard and brittle. If it rings out clearly, it is a good casting; if the sound is dull it is full of sand-holes, air-bubbles or flaws.

If the surface is smooth, even and hard, and the edges sharp and perfect it is a good casting. If the edges or surfaces are uneven and wavy it is an indication of unequal shrinkage and cooling or more likely unskilful ramming of the sand in the flask around the pattern or mould. If hollow pieces, when tapped, show uneven thicknesses on opposite sides the core has sagged or floated in the mould.[1]

In iron and steel the principal tests resorted to are for tensional strength, elasticity, ductility and limit of elasticity. These will be explained presently. The nature of the material can also be more or less ascertained by fracture and bending. It is almost impossible, however, to distinguish mild steel from wrought-iron excepting by tests for strength, and to ascertain whether or no they can be hardened or tempered, as already explained. If specimens of iron or

[1] A skilful moulder will arrange vent-holes and spaces so in the mould as to avoid dirt and blow-holes, no matter what the shape of the piece.

steel on fracture show a close, silky, fibrous grain, with few bright or crystaline specks the metal is good. If there are many crystals in the fracture, and particularly if they are coarse and large, and there are great "blotches" of color, or other evident impurities, the metal is poor. The color of the fibres is grey, that of crystals of a bright silvery aspect. It must be always remembered, however, to get the fracture by planing, or by slow bending, when cold, otherwise crystals will be formed while testing.

Condition of Fracture.

Specimens are frequently nicked and then bent cold until fractured, in such cases there will usually be a large percentage of crystals close to the nicked part, but none in the other half. All iron should be required to be free from all visible seams, blisters, buckles, cinder spots or imperfect edges.

Bending Tests.

If iron or steel pieces on fracture show the seams where the different layers have been piled and rolled together, that is, if the seams open perceptibly and are very marked, the metal is badly rolled or united. If these seams can be distinctly seen by the naked eye before fracture, the quality is even poorer. If iron or steel for a few hours be immersed in or wetted with some strong acid, such as either nitric, muriatic or sulphuric acid, the parts between the fibres will be eaten away, and the latter will be distinctly exposed. The architect can then readily judge of the nature of the rolling or working, whether the layers (or piles) are thoroughly worked together, and whether the fibres are close and all thoroughly interlaced.

Acid Tests.

Good wrought-iron should be capable of being bent double, without fracture, *while cold*, over a cylinder of diameter equal to twice the thickness of the piece. The iron should not crack under this test. If nicked, and then bent double suddenly by a blow from a sledge hammer the fracture should show but a small proportion of crystals and these nearer the nicked edge.

Beams, channels, angles and tees should be capable of having their flanges or legs bent, that is rolled up longitudinally and away from the web or other leg without opening the inside joint between web and flanges, or between legs. In practice, however, the iron usually furnished by the mills will not stand so severe a test. Its quality will be in proportion to its ability to stand this severe test.

Figure 161 shows a few illustrations of such bending tests,

the dotted lines showing outlines of original sections, before bending.

Tests for mild steel should be the same as for wrought-iron.

Ductility and Elasticity.

Ductility and elasticity are the same practically as stretching. If the material has high ductility its cross-section will be diminished greatly, when stretched, before breaking. If it has great elasticity it will stretch greatly, that is, become much longer before breaking.

Wrought-iron should be allowed a stretch $= \frac{1}{225}$ of an inch for each foot of length of specimen if subjected to a tensional strain of 10000 pounds, per square inch. Every important tensional piece should be tested up to 10000 pounds, per square inch. Those elongating more should be condemned. On removing the strain the piece should recover its original length exactly.

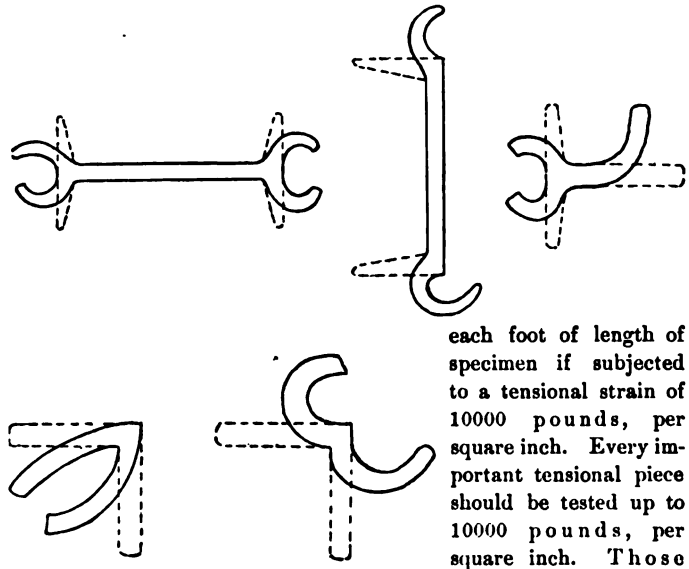


Fig. 161.

Safe strains in testing.

Test pieces should be strained up to 18000 pounds, per square inch, for compression and should then recover their original thickness, they should not crush completely under 36000 pounds, per square inch. Compression test-pieces must, of course, be short blocks or cubes. They should stand at least 25000 pounds, per square inch, in tension without

losing their capacity for recovering their original length, and should not tear apart under less than 50000 pounds tension, per square inch for small sections; if sections or plates are large they should stand at least 46000 pounds tension.

For steel the above limits must be varied, according to whether mild or hard steel is desired.

All metals, theoretically change their dimensions under the slightest strain. That is, they become longer under tensional strains, and shorter under compressive strains. When the strain is removed the material is supposed to recover its original length. This, however, is true only up to a certain strain; when this is exceeded the material loses its power to recover and remains permanently elongated or shortened. This permanent elongation or shortening is called the "*permanent set*," the strain (per square inch) which produces it is called the "*elastic limit*." This elastic limit varies in different materials, but is approximately at about *one-half* of the ultimate strain, or strain per square inch that will crush or tear apart the material.

Permanent Set.

Elastic Limit.

In very hard materials it is more than half the strain, for such materials not being elastic, will stretch (or shorten) but very little and will not show appreciable variation until a high strain is reached, when they show a set quickly, and break soon after. In soft, elastic, ductile materials they begin showing permanent stretching (or shortening) very early and continue to do so for a long time before breaking. The following will be quite safe to follow as general guides.

For cast-iron the elastic limit is about $\frac{1}{3}$ of the ultimate tensional or compressive stress.

For wrought-iron the elastic limit is about $\frac{1}{2}$ of the ultimate tensional or compressive stress in bars, and about $\frac{3}{5}$ in plates.

For mild steel the elastic limit is about $\frac{9}{20}$ and from this it varies to about $\frac{1}{5}$ for

Hard steel — both, of course, of either the ultimate tensional or compressive stress.

Table XXX gives the amount of extension and contraction, in inches, for different strains, of cast-iron and wrought-iron pieces, if each piece were one hundred feet long.

TABLE XXX.

Amount of Extension and Contraction, in inches, of Cast and Wrought Iron Bars, 100 ft. long, under different strains.

Strain, per square inch, in pounds.	CAST IRON.		WROUGHT IRON.
	Extension, under tension.	Contraction, under compression.	Extension, under tension or contraction, under compression.
1000	0,08308	0,09155	0.0444
2000	0,17150	0.18404	0,0889
3000	0,26528	0.27747	0.1333
4000	0.36442	0,37185	0.1778
5000	0.46890	0.46715	0.2222
6000	0,57874	0.56341	0.2667
7000	0.69392	0.66061	0.3111
8000	0.81446	0.75875	0.3556
9000	0,94036	0.85792	0.4000
10000	1.07160	0,95784	0.4444
11000	1,20820	1,05880	0.4889
12000	1,35014	1,16070	0.5333
13000	1,49744	1,26354	0.5778
14000	1,65010	1.36733	0.6222
15000	1,80810	1,47205	0.6667
16000	.....	1,57871	0.7111
17000	.....	1,68432	0.7556
18000	.....	1,79186	0.8000
19000	.....	1,90035	0.8444
20000	... ..	2,00978	0.8889
21000	.....	2,11994	0.9333
22000	.....	2,23145	0.9778
23000	.....	2,34370	1.0222
24000	.....	2,45690	1.0667
25000	.....	2,57102	1.1111

Up to the elastic limit it is supposed that the amount of stretch (or shortening) is exactly proportional to the amount of strain, and that the material will recover its exact original length. Reference is here made to Formula (88). Neither of the above suppositions are exactly true, though in wrought-iron and mild steels it is very nearly so. Cast-iron, however, is very variable in its extensions or contractions under strains; and all three show more or less "fatigue" and permanent set, under variable, or oft-repeated (on-and-off), or prolonged strains.

Box treats this subject very fully.

Extension of Cast-iron. He finds the extension of cast-iron, subjected to tensional strains, to be:

$$e = \frac{L}{10000000} \cdot (8{,}04.w + 0{,}0002676.w^2) \qquad (104)$$

Where e = the total amount of extension, in inches, of a piece of cast-iron, subjected to tensional strains.

Where L = the original length, in feet, of the piece of cast-iron.

Where w = the tensional strain, in pounds, per square inch, of cross-section of the cast-iron piece.

It will be readily seen, that the increase in extension is in a higher ratio than the increase in strain, which is due, as already said, to the very defective elasticity of cast-iron.

For compressive strains in cast-iron, Box's researches give the following Formula:

Contraction of Cast-iron.

$$c = \frac{L}{10000000} \cdot (9{,}108.w + 0{,}000047044.w^2) \qquad (105)$$

Where c = the total amount of shortening, in inches, of a piece of cast-iron, subjected to compression strains.

Where L = the original length, in feet, of the piece of cast-iron.

Where w = the compression strain, in pounds, per square inch, of cross-section of the cast-iron piece.

Of course, in either case the extension or contraction from changes of temperature will be independent of the above.

Table XXXI gives the length of piece required to stretch or contract exactly one inch under different strains.

By comparing these formulæ it will be seen that cast-iron yields more readily, that is, shortens more in proportion under very small compressive strains, than it extends under small tensional strains. But as the strains become greater the amount of shortening and

TABLE XXXI.

Length of Cast or Wrought Iron Bars, in feet, that will stretch or contract exactly one inch under different strains.

Strain, in pounds, per square inch.	CAST IRON.		WROUGHT IRON.
	Length, in feet, to extend one inch.	Length, in feet, to shorten one inch.	Length, in feet, to either extend or shorten one inch.
1000	1204	1094	2250
2000	583	543	1125
3000	377	360	750
4000	274	269	562
5000	213	214	450
6000	173	177	375
7000	144	151	321
8000	123	132	281
9000	106	117	250
10000	93	104	225
11000	83	94	204
12000	74	86	187
13000	67	79	173
14000	61	73	161
15000	55	68	150
16000	....	63	141
17000	....	59	132
18000	....	56	125
19000	....	53	118
20000	....	50	112
21000	....	47	107
22000	....	45	102
23000	....	43	98
24000	....	41	94
25000	....	40	90

extension become more nearly equal; they are exactly equal for strains of 4842 pounds per square inch, while for greater strains the amount of extension from tensional strains is greater than the amount of compression from the same compressive strain. With wrought-iron and mild steel the compression under small strains is even in a much more marked ratio than under small tensional strains owing to the "flowing" of the metal under compressive strains. There are no reliable experiments recorded, however, on which any formulæ could be based for wrought-iron, it may be safely assumed, however, to be perfectly elastic up to 18000 pounds, (per square inch), compressive strain and 25000 pounds, (per square inch), tensional strain. With heavier strains the shortening or lengthening will be in very much quicker ratio than the increase in their respective strains. With steel it will depend upon its nature. Mild steel will approximate nearly to the perfect elasticity of wrought-iron, under safe strains, while hard steels will become more imperfect in elasticity the nearer they approximate the nature of cast-iron.

Time plays a very important part in considering the final effect on metals of any strain.

Time Tests.

It has been found that a moderate strain on a bar, if left on a long time, will gradually increase its effect, either extending it more and more as time passes on, or shortening it more and more. This is known as the "fatigue" of the metal.

If the strain is within the elastic limit the increase in "set" will continue for a long time, but will finally become so infinitesimally small, as to be practically nothing. If, however, the strain is beyond the limit of elasticity the set will continue to grow and the piece will frequently break after many years under a strain which was considered well within the original ultimate breaking strength, but which, as time passed, fatigued, that is tired out, the material. Severe strains should therefore never be borne for a lengthened period, but should only be imposed for short periods, and they should be well within the elastic limit, to give the metal a chance to recover from the set produced by such strains.

Intermittent Strains.

Oft-repeated strains, even where put on entirely without shock, greatly weaken a material. It has been found that a load, which originally was borne with perfect safety, will, if often removed and replaced, finally break the piece.

Planat concludes by comparing many experiments on fatigue of iron: that if a strain of 58700 pounds, per square inch, can be repeated 170000 times before breaking a test piece, the same piece will break under a strain of only 48000 pounds, per square inch, if repeated 480000 times; or under a strain of 42700 pounds, per square inch, if repeated 1320000 times; or under a strain of 38400 pounds, per square inch, if repeated 4035000 times; while a strain of 32000 pounds can be repeated forever and would never break the piece.

These experiments were made by A. Wöhler and confirmed by Spangenberg, Baker, Baushinger and others: they are usually carried out by means of revolving wheels or by pistons which alternately put on and remove the strain, the number of revolutions or pressures being carefully computed. Piston-rods of engines offer good examples of oft-repeated off-and-on loads, or the driving-rods of locomotives, in fact many parts of machinery.

Impact or Rolling loads.

Where a load is put on suddenly, with shock, that is falls on, or is forced on, suddenly without regard to whether it jars or not, the effect is practically double that of a stationary (static) or dead load. This is called "impact." The subject is too extensive to enter into here, the actual effect on the beam depending upon the length of span, height of fall, and permanent load; but it will be safe enough in all such cases to simply double the factor-of-safety, or in other words allow only one-half of the safe-stress, that would be allowed for an intermittent load put on without shock. Rolling loads are considered the same as loads put on with shock; the effect depending, of course, on the velocity; all such loads, moving, rolling, jarring, etc., are called dynamic as opposed to static or dead loads.

Reversed Intermittent Strains.

If the strain is reversed at each application, that is alternately tension and then compression, the effect is equal to the sum of both, that is, double that of the same amount of strain if not reversed. To break anything we instinctively bend it one way and then the other, by doing this we reverse the strains in the fibres from compression to tension and vice-versa at each bend, the result being that the piece soon breaks.

Box has combined these rules in a Table, which is here given as Table XXXII.

TABLE XXXII.

ULTIMATE BREAKING STRENGTH OF MATERIALS UNDER DIFFERENT KINDS OF STRAINS.

Material.	If Dead Load (Static).	Intermittent Loads (off-and-on continuously.)			
		If in one direction only.		If in opposite directions.	
		Without shock.	Rolling (dynamic).	Without shock.	Rolling (dynamic).
Wrought Iron and Steel.	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$
Wrought Copper and Brass, also Slate, Timber, Masonry, etc.	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$
Cast metals: Iron, Copper, Brass, Lead, etc.	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$

By combining this table with the safe stresses given in Tables IV and V, that is, taking whatever part of the safe-stress there given for dead loads, that the nature of the load demands, we can obtain the safe-stress under any manner of loading. Where stresses in opposite directions take place, the material will yield in the direction of the weakest stress.

How to use Table XXXII.

Thus, if we have a bar of American cast-iron, according to Table IV it will be safe under a steady (static) compressive strain of 15000 pounds, per square inch, or a steady tensional strain of 2500 pounds, per square inch.

If the compressive strain were constantly and entirely removed and then put on again, but without shock, it would be safe, according to Table XXXII, to use but $\frac{1}{3}$ of this amount of 5000 pounds, per square inch; while under the same circumstances only 833 pounds would be safe in tension. If the strain were constantly removed and then put on again with shock, that is suddenly, or kept moving, only one-sixth would be safe or 2500 pounds per square inch, in compression, and 417 pounds in tension.

If the strain were alternately compression and then tension, but put on without shock the same strains would be safe; the safe strength of the bar would, therefore, be measured by the weaker of the two and would be only 417 pounds, per square inch. If the

strains alternated between compression and tension and besides this were dynamic (put on suddenly) only one twelfth would be safe or 1250 pounds in compression and 209 pounds in tension, both per square inch. The strength of the bar under these circumstances would, therefore, be only 209 pounds, per square inch.

For wrought-iron we have the same safe-stress, whether in tension or compression; for a dead, constant load we should use then, from Table IV, 12000 pounds, per square inch; from Table XXXII we should have for intermittent loads, in one direction only, that is of the same nature, but put on-and-off continuously, but without shock, two-thirds or 8000 pounds, per square inch. If put on-and-off suddenly 4000 pounds, per square inch. The same if constantly reversed from compression to tension but done slowly and without shock. If constantly reversed and it is done suddenly or with shock the safe strain per square inch would be only 2000 pounds.

When a strain on a beam is never completely removed, but changes constantly from a larger to a smaller strain, both however in the same direction, the effect is not so great as where both strains are at times constantly removed.

In such cases the effect can be found from the following Formula:

Variable Strains.

$$w = (w_i - w_{ii}) \cdot x + w_{ii} \qquad (106)$$

Where $w =$ the corresponding dead load, or constant strain in pounds, per square inch, to produce the same effect as one alternating between two unequal loads or strains w_i and w_{ii}, but both in the same direction.

Where $w_i =$ the larger of the alternating loads or strains, in pounds, per square inch.

Where $w_{ii} =$ the smaller of the alternating loads or strains, in pounds, per square inch.

Where $x = 3{,}0$ for cast-iron, and $= 1\frac{1}{2}$ for wrought-iron and mild-steel.

The above condition would frequently happen in the case of warehouse floors, bridges, and other places where there is a constant dead load, which at times is increased by other loads to be carried.

If the additional load is put on suddenly, or is a moving load, it should be doubled. In that case w_i would be equal to the doubled dynamic load plus the static load, and w_{ii} would be equal to the static load.

In all designs of metal structures great care should be taken to design all parts, not only of practicable shapes and sizes, but of dimensions that will not involve increased cost.

Economical Designing.

As a rule cast-iron, wrought-iron and steel are estimated at a certain price per pound. If the sizes are not unusually small nor unusually large the price will be the usual one. If, however, the sizes are very light the price will be greatly increased, for two reasons. The cost of preparatory work, working drawings, office expenses, models, etc., will be practically the same as for heavier work; the handling and labor will be very nearly the same, and in very light construction all this must be borne by fewer pounds and the price is consequently greater. On the other hand, if the pieces are unusually heavy, or large, or long, they may require special rolling or casting, and may require special cars and freight arrangement, special trucks, derricks, etc., or they may be very difficult to manufacture and involve much loss, many misgoes, etc.

The architect should, therefore, as far as possible design so as to use standard sizes.

In castings as well as in mill-work the standard will vary more or less with the parties doing the work. It will be impossible, therefore, to give here any universal standard. A few hints, however, may help the architect in economical designing.

Thickness of Columns.

For hollow-castings (columns) Planat gives the following as the French standard, the thicknesses being the *minimum* or *smallest* thickness possible for the length.

For columns 6′ 6″ long not less than $\frac{3}{8}$″ thick
For columns 9′ 10″ long not less than $\frac{1}{2}$″ thick
For columns 13′ 1″ long not less than $\frac{5}{8}$″ thick
For columns 19′ 8″ long not less than $\frac{3}{4}$″ thick
For columns 26′ 2″ long not less than 1 ″ thick

The practice of American iron-works is to regulate the thickness with the diameter rather than the length. Thus our iron-works make columns under 6″ diameter about $\frac{1}{2}$″ thick, those over 6″ diameter from $\frac{5}{8}$″ thick and upwards.

For castings made to carry weight the thickness should be more. In columns the maximum thickness should depend upon the possibility of making the core stiff enough to keep its place in the centre.

Large diameter with thinner shell (within reasonable limits) will give the most strength for the amount of material used, thick castings being much weaker, as already shown.

Avoid great length.

In rolled-work, beams, channels, tees, etc., of unusual length should be avoided. Most sections are rolled to 30 feet in length without extra charge; beyond this there is an additional charge per pound, for every extra five feet or fraction thereof in length, if the section is heavy, up to 40 or 50 feet which is the limit for beams and channels, or up to 90 feet for angles. Then, too, very long pieces (over 30 feet long) involve being carried on two cars, which means extra freight charges, and very heavy pieces might be refused by many railroads. Where great length is desired it will be better, as a rule, to make it up of two pieces, the extra material required for the "splicing" being, as a rule, much more economical than the cost of manufacturing and handling long and heavy pieces.

In pieced work where angles are attached to beams, etc., if it is punched and fitted together at the mill with bolts the whole will be charged at a standard rate per pound; if, however, the pieces are riveted together, there will be an extra charge on the whole, even if only one rivet were used.

If pieces are required to be cut to exact lengths there is usually an extra charge on the whole. If pieces are required to be drilled or punched, there is an extra charge on the whole, even if there is only one hole in each piece.

Cost of Fitting.

For these reasons it is usual where there is much fitting, drilling, etc., to be done to have it done at the mill for the standard rate per pound. Where, however, there is only a little to be done, as is usually the case in building construction, it is cheaper to do it at the building, using portable hydraulic punches or other similar contrivances to facilitate the labor.

A saving can also be made by doing all painting of heavy work at the building, as the mills charge a standard price per pound of metal for each coat; but where this is done, the architect cannot be too careful in having all rust removed.

Practicable Plate-sizes.

In building up plate-girders, plates as wide as eight feet can be rolled, and of thickness and length in one piece not to exceed in finished weight two tons or four thousand pounds, this weight being about the limit for single pieces of rolled-work within reasonably economical limits.

Thus, for instance, we could get plates five feet wide, one inch thick and twenty feet long, or eight feet wide, one-half inch thick and twenty-five feet long. If we require wider plates (usually for deep webs) we must take an eight-foot wide plate and shear it off, splicing the pieces. Thus if we required a ten-foot deep web, we would shear the eight-foot plate in ten-foot lengths, would place them side by side vertically, connecting them by covering plates of some kind, and our web would thus be composed of a series of panels each ten feet high and eight feet wide. Plates should not be over one inch thick on account of the difficulty of punching, nor less than one-quarter inch thick for fear of loss by rusting and failure by buckling. In steel plates, Carnegie, Phipps & Co., of Pittsburgh will roll plates up to 9′.6″ wide and 14′.2″ long, if about $\frac{1}{2}$″ thick, or 9′.2″ long by above width, if $1\frac{1}{4}$″ thick. See Table, p. 51, in their hand-book of 1889.

In round, square and flat bars almost any sizes can be made, but, here too, unusually light or heavy ones are charged at extra rates.

Sizes of Rounds, Squares and Flats.

The sizes of these vary with different mills, but are about as follows: The ordinary sizes for rounds and squares, are $\frac{3}{4}$″ to 2″ diameter, and for flats from 1″ to 4″ x $\frac{3}{8}$″ to $1\frac{1}{2}$″ and $4\frac{1}{8}$″ to 6″ x $\frac{3}{8}$″ to 1,″ these are made at regular rates.

But the mills will make, at additional rates, rounds from wire sizes up to $\frac{3}{4}$″ diameter and from 2″ to 7″ diameter; half rounds from $\frac{3}{8}$″ to $1\frac{1}{4}$″ diameter; squares from $\frac{5}{16}$″ up to 5″; flats from $1\frac{1}{2}$″ x $\frac{3}{32}$″ up to 12″ x 2″.

In designing flats the best rule to follow is to make the thickness between one-fourth and one-third of the width, except for very wide bars, when it can be between one-eighth and one-quarter.

Where long bars are in tension, and one or both ends are held by nuts and screws, it is often economical to "up-set" the screw-ends, that is, enough material is added where the screw is cut, that is the diameter is enlarged enough at ends, to make the diameter at the root or thinnest part of the screw (that is between the threads),

Up-set Screw-ends.

equal to the diameter of the whole bar. The screw itself is thus "upset," that is set upon the bar. The cost of upsetting screw-ends is more of course, than cutting screws upon ordinary bars, on account of the extra welding necessary, but in long bars, or very heavy bars, the material saved by not having to increase the diameter of the bar its whole length

(to allow for cutting away at the ends), more than offsets the extra cost of up-setting. In light bars and short bars, as a rule, it is more economical to add enough to the diameter for the whole length to allow for cutting the thread at the end. In such a case the strength of the bar, in tension is, of course, only equal to the section at the root of the thread. It is a curious fact that threads cut with old dies are stronger than those cut with new dies; the old ones apparently crushing and thus leaving more material in the threads, while the new ones cut the material right out.

Threads, nuts, etc.

Different mills have different standards for threads, nuts, heads, etc. Practically, however, they all closely resemble the standards adopted by the Franklin Institute (of Philadelphia), February 17, 1866, and which are here reproduced by their permission as Table XXXIII.

In forming eye-bars the practice of different mills again varies.

It should be remembered that they are all welded, and therefore, if of wrought-iron, only some 80 per cent in strength of the metal in the bar. All parts should therefore be designed for some 25 per cent more strength than required with the usual factors-of-safety.

Eye-bars and pins.

For ordinary sizes of flat iron the mills have dies on which these eyes are "die-forged," that is, formed by machinery. The Phœnix Co., for instance, have some fifty-six sizes of dies for flats varying from $2'' \times \frac{5}{8}''$ up to $6'' \times 2\frac{1}{2}''$. The New Jersey Steel and Iron Works have some forty seven sizes of dies for flats from $2\frac{1}{2}''$ up to $6''$. Their standards for pins are $\frac{1}{16}''$ less than even quarter-inches, as $2\frac{9}{16}''$ diameter, $2\frac{7}{16}''$, $2\frac{1}{16}''$, $2\frac{5}{16}''$, etc. All pin-holes in eye-bars should invariably be drilled, and should always be made (not over) $\frac{1}{50}$ inch larger in diameter than the pin, to allow for slipping same in.

The practice of the latter company where eye-bars are of same thickness as the flat-bar, is to make the diameter of pin (if not governed by other circumstances) four-fifths of the width of bar (or $d = \frac{4}{5}.b$). In such a case the width of metal beyond the eye (at end of bar) is made equal to the width of bar (or $a = b$) and the width each side of eye equal to three-quarters of the width of bar (or $c = \frac{3}{4}.b$), see Figure 162. Where larger pins are used the proportion must be even more increased on account of the greater amount of weakening by welding.

The above sizes are good averages. In some cases the metal around the eye forms a concentric circle with the eye. In such

cases (if not required to be larger) the radius of the eye is from three-eighths to one-half of the width of flat, and the radius of the surrounding circle is from four-fifths to nine-tenths (of the width of flat) *larger* than the radius of eye. The thickness of eye is, usually, but not necessarily, the same as that of the flat.

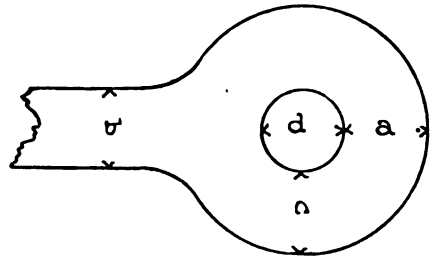


Fig. 162.

For further information as to sizes, etc., the reader should consult the hand-books issued by the different rolling-mills, taking care to get the latest issues, as they are constantly changing their rolls and stock sizes.

These books will also give the practice of each mill for locating holes in flanges of beams, channels, etc., sizes and weights of separators, manner of connecting beams by framing, etc.

Table XXXIV gives in condensed form the results of the most recently published tests on irons and steels.

POSTSCRIPT TO CHAPTER VIII.

Since writing the foregoing chapter a claim has been made by J. W. Bookwalter, an iron manufacturer of Springfield, Ohio, to have perfected a new process which will revolutionize the manufacture of iron and steel. He uses what he calls a Robert converter, which is the same in principle as the Bessemer, excepting that the air is blown in horizontally.

The claims made are; That he can produce any quality of iron or steel from the same furnace; that the impurities can be gathered and floated off on the surface; that the silicon can be controlled and burned out separately, ahead of, and without affecting the carbon, which can afterwards be reduced to any desired quantity, thus leaving the mass to be poured from the converter either pure wrought-iron, or steel of any grade, as may be desired; and that the control of the silicon and carbon is perfect.

Similar claims have been made before, but were never substantiated. Should, however, these claims prove to be genuine, it would at

once mean a large saving in the cost of manufacture of all grades of iron and steel. It will be readily seen that not only is the cost of handling the iron several times and the cost of puddling (by hand) done away with (for if this invention does all that is claimed for it, the pig-iron could be melted and run directly to the final rolls), but the cost of furnaces, etc., will be greatly reduced, as one and the same converter will make any quality of iron or steel desired.

Then, too, it would solve the problem of strong, cheap castings, as these could now be made of a high grade of steel at about the cost at present of ordinary cast-iron; this, of course, would mean a casting with the forging, welding and other properties of wrought-iron.

It is also claimed that the blast need not be nearly so strong as in the Bessemer converter, and hence the cost of this new converter is not only very much less, but its tuyères and lining will outlive many times those of a Bessemer converter.

It is further claimed that not only is the reduction in cost of steel very great, but that the cost of rolled-iron is reduced to the same level as the cost of steel.

CHAPTER IX.

RIVETS, RIVETING AND PINS.

WHEN it is necessary to secure two or more pieces of iron or steel together in such a manner that they can be readily separated, bolts are used. These are iron or steel pins with solid heads at one end and threads cut on the other end, onto which the nut is screwed, thus holding the two pieces together. How closely the two pieces may be held together depends, of course, entirely upon the man who handles the wrench. Then too, bolts or pins do not completely fill the holes through which they pass, which frequently is a cause of great weakness, besides the danger of water getting into the spaces and rusting them. Where, therefore, it is not necessary to ever separate the pieces — and the latter are of either wrought-iron or steel — rivets should be used, which, for all practical purposes, might be considered as permanent bolts or pins. Cast-iron, of course, always has to be bolted, as it would break if riveting were attempted on it. A rivet is a piece of metal (wrought-iron or steel) with a solid head at one end and a long circular shank. In its shortest description the process of riveting consists in heating the rivet, passing its shank through the two (or more) holes, while hot, and then forging another solid head out of the projecting end of the shank. The hammering forces the heated shank to fill all parts of the holes, and the shank contracting in its length, as it cools, draws the heads nearer together, thus firmly forcing and holding the pieces together.

Description of Rivet.

Rivets are made of mild steel or the very best wrought-iron, the latter being the most reliable. According to some writers, the shank is made tapering in length and circular in shape, being larger at the head and smaller at the end. In this country, however, the shank is always of uniform diameter. The length of shank from head to end varies with the thickness of pieces to be riveted together, but is long enough not only to allow for passing through the pieces, but has also enough additional length to

Length of Tail.

provide for filling of holes and forming head, the additional length being about $2\frac{1}{2}$ times the diameter of hole. The rivets are manufactured by heating rods of the diameter of rivets, which are pushed while hot into a machine, which at one operation forms the head and cuts off the shank to the desired length.

The shank before heating is about one-sixteenth smaller in diameter than the hole, to allow for its expansion when heated, *i. e.*, for $\frac{3}{4}''$ rivets, $\frac{13}{16}''$ holes are punched and for $\frac{7}{8}''$, $\frac{15}{16}''$ holes. There are four kinds of rivets, all answering the same uses, and only distinguishable by the shapes of their heads. These are the button or round headed rivet; the conical headed rivet; the pan or flat headed rivet; and the countersunk rivet. The latter is only used when a smooth surface is desirable. The first is the most used shape. Figure 163 shows the different shapes, the dotted lines indicating how the second head is finally shaped. Sometimes a rivet is countersunk on one end only.

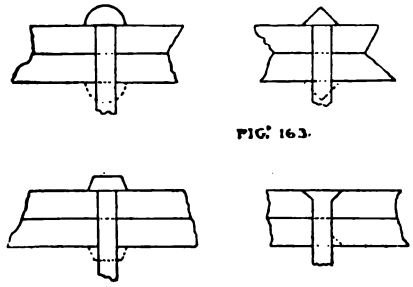


FIG. 163.

The exact sizes of heads, shapes, etc., vary in different mills.

Size of Head. The diameter of head should be from $1\frac{1}{2}$ to 2 times the diameter of shank,[1] according to shape adopted and the height of head should be about $\frac{3}{5}$ the diameter of hole. In countersunk work, the head may extend entirely through the plate or not, its diameter being accordingly smaller or larger. Where it extends through the sharp edges will shear the rivet, countersinking therefore, should be avoided in the plates. In showing riveted work the diameter of the shank is always drawn and figured where the hole is to be left open and the size of rivets is spoken of accordingly, the hole is always made $\frac{1}{16}''$ larger. Where the riveting is to be done at the shop or mills, the size of head is shown.

Pitch of Rivets. The spacing of rivets will be considered later, the direct distance from *centre of hole to centre of hole* is known as the

[1] The Franklin Institute standard for button-heads (which are usually used in the United States) is to make the head $\frac{1}{8}''$ larger in diameter than $1\frac{1}{2}$ times the diameter of shank.

"*pitch*." The pitch should never be less than $2\frac{1}{2}$ diameters; nor should the centre of any hole (if possible) be nearer to any edge than $1\frac{1}{2}$ diameters. By diameters is understood the diameter of shank. In riveted angle work the distance is frequently necessarily less. In thick plates it should be more. In drilled work the pitch might be reduced to 2 diameters. If rivet-heads are countersunk the pitch should be increased, according to the amount of metal cut away to make room for the rivet-head.

Punching of Holes.

In punching rivet-holes the position of holes are usually marked off on a wooden template and then through this marked or indented by a hand-punch on the iron plate; the plate is then passed under a punch which is usually worked by steam-power, the die and the punch being adjusted to the size of the rivet-hole wanted, the punch is usually $\frac{1}{16}''$ larger than the rivet, and the die about $\frac{1}{8}''$ larger. The thickness of plates to be punched should not, as a rule, be greater than $\frac{3}{4}$ of an inch; nor in any case should the thickness of plate be as large as, nor larger than, the rivet-hole, as, unless the diameter of the hole is larger than the thickness of the plate the punch is apt to break. Where holes are punched at the building, small screw or hydraulic (alcohol) punches are used, which can be readily carried around by one or two men, the power being obtained by screwing or pumping; or sometimes, where mechanics are not quite up to the times, a rather more clumsy lever-punch is used, the power being obtained by increasing the direct pressure by leverage. Punching makes a ragged and irregular hole, and as it gives the plate a sudden blow or shock it injures the metal considerably, unless the rivet-holes are so close, that the entire plate is practically cold-hammered. The loss in strength to the remaining fibres in a punched wrought-iron plate is about 15 per cent, this loss being, of course, in addition to the loss of area, and it is a loss that cannot be restored. In steel plates the remaining fibres are damaged about 33 per cent, but in them the loss can be restored by annealing the plate which, however, adds considerably to the expense.

Loss by Punching.

In drilled-work there is no loss, and the holes are not only accurately located but are accurately cut. But drilled-work is very expensive, as it has to be done by hand or by machine-drills, the process being slow at best and consequently meaning a very large

addition to the charge for labor. In riveted girders it would probably double the expense of the girder.

Advantages of Drilling.

The advantages of drilling are, that the holes can be cut after the plates have been partially secured together, thus assuring a perfect fit of the holes over each other; and that the holes being perfectly smooth and even bear more evenly on the rivet, and the work is less apt to fail by compression, than where the bearing of plate against rivet is ragged and uneven, as in punched work. On the other hand, the edges of drilled holes are so sharp that they promote shearing, and for this reason the edges of drilled holes in plates should be filed or reamed off. As a rule, however, the architect will find the bearing and cross-breaking strengths of rivets less than their shearing, excepting where rivets are small in comparison to thickness of plates being riveted, which is not often the case.

To settle, then, whether work should be drilled or punched, is mainly a matter of expense. Drilled-work, of course, is far preferable as regards strength and it costs accordingly.

The rule of the mills is to punch all holes, excepting for countersunk rivets, which, after punching, of course, have to be drilled, to obtain the slanting sides of the hole.

A medium course between drilling and punching would be to punch the holes smaller than desired and then drill or ream them to accurate size when partially secured together.[1] Steel should always be drilled unless annealed after punching.

In most work, however, the architect will have to be satisfied with punching, and must therefore allow sufficient material to make good the damage done and to allow for inaccuracies.

Machine-Riveting.

In riveting proper, that is, filling the holes, there are also the two methods of doing it, by hand (hand-riveting), or by machinery (machine-riveting), but unlike the making of the hole, in this case, the machine-work is both better and cheaper.

A machine-driven rivet is driven and completed by one powerful squeeze of the steam (or compressed-air) riveting-machine; this squeeze not only forces the plates more closely together, but more completely fills the hole with the rivet metal, besides the great ad-

[1] If this is not done the "drift pin" will be used to force all the holes into line, and this means crushing and possibly buckling of the plates.

vantage of doing the entire work while the rivet is hottest, and while it is, of course, at one temperature.

In hand-riveting these advantages are lost, the power being only equal to that of the mechanic's blow, and as in hand-work the process consumes some time, the rivet changes its temperature and cools considerably.

In riveting, the entire rivet, including the head, should be heated to at least a red heat. It should not be heated beyond this for fear of "burning," particularly with steel rivets. Rivets that have been heated once and allowed to cool without working should be discarded. If rivets are driven at a lower heat than a red one, they will be greatly damaged, unless riveted cold.

Hand-Riveting.

In hand-riveting at least two men are required, one to hold the head, the "holder-up," the other to do the riveting; but generally there is a boy to heat and bring the rivets, one holder-up, and two riveters, whose strokes alternate and thus accelerate the process.

The riveter puts a punch or drift-pin through the holes to clear them and force them into line; the holder-up seizes the hot rivet with his pincers and puts its shank through the hole, he then covers the head with a holding-up-iron shaped to fit it, and the riveters at the other end begin hammering down the projecting end of shank. When this is roughly shaped the use an iron (called "button set," for round-headed rivets), which is properly shaped to make the head. Before beginning to hammer down the end of shank, the riveters should always thoroughly hammer the plates around the hole, to bring the plates closely together.

How to Distinguish Riveting.

Hand-riveted work can sometimes be distinguished from machine-riveted work by the many marks at the made head. In machine-work there is but one mark, and this may be a little out of the centre with the shank and so show the squeezed material around its edge. But usually the work cannot be distinguished in this way. If hand-riveting has been conscientiously done and by careful mechanics, it is difficult to distinguish it from machine-riveting. But the shanks of hand-riveted work, as a rule, do not fill the hole as well as those in machine-riveted work, and they can more easily be "backed out" after the head has been cut off with a chisel and hammer. The only reliable test in both methods is to hold the hand one side of the head and

strike the other side with a light hammer — the hammer test — when the sound will quickly disclose loose shanks. If in machine-riveting the plate has been sprung, and the pressure is quickly removed while the rivet is still hot, the plate may settle back, lift the hot head, and so form a loose shank.

In designing riveted work, whether to be hand or machine riveted, the architect should bear in mind the necessity of placing the rivets so that they can be inserted in the holes from one side and hammered from the other, and for machine-work, that the machine can reach them.[1]

Steel rivets damaged.

Steel rivets are very seriously damaged during the process of riveting. Box gives as the average of a number of tests of bar steel a tensional-strength of 47,84 tons (gross), which, after riveting, was equal in shearing-strength to only 23,77 tons (gross), a loss of 50 per cent as between the tensional-strength of the steel and shearing-strength of the riveted work, whereas in ordinary steel work this loss never exceeds 33⅓ per cent, as between tension and shearing.

In wrought-iron the loss is about 15 per cent. In steel the strength of the rivets will depend greatly upon the composition and nature of the steel itself, but in order to be able to rivet, the steel will necessarily have to be of a mild character. The safe values given in Table IV, for compression and shearing of wrought-iron and steel, can therefore be used with perfect safety.

Use larger modulus of rupture.

When calculating bending-moments on rivets, a modulus of rupture 25 per cent greater than given in Table IV may be used, as the rivet-heads answer the same purpose as nuts and heads on pins in holding together the plates which are pulling in opposite directions, and thus reducing the bending-moment by friction. In figuring the number of rivets required an architect should err on the side of liberality, rather than to stint them, as there will necessarily be more or less of them poorly driven. He should particularly do this where strains are small and the number of rivets required are few, as one weak rivet in a small lot would quickly diminish the factor-of-safety, where in a large lot it would scarcely vary it appreciably.

[1] The minimum distance, from inside face of one leg of an angle-iron to centre of *nearest* rivet-hole, in other leg, should be at least 1¼″ for 1″ diameter rivet; 1¼″ for ⅞″ diameter rivet; 1″ for ¾″ diameter rivet; ⅞″ for ⅝″ diameter rivet; 13-16″ for ½″ diameter rivet; and if possible these distances should be increased.

Of course the more rivets there are, the more will the plates be injured and cut away; this loss, however, can be largely overcome by what is called "chain-riveting," or "zig-zag" riveting, and by making the laps or cover-plates pointed.

Chain, zig-zag or staggered.

Chain riveting consists of placing the alternate rivets on different lines instead of all on one line, see Figure 164. This again is called

FIG. 164.

zig-zag if they alternate as shown.[1] A cross-cut through this plate at any point can only pass through two rivets if chain-riveted or through one rivet-hole if zig-zag riveted; so that the plate is only weakened by two or one rivet-hole, respectively, while it may have a large number of rivets.

Where plates are lapped or joined by cover-plates, the rivets have to transfer the full strain from one plate to the other (in case of a lap); or from one plate to the cover-plate and from that to the other plate (in case of cover-plates). Of course, it can be readily seen that this means a large number of rivet-holes and a very great weakening of the plates. If it were practical to suddenly enlarge the plate at the riveting point there would be no loss, but this would be clumsy and besides it would not be practicable to roll plates with certain points enlarged or thickened. As the whole plate, however, will be equal only to its strength at its least cross-section the rivets should be so disposed as to weaken the plate as little as possible. This is done by pointing the plate ends in the case of lapping, or the ends of cover-plates where these are used, and are not covered in the construction. Where the plates are to be ultimately hidden out of sight, this expense is saved, the plate ends are left square, but the rivets are placed pointedly or pyramidically.

Plates weakened by holes.

Thus, in Figure 165 is shown a lapped joint with staggered rivets; we will suppose that calculation has shown the necessity of nine rivets to equal the tensional strain

Lapped joints.

[1] Most mills use the term "staggered" in place of "zig-zag," but for convenience in writing the writer prefers to use the term staggered to mean zig-zag rivets placed pyramidically.

on each plate. The under plate A is dotted, the upper plate B drawn with full lines. By arranging the rivets as shown in Figure 165, each plate is weakened only by one rivet-hole. (As already explained the plates themselves need not necessarily be pointed, but can be left square, if expense must be considered and looks are no object.) For, while a section at C shows plate A weakened by two rivet-holes (Nos. 2 and 3) it must be remembered that the strain on A is no longer the full strain, but has been diminished by an amount equal to the work that would have come on the one rivet-hole; for rivet No. 1 has already transferred this amount to plate B. Similarly while a cut at D shows three rivet-holes (Nos. 4, 5 and 6) the plate is really not weakened at all here, for an amount of strain equal to what would have come on the metal taken from these rivet-holes has already been transferred to plate B by rivets Nos. 1, 2 and 3.

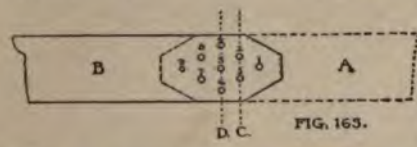


FIG. 165.

Similarly as the strain on B increases the rivet-holes in it diminish till at No. 9, plate B has got the full strain and is therefore only weakened by this one rivet-hole.

Butt joint with cover-plates.

The disadvantage of lapping plate ends is obvious, as the plate would not continue in the same plane. For this reason joints are generally made by butting the ends of the plates and covering one or both sides with cover-plates. The principle of riveting is the same as for lapped joints, but it will require a different disposition of the rivets, and twice the number of rivets, as plate A (Figure 166) has to transfer its strain to the cover-plate by one series of nine rivets, and the cover-plate transfers it to plate B by another series of nine rivets. This can be readily seen in Figure 166. In this case the disposition of the rivets requires an extra rivet each side, or 20 in all.

Two cover-plates best.

Where it is not necessary to keep plates A and B in the same plane it would be cheaper and better of course to lap them, rather than to use *one* cover-plate. If, however, two cover-plates can be used, one on top of the plates and the other under them, the advantage is very great, as the strain will be transmitted in a direct line or plane from plate A

to plate *B*, and besides this the joint is greatly strengthened by the friction between the cover-plates and plates *A* and *B*. In an experiment made by Clark with three $\frac{5}{8}$ inch thick plates riveted with one $\frac{7}{8}$ inch rivet through an oblong hole, it was found that friction added about 5 tons resistance against pulling out the centre plate. This would seem to add a safe-strength to each $\frac{7}{8}$ inch rivet of $\frac{5}{f}$ and if $f = 5$ (or a factor-of-safety of 5 were used) it would add just one ton to the calculated strength of a $\frac{7}{8}$ inch rivet. This increase, however, is a doubtful one, and would probably be lost in time by a gradual wearing of the plates due to this very friction, or possibly from slight rusting or other causes; no

Gain by Friction.

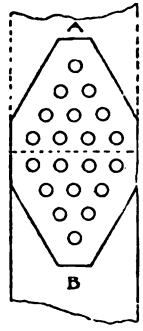


FIG. 166.

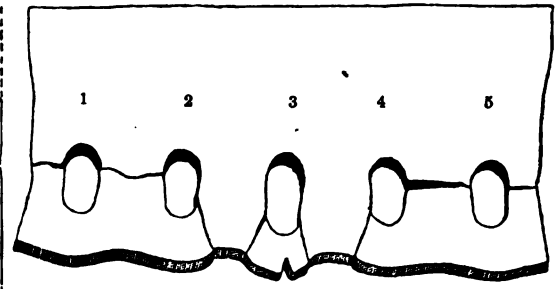


FIG. 167.

allowance should therefore be made for extra strength due to friction, but it certainly is a great advantage in making joints; and the above facts may account largely for the discrepancies in experiments on riveted joints, where no allowance is made for friction.

The pitch of rivets is, of course, governed by circumstances. The rule is to try to arrange the rivets, so that the strength of the plate between them shall equal the actual strength of the rivet.

In boiler-work, however, they must be located not only for strength, but must be placed close enough to make the joint steam-tight. For this reason, too, boiler-plates are always lapped, the joint being more easily caulked and made tight.

Boiler Riveting.

In constructional work, however, there will be a great loss and waste of material, if the rivets are placed too closely. In plate

girders and riveted joints in trusses the rule is not to make the pitch less than $2\frac{1}{2}$ diameters for punched-work, nor more than sixteen times the thickness of the least plate at the joint, or:

$$p = 16.t \qquad (107)$$

Where p = the greatest pitch, in inches, for rivets of plate-girders or riveted trusses.

Greatest Pitch.

Where t = the thickness of the thinnest plate, in inches.

The pitch is measured from centre of hole to centre of hole on a direct (straight) line.

$$p_1 = 2\tfrac{1}{2}.d \qquad (108)$$

Where p_1 = the least pitch, in inches, for rivets of plate-girders or riveted trusses.

Least Pitch.

Where d = the diameter of rivet-holes, in inches.

The exact pitch must be between these two limits; and is, of course, calculated.

Different writers have attempted to lay down exact rules for the size of rivets to be used, using for a basis for the formulæ the thickness of thinnest plate to be riveted. Such rules, however, generally do not agree with good practice, as they either make the rivets too small for thin plates, or too large for thick ones, or *vice versa*.

Diameter of rivets.

As a rule the local circumstances must control the selection of the size of rivet; the following, however, may serve as a general guide:

For plates from $\frac{1}{4}''$ to $\frac{7}{16}''$ thick, use rivet-holes $\frac{5}{8}''$ diameter.
For plates from $\frac{1}{2}''$ to $\frac{5}{8}''$ thick, use rivet-holes $\frac{3}{4}''$ diameter.
For plates from $\frac{1}{16}''$ to $\frac{13}{16}''$ thick, use rivet-holes $\frac{7}{8}''$ diameter.
For plates from $\frac{7}{8}''$ to $1''$ thick, use rivet-holes $1''$ diameter.

Of course, larger or smaller rivets can be used, but as a rule $\frac{5}{8}$ inch, $\frac{3}{4}$ inch, and $\frac{7}{8}$ inch are most desirable.

Figure 167 shows the different ways in which riveted-work will yield. This figure is made from a photograph of an actual specimen, tested and torn apart at the Watertown arsenal.

It is evident that the plate began yielding by all of the rivets compressing or crushing the plates, and finally yielded completely by tearing apart from 2 to 1 and to left edge and the same from 4 to 5 and the right edge, while rivet 3 tore its way completely out, shearing off a piece of the plate, and rivets 2 and 4 partially so.

How plate yields.

The iron plates tested were 15 inches wide, $\frac{1}{4}$ inch thick, with two

Iron cover-plates 15 inches x $\frac{3}{8}$ inch each. The rivets were $1\frac{1}{4}$ inch *of iron and filled* 1 inch drilled holes, pitch 3 inches.

The gross area of plate was 3,765 square inches. the net area 2,510. The *total* bearing-surfaces of rivets on the plates aggregated 1,255 square inches, and their aggregate shearing areas, (being in double shear) was 7,854 square inches. At 116715 pounds strain the edges contracted and scale on the specimen began to start and the plate yielded as shown at 167200 pounds strain. This was equal to 44410 pounds tension per square inch on the uncut plate, or 66610 pounds per square inch on a line at the rivet-holes (or net area).

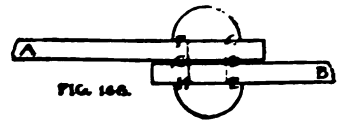


FIG. 168.

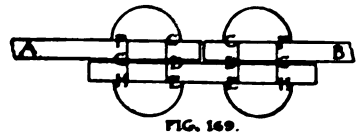


FIG. 169.

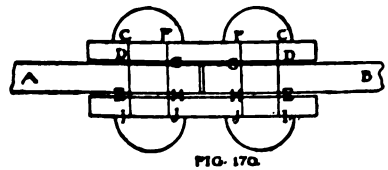


FIG. 170.

The compression from the rivets was 133230 pounds per square inch, while the shearing was only 21290 pounds per square inch.

This example shows clearly how the plate yields. Besides this the joint might yield by breaking or shearing off the rivets. We have then the following six manners in which a riveted-joint might yield.

How Riveted joint yields.

1. By crushing either the rivet or the rivet crushing the plate.
2. By shearing off the rivet — in single shear.
3. By shearing off the rivet — in double shear.
4. By bending or cross-breaking of the rivet.
5. By tearing the plate apart or crushing it between rivet-holes.
6. By the rivets shearing out the part of the plate between them and the edge.

In Figures 168, 169 and 170 are shown three kinds of joints, each with a single rivet transferring the whole strain; in Figure 168 directly from plate *A* to plate *B;* in Figure 169 transferring strain from plate *A* to cover-plate and thence to plate *B;* and in Figure

170 transferring one-half of the strain A to *each* cover-plate and thence each half is transferred back again to plate B.

It should be remarked here that cover-plates (as in Figure 169) should be at least the full width and thickness of the original plates. In practice they are made a trifle thicker (about $\frac{1}{16}$ inch or more).

Cover-plates, as in Figure 170, should *each* be the full width of original plates and at least one-half the thickness of same; in practice they too are each made about $\frac{1}{16}$ inch (or more) thicker.

The plates A and B are themselves, of course, of the same thickness.

Failure by Compression.

Now to prevent failure by the first method, compression, there must be area enough at both $G\ H$ and $C\ D$, (if plates are in tension), not to crush the rivet or the plates at these points, ($C\ D + E\ I$ and at $G\ H$ in Figure 170). This area is considered equal to the thickness of either plate A or B (or of cover-plate or their sums) multiplied by the diameter of rivet-hole.

Failure by Shearing.

To resist failure by the second method, single-shearing of rivet, the area of cross-section of each rivet must be sufficient not to shear off under the total strain on either plate A or B. It will be readily seen that only the rivets in Figures 168 and 169 are subjected to single shearing, *viz:* at their sections $G\ D$. The rivets in Figure 170 have two areas resisting shearing, $G\ D$ and $H\ E$, hence are subjected to double shearing; therefore their area of cross-section need only be sufficient to resist a shearing strain equal to only one-half of the *total* strain on either plate A or B, in order to avoid failure by the third method.

Failure by bending.

To avoid failure by the fourth method, the rivet must be sufficiently strong to resist the load as a lever in Figures 168 and 169, and as a beam in Figure 170.

In Figures 168 and 169 we can consider the part $D\ C\ F\ G$ as the built-in part of a lever, with a free end $D\ E\ H\ G$ which carries a uniform load equal to the whole strain on either plate A or B.

In Figure 170 we have a beam supported at $C\ D\ G\ F$ and $E\ I\ J\ H$, with its span or central part $G\ H\ E\ D$ loaded with a uniform load equal to the whole strain on either plate A or B.

To prevent failure by the fifth method the area of cross-section

of either plate taken at right angles across same through the rivet-hole — (that is, deducting the rivet-hole from the area of cross-section) — should be sufficient to resist the tension or compression.

Failure of plate.

To prevent failure by the sixth method the rivets must be far enough from the edges of plates (cover and original plates) not to shear out the metal ahead of them. The rule is shown in Figure 171. Make angle $A\,O\,C = 90°$ that is a right angle (0 being the centre of rivet-hole and $C\,A$ part of its circumference), and so that the directions of $O\,A$ and $O\,C$ are at 45° with edge of plate $D\,B$. Then the sums of the areas $A\,B + C\,D$ — (that is, $A\,B + C\,D$ multiplied by thickness of plate) — must be sufficient to resist the longitudinal shearing strain, which in this case would be the strain on either plate A or B (Figures 168 to 170).

Failure by rivets tearing out.

To put the above in formulæ we should have:

Bearing.
$$x = \frac{s}{d.\,h.\left(\frac{c}{f}\right)} \qquad (109)$$

Use x for lap joints only.

Use $2\,x$ in place of x for butt joints with single or double cover-plates.

Single Shearing.
$$x = \frac{s}{0{,}7857.\,d^2.\left(\frac{g}{f}\right)} \qquad (110)$$

Use x for lap joints only.

Use $2\,x$ in place of x for butt joints with single cover-plate.

Double Shearing.
$$x = \frac{s}{1{,}5714.\,d^2.\left(\frac{g}{f}\right)} \qquad (111)$$

Use $2\,x$ or butt joints with double cover-plates.

Bending-moment Lever.
$$x = \frac{s.\,h.}{0{,}1964.\,d^3.\left(\frac{k}{f}\right)} \qquad (112)$$

Use x for lap joints only.

Use $2\,x$ for butt joints with single cover-plates.

Bending-moment Beam.
$$x = \frac{s.\,h}{0{,}7857.\,d^3.\left(\frac{k}{f}\right)} \qquad (113)^1$$

[1] The fourth decimal given in formulæ is not quite right, but is made to correspond with fractions used in Table I.

Use $2x$ for butt joints with double cover-plates.

Tension on Plate. $$h = \frac{s}{b.\left(\frac{t}{f}\right)} \qquad (114)$$

Use $\left(\frac{c}{f}\right)$ instead of $\left(\frac{t}{f}\right)$ if plate is in compression.

Shearing end of plate. $$y = \frac{s}{2.h.\left(\frac{q}{f}\right)} \qquad (115)$$

(If more than one rivet use $\frac{s}{x}$ instead of s for distance of each rivet from end as shown in Figure 171.)

Where $s =$ the whole load or strain, in pounds, to be transferred from one side of the joint to the other.

Where $d =$ the diameter of rivet-hole, in inches.

Where $h =$ the thickness of plate, in inches. Where more than one plate is used, take for h the *least* aggregate sum of thicknesses of all plates acting in one direction. (The sum of cover-plates should at least equal this aggregate in thickness and should be larger, where the net b of cover-plates is smaller than the net b of connected plates.)

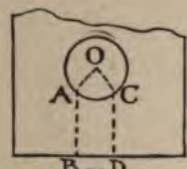


FIG. 171.

Where $b =$ the net breadth of plate, in inches, that is the breadth, less rivet-holes, at the weakest section; where more than one plate is used they should all of course be of same breadth. The net b of cover-plates will frequently be much less than that of original plates, as they lose the greatest number of rivet-holes at their centre, where they are carrying the full strain.

Where $x =$ the total number of rivets required at the joint for lap joints, and the number required *each side of joint* for butt joints with single or double cover-plates; that is in the latter two cases $2x$ will be the total number of rivets required.

Where $y =$ in inches, is the length $A\,B$ or $C\,D$ (Figure 171) from *any* rivet to free edge of plate; where more than one rivet is used, insert $\frac{s}{x}$ in formula, in place of s. It will only be necessary, of course, to calculate y for the line of rivets nearest free edge.

Where $\left(\frac{c}{f}\right) =$ safe compression stress, per square inch,

Where $\left(\frac{t}{f}\right)$ = safe tension stress, per square inch,

Where $\left(\frac{q}{f}\right)$ = safe shearing stress, per square inch,

Where $\left(\frac{k}{f}\right)$ = safe modulus of rupture, per square inch,

all in pounds, (see Table IV).

Safe Stresses on Rivets and Pins. The writer uses the following values, as a rule for *rivets and pins.*

For Wrought-iron.

$\left(\frac{c}{f}\right) = 12000$ pounds, per square inch.

$\left(\frac{t}{f}\right) = 12000$ pounds, per square inch.

$\left(\frac{q}{f}\right) = 8000$ pounds, per square inch.

$\left(\frac{k}{f}\right) = 15000$ pounds, per square inch.

For Steel.

$\left(\frac{c}{f}\right) = 15000$ pounds, per square inch.

$\left(\frac{t}{f}\right) = 15000$ pounds, per square inch.

$\left(\frac{q}{f}\right) = 10000$ pounds, per square inch.

$\left(\frac{k}{f}\right) = 18000$ pounds, per square inch.

Example I.

Lap Joint. *A wrought-iron plate, which cannot be over* 12″ *wide, is required to be so long that it has to be made up from two lengths; the joint is to be a lap joint. The plate is in tension and is strained* 65000 *pounds. Design the joint.*

We will assume that we propose to design the joint as shown in Figure 165, with staggered rivets, in that case the plate will only be weakened by one rivet-hole. We can readily see that the plate will not need to be very thick and decide to use $\frac{3}{4}$″ rivets, (that is $\frac{3}{4}$″ rivet-holes)[1]; we then shall have a net breadth of plate

Size of plate. $b = 12'' - \frac{3}{4}'' = 11\frac{1}{4}''$.

Of course $s = 65000$ in this case, and we know that

$$\left(\frac{t}{f}\right) = 12000;$$

inserting these values in Formula (114) we have:

[1] Before heating, in this case, the rivets would be $\frac{11}{16}''$.

$$h = \frac{65000}{11\frac{1}{4}.\ 12000} = 0{,}485''$$

Or we should use a plate one-half inch thick.

We next determine the number of rivets required.

In the first place there must be enough for bearing, that is not to crush the plate or get crushed by it. We use Formula (109) inserting the values, and have:

Required number of Rivets.

$$x = \frac{65000}{\frac{3}{4}.\ \frac{1}{2}.\ 12000} = 14{,}44$$

Or we should need 15 rivets for bearing. Had we figured without using the formula, we should have said, the bearing area of each rivet is $\frac{3}{4}''$ by $\frac{1}{2}'' = \frac{3}{8}$ square inches, this at 12000 pounds, per square inch, would equal 4500 pounds safe-compression for each rivet, dividing this into 65000 pounds, the strain, would, of course, give the same result 14,44 or say 15 rivets.

We next see if there is any danger from shearing. The joint being a lap joint, the rivets will have, of course, only one sectional area to resist shearing, that is, will be in single shear, so that we use Formula (110) and inserting values have:

$$x = \frac{65000}{0{,}7857.\ (\frac{3}{4})^2.\ \left(\frac{g}{f}\right)} = 18{,}39$$

Or we must use 19 rivets to prevent the shearing.

Had we figured without the use of formula, we should have said, area of a $\frac{3}{4}''$ rivet is $= 0{,}4417$ square inches. This multiplied by 8000 pounds (the safe shearing stress per square inch) $= 3533{,}6$ pounds, or each rivet could safely assume this amount of the strain without shearing. This amount being *less* than the safe compression on each rivet, would, of course, require a larger number of rivets, and should therefore be used, rather than the latter. We have, in effect $\frac{65000}{3533{,}6} = 18{,}39$ or say 19 rivets, being four more than required for bearing.

We next take up the question of bending; the joint being lapped the rivets will practically become short levers. We use Formula (112); inserting values, we have:

$$x = \frac{65000.\frac{1}{2}}{0{,}1964.\ (\frac{3}{4})^3.\ 15000} = 26{,}15$$

Or we should have to use at least 26 rivets to prevent bending;

which readily illustrates the great disadvantage of not transferring the strain in a direct plane, by using two cover-plates.

Had we not used the formula, we should have said, we have here a $\frac{3}{4}''$ circular lever, the free end projecting $\frac{1}{2}''$ and loaded uniformly with a load of 65000 pounds.

From Formula (25) or Table VII we have the bending-moment

$$m = \frac{65000 \cdot \frac{1}{2}}{2} = 16250 \text{ pounds-inch.}$$

and from Table I, section No 7 the moment of resistance,

$$r = \frac{1}{14} \cdot (\tfrac{3}{8})^3 = 0{,}04143$$

From the formula on page 49, Volume I,

$$\frac{m}{r} = s$$

we have the total extreme fibre strains on all the rivets,

$$s = \frac{16250}{0{,}04143} = 392228 \text{ pounds.}$$

This divided by the safe strain, or safe modulus of rupture $\left(\frac{k}{f}\right) = 15000$ pounds, will give the number of rivets required, *viz:*

$$\frac{392228}{15000} = 26{,}15$$

or 26 rivets as before.

We still have to decide the distance y (or $A\,B$, Figure 171). We use Formula (115). As we have more than one rivet we use in place of s the strain on each rivet or $\frac{s}{26}$, which was the largest number required as above, therefore:

$$\frac{s}{26} = \frac{65000}{26} = 2500$$

Inserting this in Formula (115) we have:

$$y = \frac{2500}{2 \cdot \frac{1}{2} \cdot 8000} = 0{,}3125''$$

This, however, being less than our rule which requires $1\frac{1}{2}$ diameters from centre of hole to edge, we will stick to the rule.

We now design the joint.

Designing the joint.

We have a plate 12″ wide, $\frac{1}{2}''$ thick, lapping, and require 26 rivets. They must be arranged not to weaken the plate by more than one rivet-hole.

If we arrange the rivet-holes as shown in Figure 172, we will find it the most economical arrangement. To be sure it allows for only

25 rivets, but that will probably be near enough, otherwise we should have to insert at least five more to keep them symmetrical. It will be readily seen that the weakest point is at section *A* where one rivet-hole is lost.

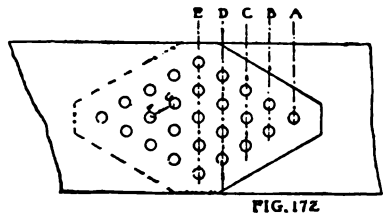


FIG. 172

Section *B* is of same strength, two rivet-holes being lost, but the strain has been reduced by an amount equal to the value of one rivet-hole.

At section *C* we lose three rivet-holes, but the strain has been reduced by the value of three rivet-holes, so that the plate practically has its full value here.

At sections *D* and *E* the plate is stronger to resist the remaining tension than required.

By figuring out *E* (12″ wide) it will be seen that the pitch on this line is more than required by the rule, Formula (108).

The pitch *F G* between two adjacent lines of rivets, measured on the slant from centre to centre of rivets, should be at least $2\frac{1}{2}$ diameters, $2\frac{1}{2} \cdot \frac{3}{4} = 1\frac{7}{8}''$ or say 2″.

It will be good practice for the student to carefully lay this joint out to scale.

Example II.

Butt joint single cover-plate. *A steel plate* 10″ *wide has to be pieced, and for local reasons this can only be done by a cover-plate on one side. The plate is subjected to a tensional strain of* 135000 *pounds. Design the joint.*

Of course, the rivets must be of steel too.

We will again assume that we can stagger the rivets, so that we shall lose only one rivet-hole. The plate will evidently have to be thick and we will decide to use 1″ rivets; this would leave us a net breadth of plate

Size of plate. $b = 10'' - 1'' = 9''$

From Formula (114) we have:

$$h = \frac{135000}{9.15000} = 1''$$

Or the plate will have to be just one inch thick. The cover-plate

should be at least the same, if there were only one rivet, but as there will evidently be more than one and we propose staggering the rivets, the cover-plate will have to be considerably thicker; we can therefore leave the cover-plate out of consideration for the present as, being thicker, or in case of one rivet equal to the plates to be joined, it will certainly be as strong, and not crush.

Required number of rivets. Now, as for the number of rivets, from Formula (109) we have for bearing:

$$x = \frac{135000}{1.1.15000} = 9$$

Or nine rivets are required not to crush the plate or be crushed by it (each side of joint).

From Formula (110) we have for single shearing (as there is evidently only one area to each rivet to resist shearing):

$$x = \frac{135000}{0{,}7857.1^2.10000} = 17{,}2$$

Or seventeen rivets are required to resist the shearing (each side of joint). The rivets will evidently be levers in this case and we have from Formula (112)

$$x = \frac{135000.1}{0{,}1964.1^3.18000} = 38{,}1$$

Or it will require thirty-eight rivets to resist the bending-moment (each side of joint). This again shows the advantage of using both a top and bottom cover-plate and so avoiding the great leverage on the rivets.

Designing the joint. It must now be borne in mind that the joint is a butt joint, therefore, unlike the case of lap joint where the whole number of rivets bear on each plate, we must here use twice the number, as only one-half, or those each side of the joint bear on one plate, or we require in all 76 rivets.

The plate is so narrow that we cannot get more than three rivets on a line across the plate, without infringing on the rules for pitch. We can, therefore, stagger the end rivets and make the rest either chain riveted or zig-zag riveted. The chain riveting will require a little longer cover-plate, but in the case of a plate-girder flange would have the advantage of using the rivets that are needed for the angle-irons.

In that case we should not stagger the end rivets, for our plate would only be weakened by one additional hole, and, of course, the

gross breadth of plate would have to be 12 inches instead of 10 inches to give same strength.

Figure 173 shows this joint chain riveted, with end rivets, staggered, to correspond to our calculated example.

We see that it takes 39 rivets each side of joint for symmetry.

Figure 174 shows this joint zig-zag riveted. It has three advantages over the other, it is shorter, takes just the right number of

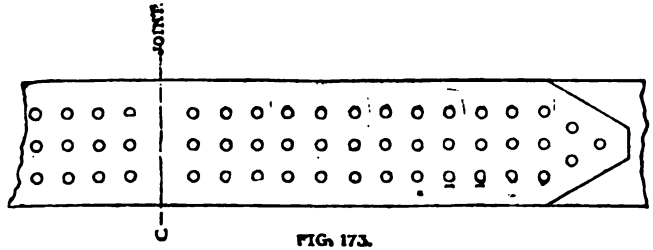


FIG. 173.

rivets and requires a thinner cover-plate. For in Figure 173 at the first line of rivets next to the joint (C), the cover-plate loses three rivet-holes and bears the full strain, or its clear breadth would be

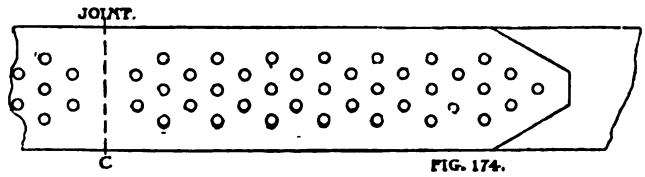


FIG. 174.

only 7 inches and from Formula (114) would require a cover-plate of thickness

$$h = \frac{135000}{7.15000} = 1{,}3$$

or say $1\frac{3}{16}''$ thick.

Whereas, in Figure 174, next to the line C we lose only two rivets and have consequently a clear breadth of 8 inches and require from Formula (114) a cover-plate of thickness equal to

$$h = \frac{135000}{8.15000} = 1{,}125$$

or only $1\frac{1}{8}$ inch cover-plate.

Had we not used the formula we should have figured out our rivets, etc. as follows:

Required net or clear area of plate

$\frac{135000}{15000} = 9$ square inches. Net breadth being 9" gives, of course, 1" thickness.

Bearing area of each rivet $= 1.1 = 1$ square inch, which will safely bear 15000 pounds or we should need

$\frac{135000}{15000} = 9$ rivets.

Single shearing area of each rivet (or area of a circle 1" diameter) $= 0,7854$ square inches, which at 10000 pounds per square inch, would give a resistance to shearing per rivet $= 7854$ pounds or we should need

$\frac{135000}{7854} = 17,2$ rivets. For bending we should have a one inch circular lever, projecting one inch and uniformly loaded with 135000 pounds.

The bending-moment would be Formula (25)

$$m = \frac{135000.1}{2} = 67500 \text{ pounds.}$$

The moment or resistance would be Table I, section No. 7

$$r = \frac{1}{12}.(\tfrac{1}{2})^3 = 0,0982$$

The strain s, therefore, on all the rivets will be [page 49, Volume I]

$$s = \frac{67500}{0,0982} = 687373$$

This divided by the safe modulus of rupture for steel rivets $\left(\frac{k}{f}\right) = 18000$ will give the required number, as before,

$$\frac{687373}{18000} = 38,2$$

Example III.

Butt-joint Two Cover-plates. *Same problem as before, but two cover-plates to be used, one above and one below the joint.*

We will again decide to stagger the rivets and losing only one rivet-hole will again require a 1" thick plate.

Now, for bearing we will have the same result as before, *viz.:* 9 rivets required, but in shearing it is evident that now each rivet has two resisting areas or is in double shear, so that we will need only one-half of the previous quantity or $\frac{17,2}{2} = 8,6$ or say, nine rivets. Had we used Formula (111) we should have obtained this result, for:

Required number of rivets.

$$x = \frac{135000}{1,5714.\ 1^2.\ 10000} = 8,6$$

For the bending-moment we use Formula (113) and have

$$x = \frac{135000.\ 1}{0,7857.\ 1^3.\ 18000} = 9,5$$

or say ten rivets required to resist cross-breaking. This shows how much better proportioned the different strains (shearing, bending and bearing) are to each other, where we use two cover-plates. Had we calculated directly without use of formula we should have obtained the same results. We have already calculated net area of plate and bearing value of rivets in *Example II*, also the single shearing value, which was 7854 pounds per rivet; as we now have two areas this would be doubled or the resistance of each rivet to shearing would be 15708 pounds and the number required $\frac{135000}{15708} = 8,6$ as before.

Advantage of Two Covers.

For bending-moment we should have a 1″ circular beam with a clear span of 1″, uniformly loaded with 135000 pounds.

From Formula (21) we have the bending-moment

$$m = \frac{135000.\ 1}{8} = 16875 \text{ pounds-inch}$$

Moment of resistance will be as before

$$r = 0,0982$$

Therefore the total strain on all the rivets

$$s = \frac{16875}{0,0982} = 171843$$

This divided by $\left(\frac{k}{f}\right) = 18000$ gives the required number of rivets as before

$$x = \frac{171843}{18000} = 9,5$$

We now design the joint, as before, remembering to stagger the rivets and to place the required number each side of joint. The greatest number required was to resist cross-breaking, *viz.*, ten.

Designing the joint.

We can design as shown in Figure 175 or as shown in Figure 176; both require eleven rivets each side of joint, but cover-plates in Figure 176 need only aggregate $1\frac{1}{8}$" in thickness, that is, be $\frac{9}{16}$" thick each; while those in Figure 175 would have to aggregate $1\frac{5}{16}$" in thickness, or be, say, $\frac{11}{16}$" thick each.

Joint shown in Figure 175 looks a little better, but otherwise there is no preference.

If cover-plates are not equal in thickness each side of plate, it would require very many more rivets. Each rivet would become a double lever, with its central part built-in and a projecting free arm each side, the length of arms being equal to their respective thicknesses of cover-plates. The load on each arm would be the proportion of whole strain, that the thickness of its respective cover-plate would be of the whole required (aggregate) thicknesses of cover-plates.

Covers of same thickness.

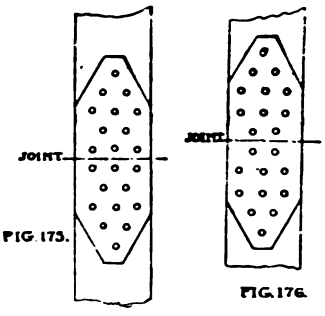


FIG. 175. FIG. 176.

There would be no sense in such an arrangement however. It would produce all sorts of unequal stresses, in shearing, bearing, cross-breaking, etc., and should be avoided. Riveted work at best is very theoretical, as the calculations depend entirely upon the accuracy and fit of each rivet. If a single rivet fails to do its share, it will at once disarrange all the strains and produce unequal stresses in different parts. Still if the above rules are followed, riveted work can be used with perfect safety. Where the result gives a fraction, a whole rivet should as a rule be used in place of the fraction. If the necessary spacing requires still more rivets, they can either be used, or, all the rivets can be reduced in size enough to bring them nearer to the allowable stresses.

No account has been taken of the loss due to punching, for this will affect the plate in tension mainly, and the safe stresses allowed for tension are very low. In compression the metal would not be strained as heavily as in tension, for the rivets will not weaken the plate so much if they entirely fill the holes, thus giving full bearing on the entire plate. Then, too, the butt joint if planed and carefully

made and joined, will transfer directly more or less of the compression.

No reliance on butt.

But in all good work it is customary to place no reliance whatever on the butt, and to calculate in compression the same as for tension, namely, sufficient *net* area in each plate, at its weakest section, to resist the whole compression strain.

Explanation of Tables.

Tables XXXV to XL inclusive, have been calculated and laid out to save most of the wearisome figuring necessary in riveted work and in connection with pins. The first three give the bearing value of pins and rivets against eye-bars or plates, and the latter three the values in tension, single and double shearing and in cross-breaking of pins and rivets. All the tables are laid out for both steel and wrought-iron.

Tables xxxv, xxxvi, xxxvii.

The full heavy straight lines in Tables XXXV, XXXVI and XXXVII represent the thicknesses of plates or eye-bars against which the different sizes of rivets or pins bear. The thicknesses given are from $\frac{1}{4}''$ to $1\frac{1}{2}''$ in Table XXXV and from $\frac{1}{4}''$ to $2''$ in Tables XXXVI and XXXVII; all by $\frac{1}{16}''$. For thicker plates or eye-bars it will only be necessary to increase the bearing value found, in proportion to extra thickness.

The columns to the left give the diameters of pins and rivets, running in Table XXXV from $\frac{1}{4}$ inch diameter (by $\frac{1}{32}$ inch) to 1 inch diameter; in Table XXXVI from 1 inch diameter (by $\frac{1}{16}$ inch) to 3 inches diameter; and in Table XXXVII from 3 inches diameter (by $\frac{1}{8}$ inch) to 6 inches diameter. The figures at the *tops* of these tables give the bearing values in pounds for wrought-iron, and those along the *bottoms*, the bearing values in pounds for average steel.

The full heavy curved lines in Tables XXXVIII, XXXIX and XL give the single and double shearing values for the same sized pins and rivets as in the previous three tables.

Tables xxxviii, xxxix, xl.

The additional vertical columns to the left in these tables give the areas of cross sections in square inches, of the different sized pins and rivets, which multiplied by ten give their weights in pounds per yard of length for wrought-iron, (for steel add 2 per cent to the weight of wrought-iron). There are also full heavy curve lines giving the strength, in tension, of tie-rods of same diameter as pins or rivets. The values selected for these curves are those *always* used by the writer in calculating pins, rivets or tie-rods.

For different values.

It sometimes becomes desirable in temporary work to use higher values, or in very important permanent structures with moving loads to use lower values. But even in such cases the tables can be used, for, as all of these curves are directly dependent on the area (or double area) of cross section of the rivet or pin, they can, of course, be used interchangeably. That is, any one who wishes to figure the safe shearing $\left(\frac{s}{f}\right)$ for steel as $= 13000$ pounds, instead of 15000 pounds, can take the curve marked "13000 pounds — tension steel," in any of the three tables.

Or, if he wishes to figure his iron at only 10000 pounds safe stress for tension, that is $\left(\frac{t}{f}\right) = 10000$ pounds, he will, instead of using the curve marked "12000 pounds — tension wrought-iron," take the curve marked "10000 pounds — single shear steel."

The writer, however, always sticks to the one set of values for tension, and for pins and rivets to those given in tables (also after Formula 115), as they are certainly *safe* values, and yet not low enough to make the work excessively costly. Iron contractors will frequently quote off-hand opinions of celebrated engineers saying these values are much too low and thus backing up their economical tendencies in trying to add lightness (and beauty) to roof trusses and plate girders, but as a rule the opinions are either not authentic, or else it is found that the celebrated engineer, when delivering the opinion, had a similar axe to grind, as had the contractor. Good engineers as well as good architects, will attempt to save their clients all they can, but hardly at the risk of taking chances in their most important constructive works. The figures along the *tops* of Tables XXXVIII to XL give the values of either iron or steel, according, of course, to which curve is used.

Curve of Bending-moment.

The dotted curved lines in the Tables XXXVIII to XL give the safe bending-moments for iron and steel pins and rivets, and for these the *lower* horizontal lines of figures are used.

How to use Tables.

The use of the tables is simple, similar to the other tables with curves. Thus if we have a $\frac{5}{8}$" steel rivet bearing against a $\frac{7}{16}$" steel plate, we use Table XXXV and pass horizontally to the right from the vertical figure (or

diameter marked) $\frac{5}{8}''$ till we strike the (fourth) slanting full bearing line, marked $\frac{7}{16}''$. This is one-third way between the vertical lines marked (*below* for steel) 4000 and the next vertical unmarked line to the right; as each vertical space at the bottom (steel) evidently is one-fourth of a thousand or 250 pounds, a third space will, of course, be 83 pounds, or a $\frac{5}{8}''$ steel rivet bearing against a $\frac{7}{16}''$ steel plate will resist safely 4083 pounds. Had we calculated arithmetically we should have had

Bearing Values.

$\frac{5}{8} \cdot \frac{7}{16} \cdot 15000 = 4101$ pounds.

Had the rivet and plate been of wrought-iron we should have used the *upper* line of figures; here the intersection is one-third way between the second and third unmarked lines after 3000; as there are five spaces above between each thousand, each space evidently represents one-fifth of a 1000 or 200 pounds for iron, so that the bearing value for a $\frac{5}{8}''$ iron rivet against a $\frac{7}{16}''$ iron plate would be $3000 + 200 + \frac{1}{3} \cdot 200 = 3267$ pounds. By calculation we should have had $\frac{5}{8} \cdot \frac{7}{16} \cdot 12000 = 3281$ pounds.

The single shearing value of a $\frac{5}{8}''$ steel rivet at $\left(\frac{q}{f}\right) = 10000$ pounds would be $= 3067$ pounds; for, the horizontal line $\frac{5}{8}''$ in Table XXXVIII strikes the single shearing curve for steel about one-third way between vertical line marked at the *top* 3000 and the next unmarked line to the right. Each space evidently represents 200 pounds, so that we should have for the steel $\frac{5}{8}''$ rivet in single shear $3000 + \frac{1}{3} \cdot 200 = 3067$ pounds. By arithmetical calculation we should have had the area of a $\frac{5}{8}''$ circle multiplied by 10000 pounds or

Shearing Values.

$0{,}3068 \cdot 10000 = 3068$ pounds.

The double shearing value of a $\frac{5}{8}''$ rivet would be double this, or $= 6136$ pounds, and this is confirmed by the Table (XXXVIII), as the horizontal line $\frac{5}{8}''$ strikes the double shearing curve for steel 20000 pounds about two-thirds way between vertical line marked *above* 6000 and its next unmarked line to the right, or

$6000 + \frac{2}{3} \cdot 200 = 6134$ pounds.

For the safe bending-moment we find the horizontal line $\frac{5}{8}''$ strikes dotted curved line marked "safe bending-moment on steel at 18000 pounds" on the second vertical unmarked line to the left of the one marked *at the bottom* 500; each space below is evidently 20 pounds, and as they increase to the left we must add the two spaces, or

$500 + 40 = 540$ pounds, which would be the safe bending-moment on a $\frac{7}{8}$" steel rivet or pin.

Or to illustrate further take the last *Example* (*III*). We had a 1" steel plate and 1 steel rivet. The actual bending-moment we found was

Bending-moment Curve.

$$\frac{u.\,l}{8} = \frac{135000.\,1}{8} = 16875$$

From Table XXXVIII we see that the safe bending-moment on a 1" steel rivet is the intersection of horizontal line 1" being one-third way between third and fourth vertical unmarked lines to the left of $1700 = 1700 + 3\frac{1}{3}.\,20 = 1767$ pounds-inch.

Dividing the actual bending-moment at the joint 16875 pounds-inch, by the safe bending-moment on each rivet, will, of course, give the number of rivets required, or

$$\frac{16875}{1767} = 9{,}55$$

being the same result as before, namely, 10 rivets. Take *Example II*, we had the same rivet, plate and strain, but there was but one cover-plate and the rivet was practically a cantilever.

The bending-moment was

$$\frac{u.\,l}{2} = \frac{135000.\,1}{2} = 62500 \text{ pounds-inch.}$$

This divided by the safe bending-moment on each rivet (1767 pounds-inch) as found in Table XXXVIII gives the number of rivets required, or

$$\frac{62500}{1767} = 35{,}4 \text{ or same as before.}$$

The use of Tables for pins or tie-rods, is, of course, exactly similar to use for rivets, as already described.

Bearing on pins. For instance, we have a 2" pin bearing against a $1\frac{1}{8}$ inch thick eye-bar or head of a tie-rod. We use Table XXXVI, the 2 inch horizontal line, strikes the full slanting line marked $1\frac{1}{8}$ inch exactly on one of the vertical lines. If both pin and eye-bar are wrought-iron — (or if either were wrought-iron we should use the smaller value) — we use the *top* line of figures, or we find our vertical line the third to the right of 24000 and as each space evidently represents 1000 pounds, the safe bearing value of our 2 inch pin against a $1\frac{1}{8}$ inch thick eye-bar would be 27000 pounds, if one or both are of wrought-iron. Had both pin and eye-bar been of steel, we should have used the bottom line of figures, where each space

evidently represents 1250 pounds, and we should have had 33750 pounds.

Shearing of Pins.

The safe shearing value for a 2 inch wrought-iron pin, would be from Table XXXIX = 25000 pounds, and for steel 31670 pounds.

By calculation we should have had for iron = 25133 pounds and for steel = 31416 pounds.

Tension on Rods.

If we had a 2 inch circular tie-rod its strength in tension from Table XXXIX would be, if of wrought-iron = 37600 pounds and if of steel = 47000 pounds.

By calculation we should have had for iron = 37752 pounds and for steel = 47190 pounds.

Calculation of Pins.

Pins are calculated similarly to rivets. The same formulæ can be used for bearing area (109) h being the thickness of head of eye-bar, in inches, and for single shear (110) and double shear (111).

For bending where there are but two eye-bars or rods, each pulling in opposite directions, Formula (112) could be used; using for h the thickness, in inches, of either rod; where there are two eye-bars or rods pulling in one direction with only one between them pulling in the opposite direction Formula (113) might be used; h in this case being the thickness, in inches, of the large (central) single rod, and twice the thickness of either of the smaller rods. In all of the formulæ x should be = 1.

As a rule, however, there are several rods at each pin-connected joint, and the bending-moment has to be carefully calculated for each special case. In designing pin-joints this should be borne in mind, and the pieces with largest strains brought as closely together, as

Designing Pin-joint.

possible, to avoid excessive bending-moments, which will require very large pins, and necessitate large eyes and heads to the rods and bars, which means, of course, very great expense; pin-connections for small trusses are more expensive than riveted joints; for large trusses they are

Pin-joint best.

cheaper. They are very much better in both cases, more easy to transport and erect, and agree much nearer with the theoretical calculation, which assumes that all members around a joint are free to move, and not rigid, as is the case in riveted trusses. Then, too, in case of any movement in the truss, it can be readily adjusted by means of nuts, swivel-links,

sleeve-nuts, etc. This cannot be done in a riveted truss, as the joints not being flexible, the tightening of any one part might throw strains on some joint not able to bear it.

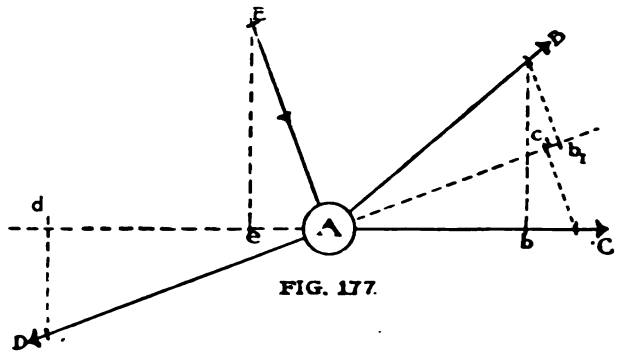


FIG. 177.

In calculating a pin-joint, the strength of pin, as a rule, should be calculated in the several lines or directions in which it is being strained, that is first along the line of one rod, then along the line of another rod (or compression piece) and so on. In doing this all the other strains must be projected (and reduced) to the same line. If this is correctly done, it will be found in every case, that the *sum* of all the strains acting in one direction along this line will be equal to the *sum* of all acting in the opposite direction along the line. Thus in Figure 177 we have the end elevation of a pin *A* strained by three forces in tension *B*, *C* and *D* and one force in compression *E*.

Strains reduced to same line.

We will first examine the pin along the line *A C*. Make, at any convenient scale, the lines *A C*, *A B*, *A D* and *A E*, each in length just equal to the amount of strain they are subjected to. Project them all on to *A C*, then will *A b* represent the resultant or equivalent of strain *A B* along the line and in the direction *A C*. Similarly *A d* will be the resultant of *A D* along the line of *A C*, but it will be in the opposite direction; *e A* will be the resultant of *E A* along the line *A C*, but it will be noticed that though on the opposite side of the pin it is in the same direction as *A C*.

Were we to examine the pin along the line $A\ D$, we should have the strain of $A\ D$ in one direction and in the opposite directions the resultant strains $A\ c_1$ and $A\ b_1$. It will be noticed that $E\ A$ has no resultant along the line $A\ D$, being normal to it; and it need not be considered, when calculating the pin along the line $A\ D$. We might still calculate the pin along the line $A\ E$; we should then have the strain $E\ A$ in one direction and in addition thereto the resultant of $A\ C$; in the opposite direction we should have the resultant of $A\ B$. The largest strain $A\ D$ has no resultant along the line $A\ E$. It is evident, however, that all of these resultants will be very small as compared to those along the other lines, and therefore need not be calculated.

Arrangement of Heads.

In Figure 178 we have a plan of the pin, showing in plan the strains and resultants acting along the line $A\ C$. As arranged in this figure the pin evidently becomes a double-ended cantilever, supported at the fulcrum d, and loaded on one free end with the load c and on the other free end with two loads b and e.

If the strains were small, this would probably be the most economical arrangement, as $A\ D$ could then be made in one piece. But if the strains were heavy it would require a tremendous pin.

In the latter case the arrangement shown in Figure 179 would be better. Here the pin becomes a beam, supported at both ends by $\frac{d}{2}$ (or one-half of the force $A\ D$); the beam carries three loads c, e and b. In this case $A\ D$ would be made up of two rods, separated. The forces c, e and b must be so distributed as to make the least

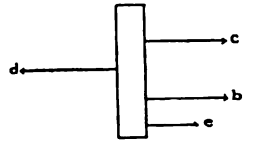


FIG. 178.

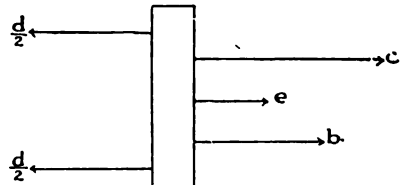


FIG. 179.

bending-moment on the beam or pin, that is, the larger ones placed nearer the supports or outer edges, and the smaller ones in the centre. Frequently it would be more economical to divide c the

larger force into two halves and place them immediately next to $\frac{d}{2}$. The small force e is frequently put on the outside, as it is not sufficient to affect the beam or pin seriously and only enlarges the span of beam, that is, length of pin or distance between supports $\frac{d}{2}$ thus greatly increasing the bending-moment. This arrangement is shown in Figure 180. Here we have a beam supported at two points $\frac{d}{2}$ with a free end carrying load e; the beam being loaded with three loads, two each $= \frac{c}{2}$ and one $= b$.

Pin with Unsymmetrical Strains.

To calculate the bending-moment at any point of this pin we have to consider the end of pin farthest from e as built in solidly (and after getting reactions) we should multiply all the forces to the right or left of the point into their distances from the point. If we select the forces on the right we deduct the sum of the moments of those (right hand forces) acting in one direction from the sum of the moments of those (right hand forces) acting in the opposite direction, the difference will be the bending-moment at the point. To check the calculation we take all the forces on the left side of the point. But the reactions will not be $\frac{d}{2}$ each as shown in the table unless e were divided and one-half put at each end of pin.

Forces act through Central Axes.

In every case of pin calculation, excepting where the forces form simple beams, as in Figure 179, the forces are supposed to act through their central axes, that is through a line drawn at half the thickness of their heads and at right angles to the pin. The heads are frequently thickened up, that is, made thicker than the rod or flat bar, in order to get the necessary bearing surface on the pin, but it will be readily seen that this lengthens the pin greatly, thus largely increasing the bending-moments. It is better, as a rule, to get the extra bearing surface by dividing the rod or flat bar into two or more parts and then distributing them along the

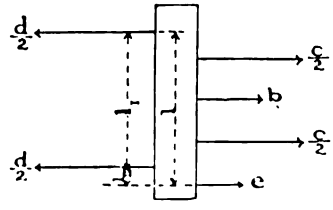


FIG 180.

pin symmetrically and at the most favorable points to avoid bending-moment.

Forces arranged Unsymmetrically.

If the pin is arranged as shown in Figure 180 and we call the different lengths along the pin, as shown, l, l, and l_{II} the reaction nearest e will be

$$= \frac{c+b}{2} + \frac{l}{l_I} \cdot e$$

and the further reaction will be

$$= \frac{c+b}{2} - \frac{l_{II}}{l_I} \cdot e$$

In practice, however, e would probably be so small and the convenience so great in making the two bars $\frac{d}{2}$ alike, that the unequal stresses caused by e in the two $\frac{d}{2}$ would probably be overlooked. But in calculating the bending-moment on pin, they will have to be considered as unequal, otherwise, the bending-moments calculated from the right hand or left hand of any point would not agree.

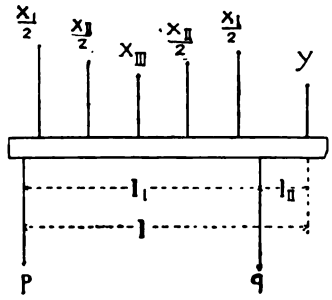


FIG. 181.

To put the above in formulæ we should have the following:

If a pin $p\,q$ is strained by a number of forces x_I, x_{II}, etc., in one direction, the forces p and q in the opposite direction will equal the reactions of a beam, see Formulæ (14, 15, 16 and 17).

If a single force y is placed beyond one of these resisting forces (say q Figure 181) the *additional reaction* on the nearer force (q) will be

Nearer Reaction Force.

$$q_I = + \frac{l}{l_I} \cdot y \qquad (116)$$

and on the further force

Further Reaction Force.

$$p_I = - \frac{l_{II}}{l_I} \cdot y \qquad (117)$$

or, we must add q_I to q and substract p_I from p to get the real strains or reactions in the tie-rods p and q.

Where q_I = the strain, in pounds to be added to nearer strain q owing to force y being placed on the (q) nearer free end of pin.

Where p_1 = the strain, in pounds, to be deducted from further strain p owing to force y being placed on the other further free end of pin.

Where q = the force (reaction) at q, in pounds, resisting the forces x_1, x_{11} etc., (see Formulæ 14, 15, 16 and 17) and to which q_1 must be added.

Where p = the force (reaction) at p, in pounds, resisting the forces x_1, x_{11} etc., (see Formulæ 14, 15, 16 and 17) and from which p_1 must be deducted.

Where x_1, x_{11} = the forces, in pounds, acting in opposite direction to p and q, and all projected to line p and q as shown in Figure 177.

Where l, l_1, l_{11} = the respective lengths, or distances, in inches, measured along pin, from *centre lines to centre lines* of respective pins, as shown in Figure 181.

As the forces x_1, x_{11} etc., should *always*, if possible, be located along pin to make the resisting forces p and q even, which is done by putting the smallest force (x_1) in the middle and dividing the others, we should have, where this is the case:

Nearer Reaction Force.
$$q = \frac{\Sigma . x}{2} - \frac{l}{l_1} y \qquad (118)$$

Further Reaction Force.
$$p = \frac{\Sigma . x}{2} - \frac{l}{l_1} y \qquad (119)$$

Where $\Sigma . x$ = the sum of all the opposing forces (x_1, x_{11}, x_{111} etc.) to and between q and p, in pounds, provided that they are divided and the halves or fractions located symmetrically with respect to q and p.

Where q = the total resisting force (reaction), in pounds, or strain on q_1, the reaction furthest from free end.

Where p = the total resisting force (reaction), in pounds, or strain on p_1, the reaction next to free end.

Where l, l_1, and l_{11} = same as in Formulæ (116 and 117); of course, all the forces must be projected to one line as shown in Figure 177.

If one of the forces, x_1, x_{11}, etc., were = y and were placed to the left of q (Figure 181) the forces q and p would be equal and each = one-half of the sum of all the other opposing forces, provided always that the forces x_1, x_{11}, etc., are symmetrically located in respect to p and q. Where there are a number of forces on both sides of pin, the pin might be treated as a continuous girder (see Table XVII.

Volume I), but the calculation would become very difficult and the parts of all rods (or compression piece) acting in the same lines and direction would become very irregular.

Locate Forces Symmetrically.

It is customary, therefore, to locate the forces along the pin symmetrically, regardless of their true resistances as they would be if treated as continuous girders, and to consider, that each takes its *proportionate load* (according to the thickness of its head) of the whole load along its respective line and direction. In each case it will require special study to obtain the most economical arrangement.

An example will more fully illustrate the method of calculating pins.

Example IV.

Calculation of Pin-joint.

The joint A of a pin-connected wrought-iron truss is strained by the following members: The tie-rod $B = -28000$ *pounds: the tie-rod* $C = -70000$ *pounds; the strut* $D = +20000$ *pounds; and the tie-rod* $E = -88000$ *pounds. All as shown in Figure* 182; *design the joint.*

We will assume that for certain reasons we wish to use a $2\frac{3}{4}''$ diameter pin. Now the first thing to do is to settle the thickness of the (eye) heads. These, of course, must have sufficient bearing against pins not to crush the pin or be crushed by it. We use therefore the following formula:

Thickness of Heads.

$$t = \frac{s}{d.\left(\frac{c}{f}\right)} \qquad (120)$$

Where t = the necessary thickness of eye-bar head, in inches.

Where s = the strain on each eye-bar, in pounds.

Where d = the diameter of pin, in inches.

Where $\left(\frac{c}{f}\right)$ = the safe-compression stress, per square inch, of the material of eye-bar or pin (whichever is the weaker in resisting crushing should be used.)

Accordingly we should have for thickness of the different eye-bar heads in our example the following:

$$A\,B = \frac{28000}{2\frac{3}{4}.12000} = 0{,}85 \text{ or say } \tfrac{3}{4}''$$

$$A\,C = \frac{70000}{2\frac{3}{4}.12000} = 2{,}12 \text{ or say } 2''$$

$$D\,A = \frac{\text{[illegible]}}{2\frac{3}{4} \cdot \text{[illegible]}} = 0.6 \text{ or say } \frac{3}{4}''$$

$$A\,E = \frac{\text{[illegible]}}{2\frac{3}{4} \cdot \text{[illegible]}} = 2.67 \text{ or say } 2\tfrac{3}{4}$$

The values for above thicknesses have been rather too broadly rounded off, but this is done to simplify the subsequent calculations.

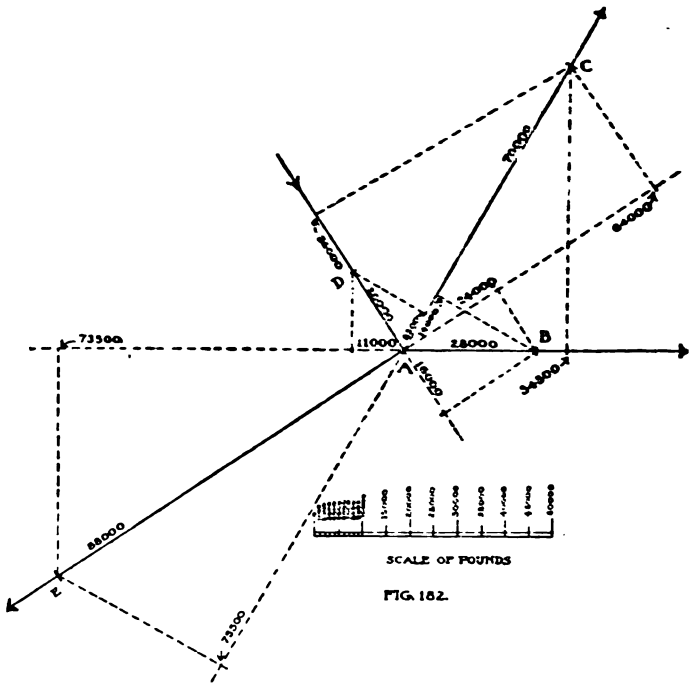


FIG. 182.

Had we used Table XXXVI we should have had the same results.

Thickness by Table XXXVI.

For *A B* (28000 pounds) the horizontal line $2\frac{3}{4}''$ and vertical line 28000 (from *above* for iron) meet between the heavy bearing lines $\frac{13}{16}''$ and $\frac{7}{8}''$, for convenience, however, we will call it $\frac{3}{4}''$ though this should not, of course, be done in a real calculation.

For *A C* (70000 pounds) horizontal line $2\frac{3}{4}''$ and vertical line 70000 from above (the second to the right of 68000) meet just be-

yond the 2″ heavy bearing line. It should be, therefore, a little over 2″ thick, but we will call it even 2″.

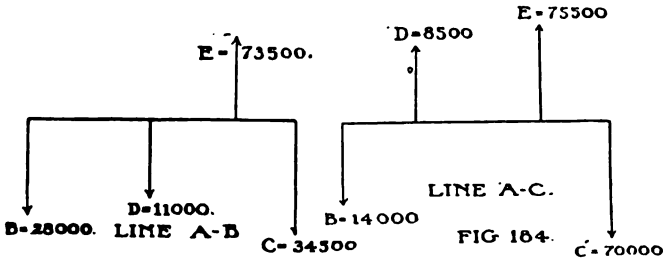
For $D\,A$ (20000 pounds) the horizontal line $2\frac{3}{4}$″ and vertical line 20000 from above, meet between heavy bearing lines $1\frac{9}{16}$″ and $\frac{5}{8}$″, we will call it $\frac{5}{8}$″.

To find $A\,E$ (88000 pounds) which is larger than $A\,C$, we shall evidently have to divide it in halves, and, of course, double the result. We find that horizontal line $2\frac{3}{4}$″ and vertical line 44000 from above, meet between the heavy bearing lines $1\frac{5}{16}$″ and $1\frac{3}{8}$″, or we will say $1\frac{3}{8}$″; this doubled or $2\frac{3}{4}$″ is the required thickness therefore, for head $A\,E$.

Of course, if we use more than one bar for either of the strains we will divide the required thickness of head accordingly. Thus, if we decide to use two bars along the line $A\,C$, each would be strained $= \frac{70000}{2} = 35000$ pounds, and the thickness of head required for each would be only $= \frac{2,12}{2} = 1,06$ or say 1″.

Arrangements of Heads along Pin.

Now to arrange the different heads along the pin, we first lay off along each line (Figure 182) the amount of strain (measuring all at the same scale) and project these strains on to the different lines. We measure these projections and have along the line $A\,B$ the strain $A\,B = 28000$ pounds pulling to the right; the projection of $A\,C = 34500$ pounds



also pulling to the right; also the projection of $D\,A = 11000$ pounds pushing to the right; the projection of $A\,E = 73500$ pounds pulling to the left, thus opposing the other three.[1]

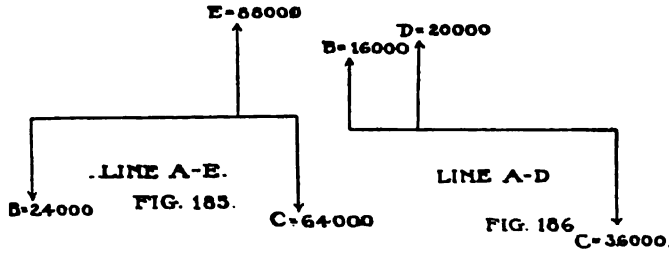
If our measurements are right the sum of the forces acting in one

[1] These figures have been rounded off to simplify the calculations.

direction along line $A\ B$, must equal the sum of their opponents, or, $A\ B + A\ C + D\ A = A\ E$ and we have in effect,

$28000 + 34500 + 11000 = 73500$

The strains on the pin along line $A\ B$ are shown in Figure 183. Those along line $A\ C$ are shown in Figure 184 and those along $A\ E$



are shown in Figure 185; it will be noticed that in the latter case $D\ A$ becomes $= 0$. In Figure 186 are shown the strains along $D\ A$, in this case $A\ E$ becomes $= 0$.

Design for largest Strain first.

As the largest strain in one direction (88000 pounds) is along line $A\ E$, we will select Figure 185 to design the joint and when we have settled the arrangement of heads along the pin to suit these strains, we will see how it affects the pin according to the strains along the other lines.

The simplest arrangement of the parts would be evidently that shown in Figure 185. We will first consider the shearing. The largest shearing strain will be between C and E and will be $= 64000$ pounds. From Table XL we find for wrought-iron, single shear (at 8000 pounds per square inch) that we should need a $3\frac{3}{16}''$ diameter pin to resist 64000 pounds single shear (as each of the vertical spaces at the top evidently represent 10000 pounds), we must pass down vertically not quite half-way between the second and third lines to the right of 50000, till we strike the single shear iron curve and then pass along the horizontal line to the left to find diameter of pin, which is between $3\frac{1}{8}''$ and $3\frac{1}{4}''$ or say $3\frac{3}{16}''$. Had we calculated arithmetically we should have had, area of cross-section required from Formula (7) transposed

Shearing strain too great.

$$a = \frac{64000}{8000} = 8 \text{ square inches.}$$

By referring to a table of areas of circles, or by calculation we find

that for an area of cross-section of 8 square inches we require a diameter of $3\frac{3}{16}''$. This is a larger pin than we want to use and besides seems a very large pin for the strains; it is evident, therefore, that our heads are badly arranged along the pin; we will decide, therefore, to divide the rods E and C each in two parts, making each head one-half the thickness as above found, and arrange them as shown in Figure 187.

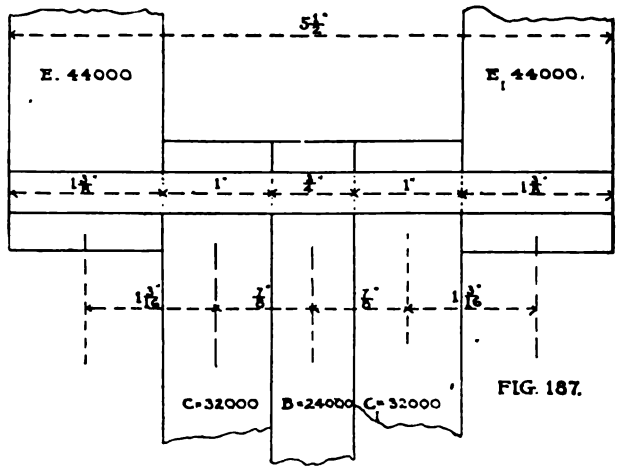


FIG. 187.

Shearing from Table XXXIX.

Now the safe cross-shearing on our pin ($2\frac{3}{4}''$) for single-shear would be from Table XXXIX = 47600 pounds, we pass along horizontal line $2\frac{3}{4}''$ till we strike iron single-shear curve and then pass upward about four-fifths way between the third and fourth vertical lines to the right of 40000: as each space is evidently 2000 pounds, we should have $40000 + 3\frac{4}{5}.2000 = 47600$ pounds. By calculation we should have had area of $2\frac{3}{4}''$ circle = 5,939 square inches, therefore safe (single) cross-shearing see Formula (7) = 5,939.8000 = 47512 pounds. The greatest cross-shearing strain on the pin with arrangement as shown in Figure 187 is 44000 pounds, and is between the heads E and C (or E_1 and C_1), so that we need not fear shearing.

The shearing area of pin being all right we now consider the bend-

ing-moment.

Bending-moment on pin.

We have marked along the pin, Figure 187, the thicknesses of heads, the length of pin required being $5\frac{1}{2}''$, to this must be added the head and nut and also sufficient for strut D ($\frac{5}{8}''$) which we remember did not come into the calculation along line $A\,E$ (Figure 182.)

Immediately under the pin, Figure 187, we have marked the distances from centre to centre of heads, which are, of course, the distances we consider when calculating the bending-moment. Accordingly our pin becomes a circular wrought-iron beam of $2\frac{3}{4}''$ diameter, with a span or length of $4\frac{1}{8}''$ and supported at both ends by forces E and E_1. The beam is loaded with the forces C, C_1 and B as shown in Figure 187.

The greatest bending-moment will be at the centre (See p. 51, Vol. I), and will be

$$m_B = 44000.\ (1\tfrac{8}{16} + \tfrac{7}{8}) - 32000.\tfrac{7}{8} - 24000.0$$
$$= 90750 - 28000 - 0$$
$$= 62750 \text{ pounds-inch.}$$

This will be much more than the pin can stand for we have, for the safe bending-moment on a $2\frac{3}{4}''$ pin, moment of resistance

$$r = 1\tfrac{1}{14}.\ (1\tfrac{3}{8})^3 = 2{,}042$$

(Table I, Section No. 7) and from Formula (18) transposed, the safe bending-moment on pin

$$m = 2{,}042\ .15000$$
$$= 30630 \text{ pounds-inch}$$

or, only about one-half of the actual bending-moment. Had we used Table XXXIX instead of calculating arithmetically we should have passed along the horizontal line $2\frac{3}{4}''$ till we struck the dotted bending-moment curve for wrought-iron at 15000 pounds and then passed vertically to the *bottom*. This would be about two-fifths way between the vertical lines 30000 and the first one to its right, each space being evidently 1500 pounds, this would mean 30600 pounds-inch safe bending-moment on a $2\frac{3}{4}''$ pin.

Bending-moment from Table XXXIX.

It is evident, therefore, that we must re-arrange the rods, trying to get the span between loads E and E_1 shorter if possible.

Try new arrangement.

We now try the arrangement shown in Figure 188, placing load B at one end. The arm end B of pin now becomes a lever and we know from Formulæ (118 and 119) that the reactionary forces E and E_1 can no longer be equal.

From Formula (118) we have

$$E = \frac{32000 + 32000}{2} + \frac{4\frac{7}{16}}{3\frac{3}{8}} .24000$$

$$= 63555$$

From Formula (119) we have

$$E_1 = \frac{32000 + 32000}{2} - \frac{1\frac{1}{16}}{3\frac{3}{8}} .24000$$

$$= 24445$$

As a check the sum of the two should equal the whole strain $A\,E$ and we have in effect $63555 + 24445 = 88000$.

This arrangement must evidently be abandoned as a bad one, for the difference in the strains on E and E_1 is altogether too great to

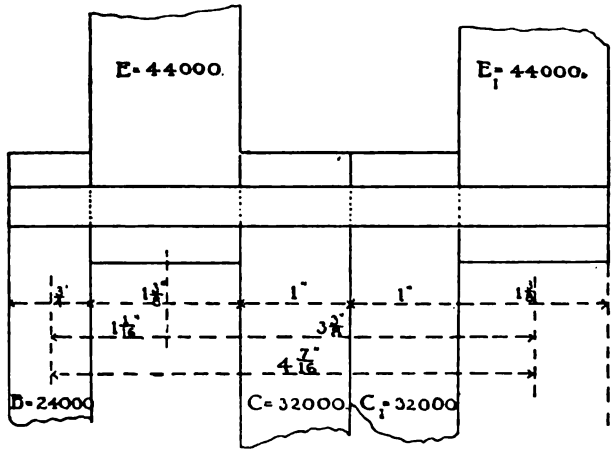


FIG. 188.

be overlooked. Besides, we can readily see that the moments at E and C_1 will exceed 24504 pounds-inch, the safe bending-moment on this sized pin.

For practice, however, we will figure out the bending-moments; they are: at E

(left side) $m_x = 24000.\ 1\frac{1}{16} - 63555.0$

$= 25500$ pounds-inch.

Check (use right side)

$$m_x = 32000.\ 2\frac{3}{16} + 32000.\ 1\frac{5}{16} - 24445.3\frac{3}{8}$$

$$= 25500$$

At C we should have:

(Left side) $m_c = 63555.\ 1\frac{3}{16} - 24000.\ 2\frac{1}{4}$

$= 21472$

Check (use right side)

$m_c = 24445.\ 2\frac{3}{16} - 32000.\ 1$

$= 21472$

At C_1 we should have:

(left side) $m_{c_1} = 63555.\ 2\frac{3}{16} - 24000.\ 3\frac{1}{4} - 32000.\ 1$

$= 29027$

Check (use right side)

$m_{c_1} = 24445.\ 1\frac{3}{16} - 32000.\ 0$

$= 29027$

Had we used rule given on p. 51, Vol. I, we should have known that the greatest bending-moment was at C_1. In applying the rule to this case the end load B should be deducted from the nearer reaction to B.

We might next try dividing the force B in two halves of 12000 pounds each, (heads $\frac{3}{8}''$ thick), leaving one at B and placing the other to the right of E_1. This will restore equality to the forces E and E_{11}, but it will be found that even this arrangement will not do, as the bending-moment at C or C_1 will still be found to be too large, namely, $= 27500$ pounds-inch.

Divide largest force.

After these numerous failures it is evident that we cannot well arrange the heads satisfactorily along the pin, unless we enlarge the pin, or else divide up the larger forces which cause us most trouble. We decide to do the latter and divide $A\ E$ into four parts of 22000 pounds each, with heads each $\frac{2,67}{4} = \frac{11''}{16}$ thick.

Forces will equalize themselves.

We now arrange the heads as shown in Figure 189. The correct way to calculate the bending-moment would, of course, be to consider the pin as a continuous girder running over four supports. This would make E_1 and E_{11} much larger than E and E_{111}. Their heads would therefore have to be thickened so as not to crush the pin, or be crushed by it. It can readily be seen that were we to do this, the calculation would be almost interminable. Besides, practically, it would be expensive to use so many different sizes of rods, heads, etc. We must, therefore, assume that all the forces E, E_1, E_{11} and E_{111} are equal. This

they will be, too, for as soon as E_{I} and E_{II} tend to take more load, they will stretch under the added tension, and this stretching will bring the pin to bear more heavily on the ends and thus the strains will even themselves up.

The bearing of these heads against the pin we know are all right, also the shearing, as the greatest shearing under this arrangement

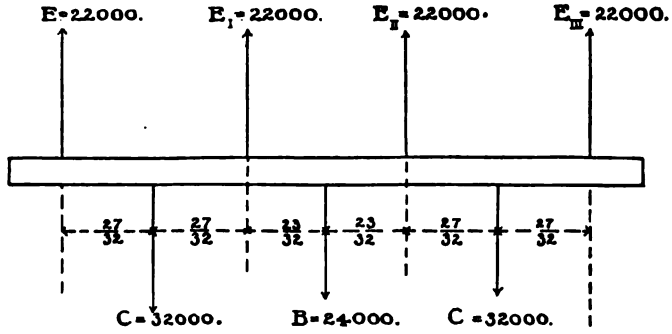


FIG 189.

will be a single shear, between E and C (or E_{III} and C_{I}) and equal 22000 pounds, or considerably less than half the safe single shearing on this pin, which we previously found to be 48000 pounds.

The bending-moments on the pin will now be, at C:

(Left side)

$$m_c = 22000.\tfrac{27}{32} - 32000.0$$
$$= 18563 \text{ pounds-inch.}$$

Check (use right side)

$$m_c = 22000.\left(\frac{27+73+127}{32}\right) - 24000.\tfrac{50}{32} - 32000.\tfrac{100}{32}$$
$$= 18563 \text{ pounds-inch.}$$

At E_{I} we should have:

(Left side)

$$m_{\text{I}} = 22000.\tfrac{54}{32} - 32000.\tfrac{27}{32}$$
$$= 10125 \text{ pounds-inch.}$$

Check (use right side)

$$m_{\text{I}} = 22000.\left(\frac{46+100}{32}\right) - 24000.\tfrac{23}{32} - 32000.\tfrac{73}{32}$$
$$= 10125 \text{ pounds-inch.}$$

At B we should have:
(Either side)

$$m_B = 22000.\left(\frac{23+77}{32}\right) - 32000.\tfrac{19}{32}$$
$$= 18750 \text{ pounds-inch.}$$

At the ends, of course, there would be no bending-moment, for take end E we should have:

$$m_E = 22000.\left(\frac{154+100+54}{32}\right) - 32000.\left(\frac{127+27}{32}\right) - 24000.\tfrac{77}{32}$$
$$= 0$$

This arrangement (Figure 189) is therefore satisfactory, so far at least as the strains along the line A E are concerned.

Examine along other lines.

We must now see if it will answer as well for the strains along the other lines (See Figure 182). The direction we shall now have to fear most, will be along the line D A for force D must be placed entirely on the outside edge of pin (not having been located yet) and being quite large,

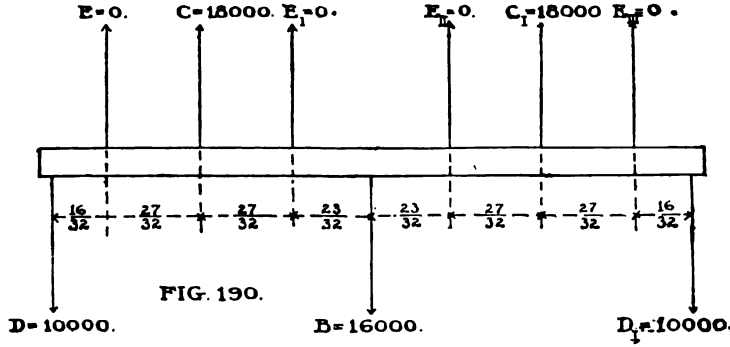


FIG. 190.

20000 pounds, may make us some trouble. In Figure 190 we have drawn the forces arranged the same as we settled on last (in Figure 189) but have added the two forces on the extreme ends D and D_I each $= \frac{D}{2} = 10000$ pounds. It will be noticed that along this line (D A Figure 182) the forces E and C are in the same direction. We have divided D into two parts for two reasons. Had we placed it entirely to one side, say to the right of E_{III}, the distance E_{III} D_I would have been one-quarter of $\frac{5}{8}''$ larger or $= \frac{21}{32}''$; the leverage of

D_1 therefore at C_1 would have been $\frac{27+21}{32} = 1\frac{1}{2}''$ and the bending-moment

$$m_{c_1} = 1\tfrac{1}{2} \cdot 20000$$
$$= 30000 \text{ pounds-inch, too much for our pin.}$$

Besides were we to calculate C and C_1 by Formulæ (118 and 119) we should find all strain on C removed and C_1 more than doubled. This evidently would not do, without special provision to meet the unequal strain by increasing C_1, which would lengthen the pin, so that we prefer to divide $D\,A$, making each head of half the thickness, or $\frac{5}{16}''$ thick. The largest shearing will be between D and C (or D_1 and C_1) and equal 10000 pounds single cross-shear or about $\frac{1}{8}$ only of the safe resistance to shearing.

The bending-moments will be (Figure 190) at C.

(Left side)

$$m_c = \tfrac{48}{32}.10000 - 18000.0$$
$$= 13437 \text{ pounds-inch.}$$

Check (right side).

$$m_c = \tfrac{148}{32}.10000 + \tfrac{50}{32}.16000 - \tfrac{100}{32}.18000$$
$$= 13437 \text{ pounds-inch.}$$

and at B

(Either side)

$$m_B = \tfrac{28}{32}.10000 - \tfrac{50}{32}.18000$$
$$= 9375 \text{ pounds-inch.}$$

The bearings, of course, are safe, as the thickness of head was originally determined by the largest strain on each rod along its own line. So that we are all right with our pin for strains along line $D\,A$.

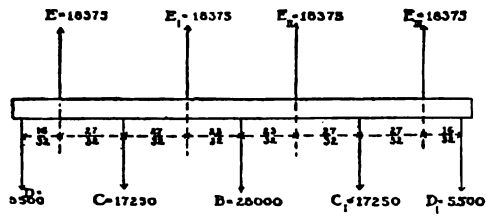


FIG. 191.

We now take up the strains on the pin along the line $A\,B$, Figure 182, which will be as shown in Figure 191.

The greatest shearing-strain here will be caused by B, and will be a double shear of 28000 pounds, or 14000 pounds on each area, perfectly safe on our size of pin ($2\frac{3}{4}''$). The moments will be:

At E

(Left side)

$$m_E = \tfrac{1}{2}.5500 - 18375.0$$
$$= 2750 \text{ pounds-inch.}$$

Check (right side)

$$m_E = 5500.\tfrac{170}{32} + 17250.\left(\frac{27+127}{32}\right) + 28000.\tfrac{77}{32} - 18375.\left(\frac{54+100+154}{32}\right)$$
$$= 2750 \text{ pounds-inch.}$$

At C:

(Left side)

$$m_C = 18375.\tfrac{27}{32} - 5500.\tfrac{48}{32}$$
$$= 8113 \text{ pounds-inch.}$$

Check (right side)

$$m_C = 18375.\left(\frac{27+73+127}{32}\right) - 5500.\tfrac{148}{32} - 17250.\tfrac{100}{32} - 28000.\tfrac{50}{32}$$
$$= 8113 \text{ pounds-inch.}$$

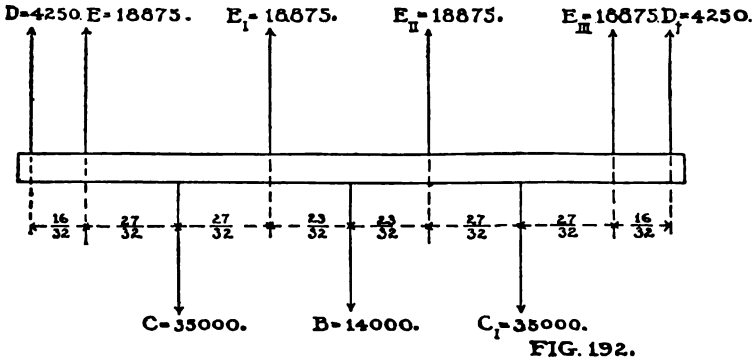


FIG. 192.

At E_1:

(Left side)

$$m_{E_1} = 18375.\tfrac{54}{32} - 5500.\tfrac{70}{32} - 17250.\tfrac{27}{32}$$
$$= 4422 \text{ pounds-inch.}$$

Check (right side)

$$m_{E_1} = 18375.\left(\frac{46+100}{32}\right) - 28000.\tfrac{23}{32} - 17250.\tfrac{73}{32} - 5500.\tfrac{116}{32}$$
$$= 4422 \text{ pounds-inch.}$$

At B:
(Either side)

$$m_B = 18375.\left(\frac{23+77}{32}\right) - 17250.\tfrac{50}{32} - 5500.\tfrac{28}{32}$$
$$= 14484 \text{ pounds-inch.}$$

So that the arrangement of heads along pin is all right so far as strains along line $A\ B$ (Figure 182) are concerned.

We finally examine for the strains along line $A\ C$ and have strains

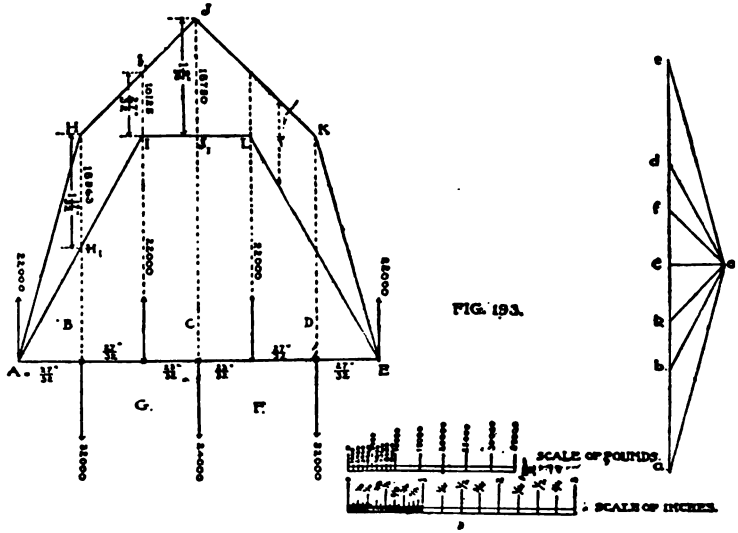


FIG. 193.

on the pin accordingly as shown in Figure 192. The greatest shearing here will be between E and C (or C_1 and E_{III}) and will be

$$= 4250 + 18875 = 23125 \text{ pounds,}$$

single shear, still, less than one-half of the safe single-shear on the pin.

By calculation the moments will be found to be at the different points, as follows:

$m_E = 2125$ pounds-inch.
$m_C = 21637$ pounds-inch.
$m_{E_1} = 11617$ pounds-inch.
$m_B = 16648$ pounds-inch.

So that this arrangement of the different bars and strut along the pin is in every way satisfactory.

Graphical method more simple. The graphical method of obtaining bending-moments is frequently much more simple than the arithmetical method; in important calculations both should be used so as to check each other.

All the rules for formulæ given in Chapter VII for the calculation of transverse strains by the graphical method will apply equally well for pins; the only difference will be that where there are more than two forces on each side of the pin, the base line of the polygonal figure between reactions p and q will no longer be straight, but will be composed of several lines.

Thus, if we take the pin and forces shown in Figure 189, we should change the notation to that adopted for the graphical method, which would be as shown in Figure 193. That is force E (22000 pounds) of Figure 189 would be known as $A\ B$ in Figure 193; again force C, (32000 pounds) of Figure 189 would be called force $E\ F$ (*not* $F\ E$) in Figure 193.

Example solved graphically. Proceeding now to the calculation, we lay off along the vertical load line $a\ e$, the following forces:

$a\ b = A\ B = 22000$ pounds.
$b\ c = B\ C = 22000$ pounds.
$c\ d = C\ D = 22000$ pounds.
$d\ e = D\ E = 22000$ pounds.

and in the opposite direction, we lay off:

$e\ f = E\ F = 32000$ pounds.
$f\ g = F\ G = 24000$ pounds.
$g\ a = G\ A = 32000$ pounds.

which will, of course, bring us back to the starting point a, as the opposing forces must aggregate the same sum.

We now select our pole o. This we remember can be arbitrarily located, or else at a distance $o\ c = \left(\frac{k}{f}\right)$; in our case we will make the distance, say, 12000 pounds.

Pole distance. The distance (of pole from load line) being arbitrary we shall multiply the verticals v (inch-scale) in Figure 193, by this pole distance (pounds-scale) to obtain the bending-moments at the points of pin immediately below verticals. If the pole distance from load line had been made $= \left(\frac{k}{f}\right)$, then the

length of verticals v measured at inch-scale would have been the *required* moments of resistance of the corresponding points of pin below verticals; and each respective v multiplied by $\left(\frac{k}{f}\right)$ would be the bending-moment at each point. We will, however, make in our case the pole distance, arbitrary, *viz:* $c\,o = 12000$ pounds. We now begin at any point of line $A\,B$ and draw $A\,I$ parallel $b\,o$ till it intersects $B\,C$ at I; next draw $I\,L$ parallel $c\,o$ to intersection with $C\,D$ at L; and similarly draw $L\,E$ parallel $d\,o$; $E\,K$ parallel $o\,e$; $K\,J$ parallel $o\,f$; $J\,H$ parallel $o\,g$ and $H\,A$ parallel $o\,a$; the last line must intersect the first at point of starting A or some error has been made.

Strain diagram.

It will be noticed that the individual outlines of $A\,I\,L\,E\,K\,J\,H\,A$ cover the capital letters in Figure 193, corresponding to small letters from which their respective parallel lines started in strain diagram. Thus, for instance $A\,I$ covers letter B and is parallel to $b\,o$; similarly $I\,L$ covers C and is parallel to $c\,o$; $L\,E$ covers D and is parallel $d\,o$; $K\,J$ covers F and is parallel $o\,f$; $J\,H$ covers G and is parallel $o\,g$; similarly we can consider $E\,K$ as covering E and it is parallel $o\,e$; and $H\,A$ as covering A and it is parallel $o\,a$.

We now measure the verticals through the figure $A\,I\,L\,E\,K\,J\,H\,A$, the longest, of course, will give the greatest bending-moment. This happens to be the central one $J\,J_1$ it measures $1\frac{9}{16}''$, therefore

Curve of bending-moments.

$$m_J = 1\tfrac{9}{16} \cdot 12000 = 18750$$

which corresponds to m_B of Figure 189.

Similarly we should have:

m_{J_1} (formerly m_{E_1}) $= \frac{27}{32} \cdot 12000 = 10125$ pounds-inch.

m_H (formerly m_C) $= 1\frac{7}{32} \cdot 12000 = 18563$ pounds-inch.

Had we analyzed the strains on pin as shown in Figure 192 graphically, our verticals would have measured,

at E:

$$v_E = \tfrac{3}{16}''$$

at C:

$$v_C = 1\tfrac{3}{16}''$$

at E_1:

$$v_{E_1} = \tfrac{31}{32}''$$

and at B:

$$v_B = 1\tfrac{3}{8}''$$

The corresponding bending-moments would have been:

at E:

$$m_E = \tfrac{3}{16}.12000 = 2250 \text{ pounds-inch.}$$

at C:

$$m_C = 1\tfrac{1}{16}.12000 = 21750 \text{ pounds-inch.}$$

at E_1:

$$m_{E_1} = \tfrac{1}{32}.12000 = 11625 \text{ pounds-inch.}$$

at B:

$$m_B = 1\tfrac{3}{8}.12000 = 16500 \text{ pounds-inch.}$$

which are very close to the correct moments, which we found arithmetically to be:

$$m_E = 2250 \text{ pounds-inch}; \quad m_C = 21637 \text{ pounds-inch};$$
$$m_{E_1} = 11617 \text{ pounds-inch}; \text{ and } m_B = 16648 \text{ pounds-inch.}$$

Simplest method use curve of moments of resistance.

The simplest method of calculating pins, as a rule, will be — after calculating (or ascertaining from Tables) the safe bearing and shearing stresses of the pin, — to calculate the actual moment of resistance of the pin, see Table I, Section No. 7, fourth column. Now proceed graphically, being sure to make the pole distance in every case equal to the safe modulus of rupture $\left(\frac{k}{f}\right)$ of the material of pin. After this it will only be necessary to see that none of the verticals through the different polygonal figures (corresponding to $A\ I\ L\ E\ K\ J\ H\ A$ of Figure 193) — that none of these verticals measure at inch-scale *more* then the actual moment of resistance of the pin. If this is done the calculation and selection of the best arrangement becomes very simple and easy. After the final and best arrangement has been determined on, it would be well to calculate arithmetically the moments as per this final arrangement, thus checking the graphical solution.

Contractors claim of no bending-moment ridiculous.

The writer has frequently been told by contractors that owing to the friction due to the pressure of the nut and head, that it was impossible for any bending-moment to take place on a pin. As well it might be claimed that owing to the pressure of the walls, there is no bending-moment on a built-in beam. The safe modulus of rupture of a built-in beam can be assumed higher same as we do for pins, but the beam will break across if too heavily strained and so will the pin. Besides it must be remembered that the heads of eye-bars necessarily cannot fit the pins perfectly; and even if the argument

were correct, (which it is not,) that friction offsets the bending-moment, the least rusting of the joints would diminish the pressure between nut and head, thus destroying its value.

Again, contractors will admit that there exist bending-moments on pins, but will deny it in the case of rivets, though the cases are precisely analogous; this latter argument though, if sifted, will generally lead the contractor to admit that its real basis is the large number of rivets it frequently requires; and the less rivets he can get along with, the happier will your contractor be.

CHAPTER X.

PLATE AND BOX GIRDERS.

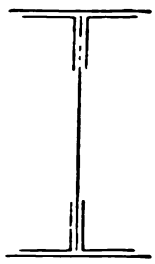


FIG 194.

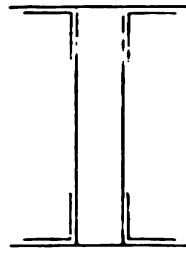


FIG. 195

WHEN it becomes necessary to cover floors or spaces of such large span, or to carry loads so heavy, that rolled-beams will not answer the purpose, girders are made up of plates and angles riveted together, and are known according to their section as "riveted plate girders" with single webs (Figure 194), "or riveted box girders" with two or more webs (Figure 195). As a rule, too, riveted girders of equal strength can be more cheaply manufactured than the heavier sections of rolled beams.

Single web girders.

In the case of the former the vertical plate or web has two angle irons riveted horizontally along its entire top edge and the same along its bottom edge, or four in all. In very light construction the free legs of the angles might answer for the top and bottom flanges, but, as a rule, a plate is riveted to these free legs, at right angles to the web, both top and bottom, thus forming the flanges of the girder. This plate need not necessarily extend the entire length, but it usually does. Where the thickness of flanges required is very great, say one inch or more, each flange is made up of two or more thicknesses or layers of plates.

In such cases only the layer nearest to the web is carried the entire length, the other layers gradually decreasing from the point of greatest bending-moment (usually the centre) towards the ends.

To carry all the layers to the ends in heavy work would be a great extravagance, the only advantage gained being a slight increase in stiffness, which can be very much more readily and economically gained by increasing the depth of web.

Box girders.

In double web box girders only two angles are attached to each web, one at the top and one at the bottom, both on the outside surface of each web. To place angles on the inside surface is impracticable, as webs would have to be placed sufficiently far apart for the "holder-up" to crawl in, and the riveting would not only be weak, having to be done by hand, but it would weaken the flange by just so many additional rivet holes. In short girders with heavy loads, where shearing is the main danger, box girders with three webs are sometimes made; in that case the middle web has the usual four angles, but the two outside webs only two angles each.

Beyond the additional stiffness sideways, in resisting lateral flexure, there is no particular advantage in using a box girder. It is more clumsy to handle and to make, and not readily painted on all exposed surfaces, and besides is more extravagant of material in proportion to its strength. Where the flange is of great breadth and it becomes desirable to have two or more webs, the writer always prefers to use two or more single web plate girders, and to secure them together with bolts and separators, or by latticing the top and bottom flanges, together.

The angles need not necessarily be even-legged; nor need the web necessarily be of same depth throughout, nor of same thickness throughout. It will, however, greatly simplify the calculation to keep the web uniform throughout, and in most cases the extra labor involved in varying the thickness of web, would more than offset the cost of the unnecessary material at the centre.

Where girders are very deep, the web is made in sections or panels, as already explained. In such cases the web can readily and economically be thickened towards the ends.

Cambering girders.

Whenever possible, the girder should be cambered up at the centre an amount equal to the calculated deflection. But as girders are usually made of straight plates, and machine riveted and punched, the cambering is rarely practicable. Should the girder, however, show any bending or cambering in transportation, the architect should be sure to have the cambered side placed on top.

The calculation of riveted girders is exactly the same as for iron beams, but has the additional element of the number and location of rivets to be looked into, also the stiffness of web and overhang of flanges.

The reactions, vertical shearing, bending-moments, actual and required moments of resistance, deflections, etc., can be calculated arithmetically by the rules given in Chapters I and VI; or graphically by the rules given in Chapter VII.

The rules for calculating riveted work were given in the previous chapter (IX).

Horizontal flange strain.

The only new matter is to find what the strain on the rivets will be. It will be readily seen that when a plate girder is loaded the tendency of the flanges and angle irons is to slide horizontally past the web (see Figures 120 to 125).

This tendency to slide is called the horizontal flange strain. The rivets connecting angles to web resist this tendency and there must be sufficient rivets to do this safely.

The total amount of this tendency to slide or horizontal flange strain between any selected point of girder and the *nearer* end of girder, is equal to the bending-moment at the selected point, divided by the depth of web of girder at the point, or

$$s = \frac{m}{d} \qquad (121)$$

Where s = the total strain, in pounds, coming on all the rivets connecting *either* top or bottom flange to web, between any selected point of girder and the nearer end.

Where m = the bending-moment in pounds-inch, at the selected point of girder.

Where d = the total depth, in inches, of the web of girder at the selected point.

The above strain s will exist in both top and bottom flanges and will be resisted by all the respective rivets in *either* top or bottom flange that connect the angles to the web.

Number of rivets in web leg of angles.

It should now be ascertained which is the weakest resistance of each rivet, whether it be to bearing (compression), to shearing, or to bending, and this weakest resistance divided into strain s, as found by Formula (121) will, of course, give the number of rivets required along *each* edge of web between the selected point and the nearer end of girder.

Frequently many more rivets will have to be used than are required by calculation in order not to exceed the greatest pitch for rivets given in Formula (107).

To ascertain the number of rivets required (along either web edge) between any two points of girder, we will, of course, take the difference between the numbers required from each point to end of girder.

In the flange leg of angles usually fewer rivets can be placed than in the web leg, though many good engineers frequently make them equal in number. But this is really unnecessary, for even if the strains on the flange rivets were the same as those on the web rivets (which they are not) we should still have two rivets in the flange to one in the web on all single plate girders.

Number of rivets in flange leg of angles.

There seems to be considerable difference of opinion as to just how to figure the strain on the flange rivets. The best course would seem to the writer to be, to assume that each flange cover plate must transfer at each of its ends, by rivets, to the angle iron and parts of flange plates between it and the angle iron, an amount equal to the safe stress the plate is capable of exerting (that is, net area of cross section of the plate multiplied by either the safe compression stress $\left(\frac{c}{f}\right)$ or by the safe tensional stress $\left(\frac{t}{f}\right)$ as the case may be).

This amount should be transferred by sufficient rivets, between the end of each plate, and the point of girder at which the full thickness of the plate is required to make up the required moment of resistance. From this point to centre the rivets can be spaced according to the rule for greatest pitch, Formula (107), but when rivets are so spaced the pitch of the rivets immediately nearest the ends of any cover plate should be greatly decreased for a distance of three or four rivets at each end.

By the above method the amount of strain on rivets can be quite accurately computed.

Locating flange rivets.

The simplest method of locating rivets is to construct what might be called the curve of moments of resistance. This can be done as shown in Figure 151, Chapter VII (where $C\,D\,E\,F\,G\,C$ is the curve of moments of resistance), or we can calculate arithmetically the required moments of resistance at several points of girder, and lay out the curve as

shown in Figure 196, where $A\ B$ represents the length of girder, and $M\ C$, $I\ D$ and $N\ E$ the calculated required moments of resistance at points M, I and N.

The curve of moments of resistance is, of course, $A\ C\ D\ E\ B$, and its axis or base $B\ A$.

We now make $I\ H = a, d$ see Formula (99); a, being the net area of cross-section in square inches of two angles, and d the total depth of girder in inches.

$H\ D$ will now represent the total required thickness of flange plates, which we can find from Formulæ (36) or (98).

We will decide to divide it into three layers $H\ G$, $G\ F$ and $F\ D$. We draw the lines as shown and find that plate No. 1 can stop at K, though it would be better to run it full length, it is, however, needed of full thickness at J. Again plate No. 2, can stop at J, but is

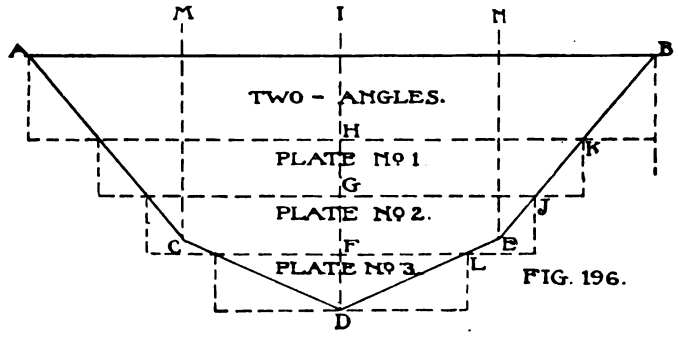


FIG. 196.

needed full thickness at L. The top plate, of course, will run from L to centre. The left half of girder, will of course be similar, the loads evidently being symmetrical each side of centre. In practice the plates rarely are stopped at the exact points calculated, but are usually extended beyond these points a distance equal to from once to twice the width of plate.

There must now be rivets enough between D and L to equal the efficiency of plate No. 3, between L and J to equal the efficiency of plate No. 2 and between J and K to equal the efficiency of plate No. 1. If there is not room to get them in the plates must be sufficiently extended to get them in, that is No. 3 must be lengthened beyond L and towards J; No. 2 must be lengthened beyond J towards K and No. 1 carried on towards end.

Regularity of spacing. In laying out the rivets they should be as regular as possible, the best method is to lay out the total number of rivets required from centre to end, gradually decreasing the pitch towards ends, and then to make each of the plates No. 3, No. 2 and No. 1 of sufficient length beyond their respective point D (for No. 3) L (for No. 2) and J (for No. 1) to take in the number of rivets required. The length of plates may always be more than shown in Figure 196 without harm, but *never less*.

We should have then for the bottom flange:

Number rivets in end of each flange plate.

$$x = \frac{a.\left(\frac{t}{f}\right)}{v} \qquad (122)$$

Where $x =$ the number of rivets required in each end of each flange plate between its ends and the nearer points to ends at which its full strength is required by the girder.

Where $a =$ net area of cross-section of the flange plate, in square inches (less rivet holes), at its weakest section.

Where $\left(\frac{t}{f}\right) =$ the safe tensional stress, per square inch of the material. For top flange use $\left(\frac{c}{f}\right)$ and look out that rivets are not so far apart as to cause bending or wrinkling of plates.

Where $v =$ the safe stress, in pounds, or least value of each rivet. That is the bearing, shearing or cross-breaking value of the rivet, whichever is the smaller.

Value of rivets. The rivets in the flanges will, of course, be cantilevers, loaded with their respective amounts of $a.\left(\frac{t}{f}\right)$ or $a.\left(\frac{c}{f}\right)$ respectively, a being the area as given in Formula (122). The free end of cantilever will be of a length equal to the thickness of the respective flange plate, or equal to the thickness of leg of angle iron, whichever is the smaller should be used.

The bearing area will be the diameter of the rivet multiplied by the same smaller thickness.

In single plate girders the shearing of *rivets will* rarely determine their number, this will generally be more than the bearing or bending value.

Figure 197 shows clearly the way in which rivets are strained.

The web-rivet, No. 3, is *bearing* against web on the surface $E\,F$ and against angles on the two surfaces (sum of) $D\,E$ and $F\,G$

The rivet has two cross-shearing areas, at E and F. This rivet is a beam supported at $D\,E$ and $F\,G$ and loaded uniformly with its share of horizontal flange strain, which it is transferring to web.

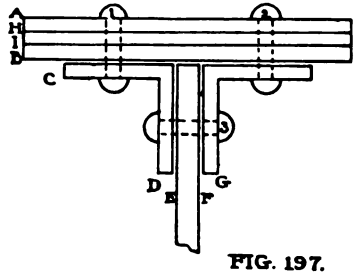


FIG. 197.

The flange rivets, Nos. 1 and 2, we will suppose are connecting the flange plate $A\,H$ to the angle iron. Their bearing then is against $A\,H$ and in the opposite direction against $B\,C$, the lesser should be used. Their shearing area is *either* along the line H or the line B according to which end is considered the cantilever, so that they have only one shearing-area practically in the calculation, instead of two as with the web rivets.

Then rivets Nos. 1 and 2 are cantilevers and are built in *either* from H to C and loaded uniformly on the free end $A\,H$, or built in from A to B and loaded uniformly on the free end $B\,C$, whichever projection $A\,H$ or $B\,C$ is smaller should be used. The load on the cantilever being as already explained equal to each rivet's share of an amount equal to the net-area, of top plate $A\,H$ multiplied by the safe tensional or compressive stress per square inch of the material.

There is, of course, a tendency of the plates $H\,I$, $I\,B$, etc., to slide past each other and past angles.

Plate tendency to slide.

This tendency will exist particularly at the centre of girder and in those parts of rivets which simply tend to hold the plates together after the plates have once transferred their strength and become a permanent part of the flange. But this tendency rarely amounts to much, unless the plates are very thick; and if the rivets are spaced according to rules given can be overlooked. If it is desired to calculate the strain on each rivet, due to this tendency of the flange plates to slide past each other, it can be done by the following formula, which assumes that at any right angled cross-section of flange through rivets there are always two rivet holes.

$$v = \frac{b}{d} \cdot (x-y)^2 \cdot \left(\frac{k}{f}\right) \qquad (123)$$

Where $v =$ the safe value or stress on any flange rivet, in

pounds, to resist the tendency of any two flange plates (or plate and angle leg) to slide past each other.

Where b = the total breadth of flange plate, in inches.

Where d = the total depth of girder, in inches.

Where x = the distance, in inches, from the horizontal neutral axis of girder to centre of flange plate, further from neutral axis.

Where y = the distance, in inches, from the horizontal neutral axis of girder, to centre of flange plate (or centre of flange leg) immediately next to other plate, but on the neutral axis side of same.

Where $\left(\frac{k}{f}\right)$ = the safe modulus of rupture, in pounds, of the material.

Splicing girder plates.

If any part of a girder, either web or flange, is spliced, made of two parts, the number of rivets *each side of splice*, and the amount of *additional* cover plates, etc., should be made sufficient to transfer the full strength of original plate across the joint.

In locating the rivets of a splice care should be taken not to weaken the original plate by holes not allowed for in the original calculation of moment of resistance of the section. There is no difficulty in splicing webs, as cover plates can be put on each side, and the strains in the web are comparatively small.

Web-splice.

These (web-splice) plates and their rivets *each side of* joint should be of sufficient strength to transfer the amount of the vertical shearing strain at the joint from one side of (spliced) web joint to the other side of joint. In the flange, however, it is more troublesome.

Flange-splice.

In heavy girders, however, (the only ones usually, where it is necessary or where it pays to splice the flange plates), it is best to carry the upper or outside layers of flange plates a longer distance from the centre (or point of greatest bending-moment) than required by calculation, thus gaining extra material in the flange, and more than required there by calculation, and then using this extra material to offset the loss suffered by making the additional rivet holes and by cutting or joining one of the flange plates at the point. For instance, Figure 198 represents the side-view of part of the top flange of a plate girder. $A\,B$ is the first flange plate running entire length of girder, A being towards end and B towards centre.

This plate has to be spliced. We have previously found that we can thin down our flange at the points *F, E, D* and *C*. We will decide to piece plate *A B* between *D* and *E* say at *G*. Of course the flange will thus be weakened at the point *G* by the entire loss of

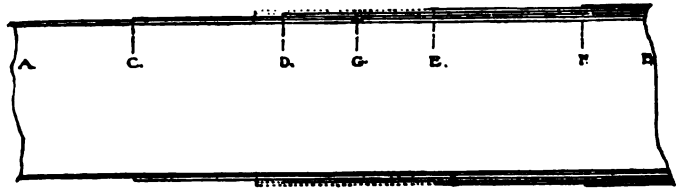


FIG. 198.

plate *A B* and if we attempt to regain this by cover plates it will lose the additional rivet holes. But by prolonging the upper plates as shown by dotted lines this loss can be made good and without any additional rivet holes.

For by the time the plate which originally ended at *E* has been extended to *G* the girder is considerably stronger than needed, that is, stronger by the amount of thickness of this extended plate, and the girder can therefore bear the loss suffered by the cutting of the lower plate. Providing, of course, the plates are of equal thickness. If there are not enough rivets between *G* and *D* to take up the strength of the spliced plate, the plate which ends at *D* will also have to be extended, as shown by dotted lines, till the number of rivets desired have been covered.

In many cases the extending of flange plates is sufficient to form the splice, but frequently an additional cover plate over the extended flange plate may simplify and cheapen the cost. The arrangement in each case will depend upon the number of rivets required, the respective thickness of plates and other local circumstances.

Angles in one piece.

As a rule the angles are made in one piece from end to end, as they can easily be obtained of great length, and are awkward to splice. Angles should be used as heavy as possible, but if very thick they are difficult to straighten, and besides reduce greatly the value of flange rivets, owing to the bending-moment.

In determining the thickness of web it has to have sufficient area

of vertical cross-section at all points to resist the vertical cross-shearing, and must be stiff enough not to buckle under its load, which will be equal to the vertical cross-shearing at each point of web.

As this vertical cross-shearing is always greatest at one or both supports, we should have for thickness of web:

Web-thickness. $$b = \frac{p}{d.\left(\frac{g}{f}\right)} \qquad (124)$$

Where b = the required thickness of web, in inches, (should never be less than $\frac{1}{4}''$ thick).

Where d = the depth of web, in inches, this should be the *net* depth d, that is depth *less* all rivet holes coming on any vertical section at or near reaction.

Where p = the reaction, in pounds, at either end, (Formulæ 14 to 17). The larger reaction should be used, where they are unequal.

If web is not to be of uniform thickness throughout, use, in place of p, the amount of vertical shearing-strain, in pounds, at the point for which thickness of web is being calculated.

Where $\left(\frac{g}{f}\right)$ = the safe cross-shearing stress of the material, in pounds, per square inch.

Danger of buckling. In many plate girders the web will be so thin in comparison to its depth, that there will be serious danger of the web buckling, particularly towards the ends where the vertical cross-shearing (except in case of single concentrated loads) is always greatest.

To avoid this danger the web is stiffened by riveting upright angle irons, or T-irons to same, between the flanges. The ends of these stiffeners should *always be planed* and bear truly against each flange.

Use of stiffeners. At the very ends of girders there should always be stiffeners over the reactions, of sufficient strength, as columns, to carry the amount of reaction, less the amount of bearing of web on reaction. As the length of these columns will be only equal to the depth of girder, and the column will generally consist of the bearing amount of web, plus two angles, they, the stiffeners, can safely be considered as short columns and the full safe compression stress, per square inch, allowed on them. The number of rivets connecting any of the stiffeners to web, should equal the amount of cross-shearing being carried by the stiffener,

that is vertical cross-shearing at the stiffener, less the amount borne by web. This latter amount at the ends is the safe load on a column or section of web, equal to its bearing on reaction; between reactions a section of web equal to its depth is taken as assisting or being assisted by the stiffener, that is as acting together with the stiffener. While the web really receives its load from the flanges by pin-connected ends — rivets — it is, nevertheless, assumed by most engineers to have planed ends, presumably to avoid too many stiffeners, the whole calculation, as it is, being but very theoretical anyhow.

We should have, then, amount of strain on end stiffeners,

Strain on end stiffeners.

$$s = p - \frac{12000.\, b.\, x}{1 + \frac{0{,}0003.\, d^2}{b^2}} \qquad (125)$$

and amount of strain on intermediate stiffeners,

Strain on intermediate stiffeners.

$$s = y - \frac{12000.\, b.\, d}{1 + \frac{0{,}0003.\, d^2}{b^2}} \qquad (126)$$

Where s = the total compression strain on stiffeners in pounds; the stiffeners should have sufficient area of cross-section to resist this strain, considered as short columns, and sufficient rivets connecting them to web to $= s$ in value.

Where p = the *greater* reaction, in pounds, where the reactions are unequal; or *either* reaction where they are equal, see Formulæ (14 to 17).

Where y = the amount of vertical cross-shearing, in pounds, at the point of girder, see Formula (11), at which stiffener is applied.

Where x = the distance, in inches, that girder rests on (selected) reaction p.

Where b = the thickness of web, in inches, at end or at point y, as the case may be.

Where d = the depth of web, in inches, at end or point y, as the case may be.

To decide whether the web needs or does not need stiffeners, and if so, at what points, use the following formula.

Where stiffeners are necessary.

$$y = \frac{12000.\, b.\, d}{1 + \frac{0{,}0003.\, d^2}{b^2}} \qquad (127)$$

Where b and d same as in Formulæ (125 and 126). Should y be

larger than the greater reaction p no stiffeners are required, except at the very end.

Should y be *less* than either reaction, stiffeners will be required up to the point of web where the *vertical shearing* — (as found by Formula 11) — *just equals* y.

At this point place stiffeners a distance apart equal to the depth of web.

Stiffeners should always be placed under concentrated loads. At end of web place stiffeners and again just inside of reaction, and between end and point where y equals shearing place stiffeners, not less than the depth of web apart, and gradually diminishing the distance between them towards end; this distance should be regulated by the amount of increase in vertical shearing towards end.

Deflection of plate girders.

In regard to the deflection of plate girders, the same rules apply, as for beams, that is Formulæ (36) to (42), Table VII, and Formulæ (95) to (97). It should be noted, however, that owing to more or less imperfections in riveting, fitting of parts, etc., the plate girders will deflect very much more than if calculated by these rules, with a modulus of elasticity same as for perfectly rolled beams.

To allow for these imperfections in manufacture a lower modulus of elasticity should be used, to be varied according to the care exercised in manufacturing the girder. Experiments on riveted girders have given moduli of elasticity for steel as low as 5000000 pounds-inch.

This, however, is probably an extreme case. The writer would recommend that the following be used, where no experiments can be made:

Decreased modulus of elasticity.

For wrought-iron plate girders $e = 18000000$ pounds-inch.

For mild-steel plate girders $e = 20000000$ pounds-inch.

Where e = the modulus of elasticity, in pounds-inch, to be used in calculating the deflection of plate girders.

Before giving an example, Tables XLI, XLII and XLIII should be explained. They have been calculated to enable architects to lay out the required size of plate girders by their use, and without elaborate calculations. They will be found to be very accurate and valuable for preliminary estimates, quick designing of girders, and checking of final calculations.

Use of Tables XLI, XLII and XLIII.

Table XLI gives the value of the web in resisting the bending-moment. It should be remarked here, that some engineers do not include the web at all; others include only one-sixth of the web at top and bottom. This is practically reducing the web to the same level as if the top and bottom flanges were merely latticed together. The writer believes, that in house-work at least, it can and should be safely included, particularly as it does not greatly affect the final result anyhow. In box girders the two webs should be considered as one web of thickness equal to the sum of the two; except when calculating for buckling, when, of course, each web is taken separately.

Table XLII gives the value of the four angle-irons, for six different sizes of angles, and Table XLIII the value of *each inch of effective* width of flange. In all of the Tables the horizontal column of figures at the top indicates the length of span of girder, in feet; the vertical columns of figures to the left indicate the respective values in tons (of 2000 pounds each); while the figures on the curves indicate the *depth of web* of the plate girder.

The tables are calculated for a safe modulus of rupture $\left(\frac{k}{f}\right)$ or extreme fibre strain of 12000 pounds per square inch, and intended, of course, for wrought-iron.

For different fibre strains.

For those desiring to use a smaller or greater strain it will only be necessary to increase or reduce the actual load (respectively) in the calculation. Thus, if it is desired to use a fibre strain of 15000 pounds, this will be one-quarter more than allowed for in Tables. We therefore use but four-fifths of our load in the calculation and find by the Tables, what sized girder will safely carry four-fifths of our load with an extreme fibre strain of 12000 pounds. When we then add one-fifth of the original load (or a quarter additional to the calculated load) it will, of course, add also one-quarter or 3000 pounds to the extreme fibre strain. Or, we wish to use an extreme fibre strain, of only 10000 pounds. We add one-fifth to our load making it $\frac{6}{5}$ (or one and one-fifth) and find from Tables the size of plate girder to carry this increased load at 12000 pounds fibre strain; when now we deduct one-fifth of the original load (or one-sixth of the calculated load), we will, of course, at the same time diminish our extreme fibre strain one-sixth to 10000 pounds.

The use of the Tables is very simple and easy. For loads other than uniform, and for steel, the data at bottom of Table XLI shows their respective values as compared to those given in Tables.

It should be noted that the "greatest deflection" has been calculated for the most perfect work. For ordinary work this deflection will be increased, according to the quality of the workmanship, to one-half more than for perfect work.

In using the Tables, first settle the size of web, then of the angles, and finally the size of flange plates.

Example I.

Designing girders by Tables. *We will suppose that we have a wrought-iron plate girder of* 60 *feet span, which is to carry a uniform load of* $89\frac{3}{8}$ *tons and two loads of* $44\frac{1}{16}$ *tons each, one concentrated near each end, and one quarter span from reactions. We are to use a web* 36″ *deep and flange* 21″ *wide. Design the girder parts by use of Tables.*

From the arrangement of loads W_{III} and W_{IV} (at bottom of Table XLI) we see their sum is equal to a uniform load, or

$$44\tfrac{1}{16} + 44\tfrac{1}{16} = 89\tfrac{3}{8}$$

that is, our two concentrated loads will have the same effect on the girder as a uniform load of $89\frac{3}{8}$ tons; our total load on the girder, therefore, will be equal to $178\frac{3}{4}$ tons uniform load, which we will assume includes the weight of the girder itself. We will decide to use four 6″ x 6″ x $\frac{7}{8}$″ angles, as the loads are very heavy, and $\frac{7}{8}$″ rivets throughout.

We now settle the thickness b of web, from formula given in right-hand corner of Table XLI, namely:

$$b = \frac{U}{8.d} = \frac{178\frac{3}{4}}{8.36} = 0{,}621$$

or, say, web should be $\frac{5}{8}$″ thick.

Value of web. We now find the value of web in resisting the bending-moment from Table XLI. Pass down the vertical span line marked 60′ 0″ till we strike the curve marked 36″, this is two-thirds way between the horizontal lines marked on the left — (in the $\frac{5}{8}$″ thick vertical column, the second from left) — 7,5 and 10,0 respectively, or our web would safely carry about 9 tons. We next take Table XLII, remembering that we selected the 6″ x 6″ x $\frac{7}{8}$″ angles, the values for which are in the extreme left column.

We again pass down the 60′ 0″ vertical line till we strike the 36″ curve, which is a little more than half-way between the horizontal lines marked in the extreme left — $\frac{3}{8} \times 4 \times 4\frac{1}{2}$ — values 28.2 and 32.9 respectively; or, the four angles together will take care of about 30⅞ tons, this added to the web value 2 tons makes 32⅞ tons of the 175⅝ tons to be cared for, or a value of 142¾ tons to come on the flange.

Value of four angles.

The flange is to be 24″ wide; from this we must deduct the two ⅞″ rivet holes[1] in our effective width of flange would be = 19¼″, therefore each inch must carry

$$\frac{139}{19\frac{1}{4}} = 7.22 \text{ tons.}$$

We now take Table XLIII. pass down the 60′ 0″ vertical line till we strike the curve 36″ and then pass to the left to find the above value 7.22 tons. We strike the curve on the fifth horizontal line from the top and passing to the left find that we cannot find any such value as 7.22 tons, in other words the flange will have to be thicker than two inches. The value of 2″ we find is 4.8 tons, leaving us 7.22 — 4.8 = 2.42 tons to care for in addition to the 2″ thickness: this, we find (still on the fifth horizontal line) is under the thickness marked 1 or our flanges will have to be exactly 3″ thick at the centre of girder.

Value of flange.

In regard to the web, should we decide to make a box girder, each web should be at least one-half the calculated thickness or $\frac{5}{16}$ thick. In practice it would be better to make them a little heavier, for such heavy girders, say about ⅜″ thick each.

Example II.

Detailing a riveted girder.

A single web riveted plate girder is of 59′,0″ *span. It four feet from left support, and thence every five feet to five feet from right support it carries a concentrated load of* 19500 *pounds, the third loads from each end being increased (by columns) to* 91000 *pounds. These loads include the allowance for weight of girder. The web must not be more than* 36″ *deep. Detail the girder.*

This girder is one of some twenty-five used by the writer in a large public building in New York City, hence the limitation as to depth of girder.

[1] Many engineers deduct in addition to size of rivet, 1-16″ for punching and 1-16″ for reaming, which in our case would make the rivet holes 1″ instead of ⅞″.

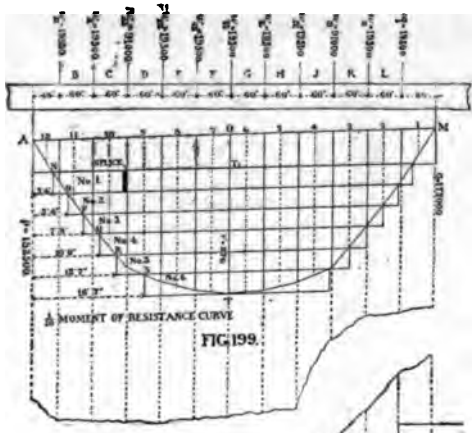


FIG. 199.

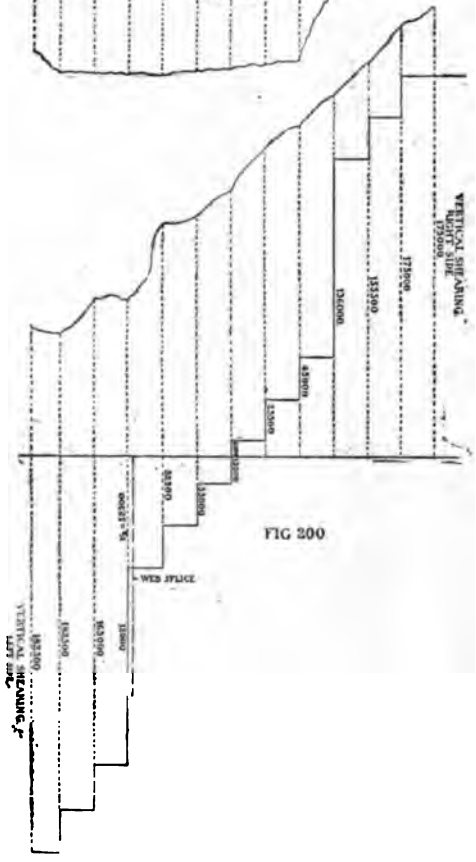


FIG 200

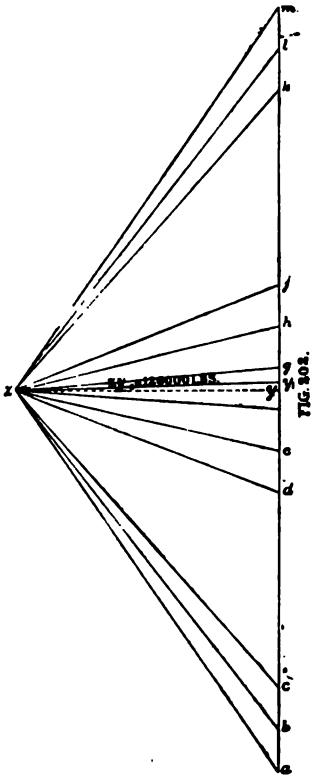


FIG. 202.

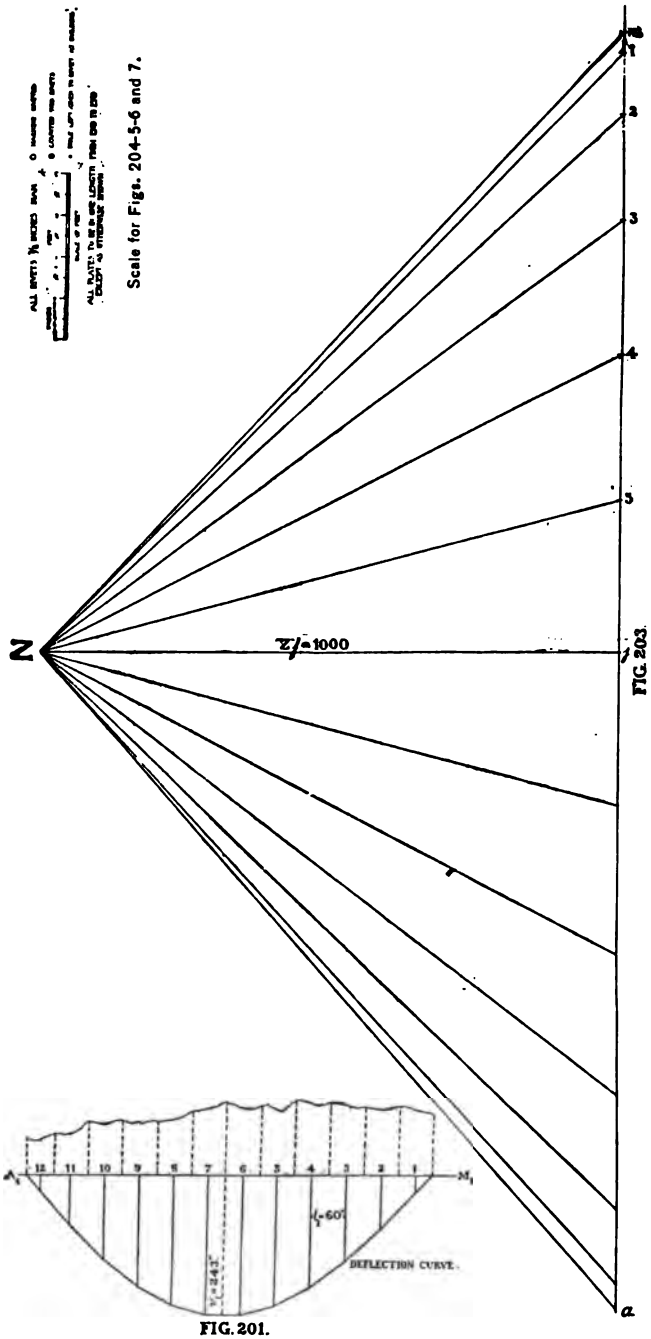


FIG. 201.

The working-drawings, as they were furnished to the contractors by the writer, are given in Figures 204 to 210 inclusive.[1] The only objection (on the score of cost) was to the length of some of the flange plates, but this could not be avoided, as the level of beams resting on the girders could not be disturbed, and there was not room enough between beams to get in the necessary length of cover-plates for splices.

The reader will readily see that this is practically the same example as the former one (*Example I*), so that we need not refer to the Tables for preliminary designing. We know then from the Tables that the girder, at the centre, will need to have a $36'' \times \frac{5}{8}''$ web, a $3'' \times 21''$ flange, four $6'' \times 6'' \times \frac{7}{8}''$ angles, and that we will use $\frac{7}{8}''$ rivets.

Size by Tables.

We will now see whether this is confirmed by calculation. We will first use the graphical method (see Chapter VII) on account of the large number of loads.

In Figure 202 (p. 121) we lay off our vertical load line $m\,a$, where $m\,l = l\,k = j\,h = h\,g$, etc., $= 19500$ pounds, and $k\,j = d\,c =$ 91000 pounds at pounds-scale. We would select the pole distance $x\,y = \left(\frac{k}{f}\right) = 12000$ pounds, but that it will make the moment-of-resistance curve too deep for convenience. We will, therefore, decide to make the distance $x\,y = 10.\left(\frac{k}{f}\right) = 120000$ pounds at pounds-scale. We shall, therefore, have to multiply all the verticals through the moment-of-resistance curve in Figure 199 by ten to get their actual values. We draw the moment-of-resistance curve Figure 199 (see Chapter VII) and find its base line $A\,M$. As our loads are not symmetrical on the beam, being four feet distant at one end and five feet at the other, we draw in Figure 202 $x\,y_1$ parallel $A\,M$ of Figure 199, and find our reactions

Curve of moment of resistance.

$$y_1\,m = q = 175000 \text{ pounds and}$$
$$a\,y_1 = p = 182500 \text{ pounds.}$$

The greatest bending-moment will be at load w_v, where the greatest vertical v through the moment-of-resistance curve (Figure 199 on p. 121) measures 276 inches (by inch-scale). Not having used the pole distance $x\,y = \left(\frac{k}{f}\right)$ in Figure 202 we

Greatest bending-moment.

[1] It is to be regretted that some of the illustrations are necessarily very small; the reader is advised to use a magnifying glass.

must multiply this by ten to get the actual required moment of resistance which would be = 2760. For the same reason we cannot use Formula (98) to calculate the flange thickness and therefore refer to Formula (36) and have for thickness of flange at centre

$$= \frac{\frac{2760}{37} - 16{,}4}{19{,}25} = 3{,}02$$

Required flange thickness.

Or we need, as found by Tables, a flange thickness of three inches at the centre of girder. In the above formula, it should be explained, 2760 was the *required* moment of resistance at the point (that is at load w_{v_1}) for which we were calculating flange thickness; 37 represented the approximate total depth of girder, allowing say for one half-inch plate to each end of girder; 19,25 was the net width of flange, after deducting two rivet holes; and 16,4 was the net area of cross-section (after deducting four rivet holes) of two 6″ x 6″ x ⅞″ angles.

Where to diminish flange thickness.

We will decide to use six half-inch thick plates in each flange and must next decide where to break them off. Accordingly we use Formula (99), and have value of two angles

$$= 16{,}4 \,.36 = 590{,}4$$

or of the whole *required* moment of resistance (2760) the angles furnish an amount = 590; now, as our moment of resistance curve is only of one-tenth the required depth, we divide this by 10 and make $O\,T_1 = 59$ at inch-scale. We now divide $T_1\,T$ into six equal parts and draw the parallel lines to base $A\,M$, their intersections N, O, P, Q, R, and S with the curve are the points at which the respective plates can be broken off.

We shall, however, carry the first plate $N\,O$ the entire length, and as this plate is spliced inside of the curve, we shall have to carry plate No. 5 over the joint to make up for the lost section, and we shall have to carry both plates Nos. 4 and 5 sufficiently far to the left of the splice to get in the necessary number of rivets to equal the value of cut plate.

Number of flange rivets required.

It will also be necessary to prolong some of the plates to get in the necessary rivets *beyond* their points of contact with curve. Thus between O and N we must get enough rivets to equal in value plate No. 1, or else prolong plate beyond N; between P and O enough rivets to equal plate No. 2, or else prolong plate beyond O; and so on till in the last

plate No. 6 we must get enough rivets between T and S to equal plate No. 6. Now these plates are all of equal value $= \frac{1}{2}'' \times 19\frac{1}{4}'' = 9\frac{5}{8}$ square inches of cross-section, which multiplied by $\left(\frac{k}{f}\right) = 12000$ pounds gives the real strain s on the rivets, or

$$s = 9\tfrac{5}{8}.12000 = 115500 \text{ pounds.}$$

We will, therefore, lay out rivets enough, in each flange, between S and the end A to take this strain five times, gradually decreasing the pitch towards the end of girder. We will then carry each plate sufficiently far beyond the length required by curve, to get its respective number of rivets.

Spacing flange rivets.

Now the value of rivets ($\frac{7}{8}''$) in flange will be for shearing (single area) $= 4800$ pounds each. For bearing and bending the rivets will evidently get their value from the $\frac{1}{2}''$ plate, this being thinner than the angles, and we have bearing value $= 5250$ pounds per rivet. Either of the above can be found by calculation, or from Tables XXXV and XXXVIII.

Value of each flange rivet.

For bending we have from Table XXXVIII for a $\frac{7}{8}$ rivet, the safe bending-moment $= 990$ or say 1000 pounds-inch.

The actual greatest bending-moment will be, Formula (25)

$$m = \frac{u.(\frac{1}{2})}{2} = \frac{u}{4}$$

and as the safe

$$m = 1000 \text{ pounds-inch,}$$

we have

$$1000 = \frac{u}{4} \text{ or}$$

$$u = 4000 \text{ pounds.}$$

The value of the rivets against bending — (4000 pounds each) — being their least value, will control the design. Each cover-plate therefore requires

$\frac{115500}{4000} = 29$ rivets, and from S to end we shall require 145 rivets in each flange. From S to T we require only 29 rivets, but they will have to be spaced more frequently to comply with the rule for greatest pitch, or Formula (107), accordingly the pitch of the latter should not exceed $= 16.\frac{1}{2} = 8$ inches.

Figure 207 shows a plan of the top flange of left half of girder. Plate No. 6 might have stopped at S, but is carried two rivets

further to avoid breaking under a beam, which rested on the girder at this point. The splice of plate No. 1 has been made just to the left of R, where plate No. 5 might have stopped; we must, therefore, carry plates Nos. 5 and 4 at least 29 rivets beyond the splice, which has been done. Plate No. 4 might have stopped two spaces nearer R, but for the splice; Plate No. 3 we stop at the right number of rivets to the left of Q, and plate No. 2 to the left of P. Plate No. 1, which might stop 29 rivets to the left of O, we decide to carry to the end.

Splicing the flange.

The countersunk rivets shown come under beam or column ends. The blank spaces were to bolt column plates to. The blank spaces for beams were marked on later from memoranda in the contractor's shop.

Having detailed our flange, which will be the same both for top and bottom flange, we will now consider the web.

The size of web we settle from Formula (124) and have for thickness b, assuming that there will be no more than six rivet-holes in any vertical section, or

Thickness of web.

$$d = 36 - 6.\tfrac{7}{8} = 30\tfrac{3}{4}'';$$

$$b = \frac{182500}{30\tfrac{3}{4}.8000} = 0{,}74$$

or nearly $\frac{3}{4}''$. The web, however, was made $\frac{5}{8}''$ as the above was required only at the one extreme end and through its rivet-holes. The effect of decreasing the breadth of web being, of course, to raise the actual shearing per square inch at this point to a little over 9000 pounds per square inch.

We next decide where stiffeners are required; we use Formula (127) and have

Where stiffeners are required.

$$y = \frac{12000.\tfrac{5}{8}.36.}{1 + \dfrac{0{,}0003.36^2}{(\tfrac{5}{8})^2}}$$

$$= 135000 \text{ pounds},$$

Or we require stiffeners from the end to the point where the vertical shearing is less than 135000 pounds.

By referring to Figure 200, we find this would be about ten feet from the end.

The stiffeners, however, were placed more frequently, as shown in Figure 206, both for looks, and as there was some danger of heavier loads being placed on the centre of girders, which would, of course, increase the vertical shearing near centre. These stiffeners were made of $6'' \times 6'' \times \frac{1}{2}''$ angle irons, with $6'' \times \frac{7}{8}'' \times 24''$ filler plates

behind them, so as not to bend the stiffeners. The filler plates being cheaper than would be the cost of blacksmith work involved in bending these angles around the vertical legs of flange angles. Their upper and lower ends were "milled" off and made to bear firmly.

Now as to value of rivets through web, we should have for bearing $\frac{5}{8}''.\frac{7}{8}''.12000 = 6562$ pounds; for shearing, being in double shear, twice the value previously found for single shear or, $2.4800 = 9600$ pounds; and for bending we have a $\frac{7}{8}''$ circular beam of $\frac{5}{8}''$ span. The safe bending-moment we previously found to be 1000 pounds-inch, the actual bending-moment is $\frac{u.l}{8}$, therefore

Value of web rivets.

$$\frac{u.\frac{5}{8}}{8} = 1000 \text{ and}$$

$$u = 12800 \text{ pounds,}$$

Or the value against bending would be 12800 pounds. As the bear-

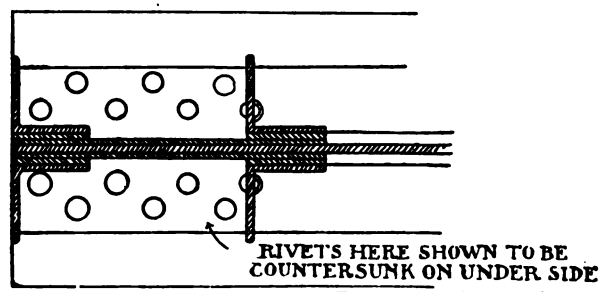


FIG. 208.

ing value (6562 pounds) is the smallest we will use that in determining the number of rivets in web.

For end stiffeners we use Formula (125)

Number of rivets in end stiffeners.

$$s = 182500 - \frac{12000.\frac{5}{8}.16}{1 + \frac{0{,}0003.36^2}{(\frac{5}{8})^2}}$$

$$= 122500 \text{ pounds.}$$

Or we should need

$$\frac{122500}{6562} = 18{,}6$$

Or we should need some 19 rivets in the end stiffener, we therefore decide to use a filler plate 24" x $23\frac{5}{8}$" x $\frac{3}{8}$" each side, which will not

only help stiffen the web, but affords us the room to get in the necessary number of rivets, without cutting more than six rivet-holes on any one vertical line. Figure 208 gives a plan of arrangement of web at end. We next take the stiffener located some 4′ 6″ from the end. The vertical shearing here (see Figure 200) is 163000 pounds.

Number of rivets in central stiffeners.

From Formula (126) we have therefore strain on this stiffener

$$s = 163000 - \frac{12000.\frac{5}{8}.36}{1+\frac{0{,}0003.36^2}{(\frac{5}{8})^2}}$$

$$= 28000 \text{ pounds.}$$

Or we should need

$$\frac{28000}{6562} = 4{,}3$$

or say five rivets, we must, however, locate them oftener, see Formula (107).

We next decide to splice the web at the point shown in Figure 206, and as shown in plan Figure 210.

Splicing the web.

The vertical shearing at this point (see Figure 200) is 163000 pounds, we need, therefore, *each side of joint*

$$\frac{163000}{6562} = 24{,}8$$

or say 25 rivets. Including those in the angles, which, of course, help splice the joint, we have 26 each side.

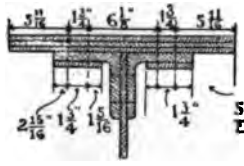
We next settle the size of splice plate by Formula (114). We shall have for its neat breadth

$$b = 24'' - 6.\tfrac{7}{8} = 18\tfrac{3}{4}''$$

and have for thickness of splice plates

$$h = \frac{163000}{18\frac{3}{4}.12000} = 0{,}72$$

As there are two plates, one each side of the web, we shall make each one-half the above, or say $\frac{3}{8}''$ thick, remembering, however, to fill out behind the angle with an additional $\frac{1}{2}''$ thick (filler) plate.



SECTION OF FLANGE SPLICE.

FIG 209.

Number of rivets connecting web and angles.

We must next settle the number of rivets connecting the angle and the web.

The vertical through moment of resistance curve (Figure 199) at the centre measures 276″ and the axis $x\,y$ in (Figure 202) we made

$= 120000$ pounds,

therefore, from Formula (93) the bending-moment at centre, or:

$m_{centre} = 276.120000$

$= 33120000$ pounds-inch.

This divided by the depth will give the horizontal flange strain from centre to end of girder, see Formula (121), or

$$s = \frac{33120000}{36} = 920000 \text{ pounds.}$$

This again divided by the least value of web rivets, which we

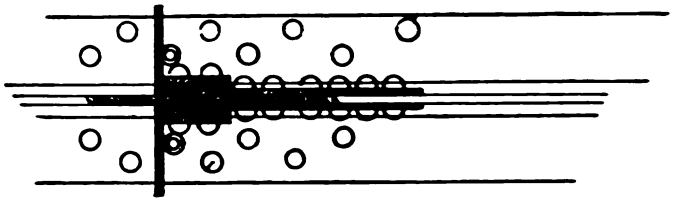


FIG. 210.

previously found to be 6562 pounds, gives the total number of rivets required, or

$$\frac{920000}{6562} = 140$$

In reality we have placed 143 rivets from centre to end, so as not to place the central ones too far apart. Again take a point just under the column (or w_{IX}) say fourteen feet from the left reaction. The vertical (Figure 199) measures 225″, therefore bending-moment at w_{IX}, or

$m_{w_{IX}} = 225.120000$

$= 27000000$ pounds-inch

and horizontal flange strain

$$s = \frac{27000000}{36} = 750000$$

and required number of rivets

$$\frac{750000}{6562} = 114$$

In reality there are only 113 between this point and end, but that is near enough, as it spaces more evenly so. Again, take a point at the first load to the left, or w_{x1} which is four feet from left reaction. The vertical (Figure 199) measures 75″, therefore bending-moment,

$$m_{w_{x1}} = 75.120000$$
$$= 9000000 \text{ pounds-inch}$$

and horizontal flange strain to end

$$s = \frac{9000000}{36} = 250000$$

Therefore number of rivets required,

$$\frac{250000}{6562} = 39$$

In reality there are 42 rivets.

Length of bearing on reactions.

It will be noticed that in allowing for horizontal flange stress we take all the rivets to the very end of girder, this, of course, is right; although before right through for convenience we have considered the end as at the reaction. The amount the girder will run over the reaction will be determined by the crushing strength of the wall, or pier, or column it is supported by.

In our case we have a bearing $21'' \times 16 = 336$ square inches, and therefore load per square inch on masonry

$$= \frac{182500}{336} = 543 \text{ pounds}$$

per square inch. This was distributed onto the brickwork by heavy, ribbed, enlarged cast-iron plates.

The only thing remaining to be done now is to figure the deflection.

Deflection found graphically.

In Figure 199 we draw the vertical lines, 1, 2, 3, 5, 6, etc., through the moment of resistance curve, the distance between them (l_1) being practically 60″, the first one being a half distance. In Figure 203 we carry down these lengths in succession on line m 1, 2, 3, etc., to a. We select our pole z arbitrarily at a distance

$$z\,j = 1000''.$$

In Figure 201 we now construct the deflection curve. The longest vertical is in the centre of girder and

$$= v_1 = 243''.$$

The moment of inertia of the section of the girder at the centre will approximate very closely to 58000 (for exact amount see Table I,

section No. 14) and remembering that for built-up plate girders of wrought-iron we must use a modulus of elasticity equal to only 18000000 pounds-inch, we have the central deflection, in inches, of the girder (see Formula 97)

$$\delta = \frac{243.60.1000.120000}{18000000.59000} = 1{,}66''$$

The safe deflection, not to crack plastering, would be, (see Formula 28)

$$\delta = 59.0{,}03 = 1{,}77''$$

Deflection by Table XLI.

Or, our girder is amply stiff. Were we to consider our load as equal to a uniform load of 357500 pounds we could use the approximate formula for deflection given in Table XLI, and should have had

$$\delta = \frac{59^2}{75.42} = 1{,}105''$$

We must add one-half to this for a modulus of elasticity of only 18000000 (the approximate formula being based on 27000000) and would have

$$\delta = 1{,}105 + 0{,}552 = 1{,}657''$$

or the same as by the graphical method.

Deflection found arithmetically.

Or, we might have calculated the deflection by Formula (39) again considering the load as a uniform load, and should have had

$$\delta = \frac{5}{384} \cdot \frac{357500.708^3}{18000000.58000}$$

$$= 1{,}62''$$

Or, practically the same result, and showing how closely the different methods agree. Had we figured the girder arithmetically we should have obtained practically the same results throughout. We should have considered our load as a uniform load of 357500 pounds, which would give us equal reactions, of 178750 pounds each, an error of hardly 2 per cent.

Bending-moment arithmetically.

The bending-moment at the centre would be, Formula (21),

$$m = \frac{357500.708}{8} = 31638750 \text{ pounds-inch.}$$

The required moment of resistance therefore, would be, Formula (18),

$$r = \frac{31638750}{12000} = 2636{,}5$$

The actual moment of resistance (by section No. 14, Table I) will be found to be at the centre

$r = 2740$ or considerably more than required.

The load, however, is not strictly equal to a uniform load, hence the discrepancy, we should, of course, use the result found graphically, which was based on the actual conditions.

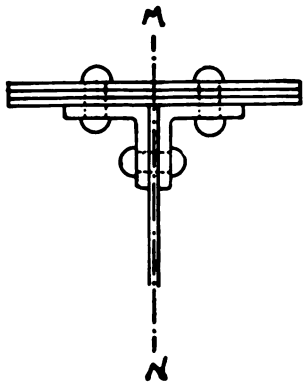


Fig. 211.

In figuring the girder arithmetically the required moments of resistance at different points along the girder should be ascertained; after which the curve of moments of resistance can be laid out and the flanges, web, rivets, etc., of girder, calculated the same as already explained. If our girder were not braced sideways we should have to calculate for lateral flexure, using Formula (5). For the area a we should take the area of top flange at centre, plus two angles and the part of web between angles. For the square of the radius of gyration we should take the same parts around an axis $M. \ . \ . \ N$ at right angles to flange, or as shown in Figure 211. We omit rivet-holes in this case, for ease of calculation, and as all parts are in compression.

Lateral flexure, top flange.

We have then

$$a = 3.21 + 2.9{,}73 + 6.\tfrac{5}{8}$$
$$= 86{,}21 \text{ square inches.}$$

Now for ϱ^2, not finding the exact section in Table I, we must find the moment of inertia i and divide this by the area.

We have then

$$i = \frac{3.21^3}{12} + \frac{7.12\frac{5}{8}^3}{12} + \frac{5\frac{1}{8}.21\frac{3}{8}^3}{12}$$
$$= 2468$$

Therefore $\varrho^2 = \frac{2468}{86{,}21} = 28{,}6$

and from Formula (5), l being, of course, the span of girder in inches:

$$w = \frac{3.86{,}21\,.12000}{1 + \frac{4.708^2.0{,}000025}{9.}}$$

$$= 1149600 \text{ pounds.}$$

One-third of this would be safe, therefore

$$\frac{w}{3} = 383200 \text{ pounds}$$

would be the safe stress in flanges not to cause lateral flexure, or the safe stress per square inch should not exceed

$$\frac{383200}{86{,}21} = 4445 \text{ pounds.}$$

The actual stress will, of course, equal the area 86,21 multiplied by the average fibre stress, per square inch. To find the average fibre stress, use the following Formula:

Average fibre stress in flanges.

$$v = \frac{2.x.\left(\frac{k}{f}\right)}{d} \qquad (128)$$

Where v = the fibre stress, per square inch, in flanges of plate girders.

Where x = the distance of the centre of gravity of part of flange being strained — (flange, angles and part of web between them) — from the centre of depth of web, in inches.

Where $\left(\frac{k}{f}\right)$ = safe modulus of rupture, in pounds, per square inch, or stress on extreme fibres, in pounds per square inch.

In our case this distance x would be found by rule given on p. 7, (Vol. I), and would be (see Figure 211)

$$x = \frac{21.3.19\frac{1}{2} + 12\frac{5}{8}.7_{\frac{1}{8}}.17\frac{9}{16} + 2\frac{3}{8}.5\frac{1}{8}.14\frac{9}{16}}{86{,}21}$$

$$= 18{,}63$$

Therefore the average fibre stress from Formula (128)

$$v = \frac{2.18{,}63\,.12000}{42}$$

$$= 10645 \text{ pounds.}$$

The actual total compressive stress on flange will therefore be

$$= 86{,}21\,.10645$$

$$= 917705 \text{ pounds.}$$

This result should, of course, be the same as our horizontal flange stress, previously found, and by referring back, we see that this was practically the same (920000 pounds.)

Our actual compressive stress we see therefore is about two and one-half times larger than the safe stress to resist lateral flexure as found above (383200 pounds.)

We should therefore, either brace the girder sideways — which was done by the beams in our case — or we should have to broaden the top flange.

Wrinkling of flange. We can readily see that there is no danger of wrinkling in so heavy a flange, but did we wish to calculate it, we would do so by Formula (4) or Table III.

A new deep beam. Since the publication of Table XX, the Homestead Steel Works of Pittsburgh (*E*) have begun rolling 24″ deep steel beams from 240 to 300 pounds per yard in weight.

The data in regard to these beams is as follows:

	Depth of beam (*d*)	24″	24″
	Weight per yard	300	240
	Width of flanges (*b*)	7,20	6,95
	Thickness of web	0,75	0,50
	Area of each flange	6,83	6,55
	Area of web	16,34	10,90
	Total area (*a*)	30,00	24,00
Neutral axis normal to web.	Moment of inertia (*i*)	2349,00	2061,00
	Moment of resistance (*r*)	195,75	171,75
	Sq. of rad. of gyration (ϱ^2)	78,30	85,88
	Transverse value (steel)	1958000	1718000
Neutral axis parallel to web.	Moment of inertia (*i*)	47,13	41,65
	Moment of resistance (*r*)	13,10	12,00
	Sq. of rad. of gyration (ϱ^2)	1,57	1,74
	Transverse value (steel)	131000	120000

CHAPTER XI.

GRAPHICAL ANALYSIS OF STRAINS IN TRUSSES.

THE same general rules which apply to beams and girders apply equally well to trusses; but as the latter are made up of a large number of parts, some sustaining the loads directly, others transmitting the consequent strains and thus helping indirectly to sustain the loads, it becomes difficult and often very complex to follow out all the strains arithmetically. For this reason the graphical method is generally used, and for the architect, who has many other things to remember, besides strains and stresses, will always be found to be the most convenient.

There are three steps necessary in designing a truss:

1st. Ascertaining the amounts of loads on each part, and their points of application.

2d. Ascertaining the consequent strains on each member of truss.

3d. Designing the members and joints of truss.

In calculating trusses it is always assumed that all the members meeting at any joint are connected by a single pin, and are, therefore, at liberty to move around this pin, until they assume equilibrium towards each other, when of course, they will all counterbalance each other and remain stationary. All loads are, therefore, assumed to act directly on the joints, and are considered as vertical forces at these points (except where wind is allowed for separately). It will frequently happen, however, that the loads are not placed directly over the joints.

Transverse strain on members.

For instance, the load might be uniformly distributed over the entire rafter (or chord): in that case, we should have to assume one-half the load on each panel as coming directly (and vertically) on the joint (that is, each joint would act as a vertical reaction, made up of several parts), and afterwards when designing the truss members, we should have to add sufficient material to the rafter (or chord) to take care of the *transverse strain*, due to the uniform load.

Or, again, we might have a load w_{I} as shown in Figure 212. The amounts of this load coming on joints Nos. 2 and 3 would be figured exactly the same as reactions (Formulæ 14 and 15).

Joints act as reactions.

In these formulæ p would be load No. 2; q load No. 3; l would be the length of rafter from No. 2 to No. 3; m would be the length of C or from No. 2 to load; and n would be the length of D or from load to No. 3.

All these lengths can be measured either along the rafter, or horizontally between the vertical lines, the result will be the same.

Where loads are suspended from the lower chord, or what amounts to the same thing — rest on same (as ceilings for instance), the load

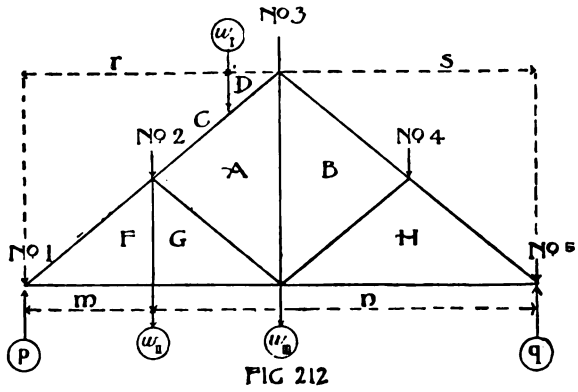


FIG 212

can be considered as hanging from the bottom chord as shown at w_{III} Figure 212.

Loads on lower Chord.

This, however, seriously complicates the strain diagram, it is better therefore to consider (which is also the fact) that the load w_{III} is transferred directly to No. 3 by the tie-rod A B. We will therefore in making our strain diagram add the amount of w_{III} to load No. 3, and *must remember later to add an amount of tension (equal to w_{III}) to rod A B over and above that found by strain diagram.* If we had a load w_{II} placed half-way between p and w_{III} we should have to make the tie-beam or lower chord sufficiently strong to bear this transverse load in addition to the tension existing in it. In this case, the point w_{III} and p would be the reactions for load w_{II}, and the rod A B would

transfer its share up to No. 3 in addition to load w_{III}. But as a rule it is more economical to transfer the load w_{II} up to joint No. 2 directly, by means of the tie-rod. When making the strain diagram, however, this rod should be omitted and the truss shown as at H. Otherwise it will be found that the corresponding points f and g of strain diagram would coincide. This would mean that there was no strain on F G due to the strains in truss; and this is a fact, as the only stress in the rod is in resisting the direct tension due to the load hanging from No. 2 by the rod.

Reactions at Ends.

When figuring the reactions p and q they should be figured by the Formulæ (14) to (17) inclusive. If all the loads are uniformly or symmetrically placed along the truss, each reaction will be just one-half of the total load. If not, then the truss is considered the same as if it were a beam and the reactions figured by the formulæ, the distances m, n, r and s are measured horizontally between verticals as shown; the distance l is, of course, the entire (horizontal) distance from p to q.

Allowance for Wind and Snow.

General Rule.

Wind can safely, as a rule, be assumed to act vertically on the truss and to simply add just so much to the calculated dead load. The writer generally adds for this climate (New York City) 30 pounds per square foot of surface of roof (measured on the slant, not horizontally). This will do for small roofs and approximate calculations of large roofs. This allowance will include the necessary allowance for snow, for, if the roof is steep, the snow will either slide off, or be blown off, and if the roof is flat the wind pressure will be very much smaller and the reduction in wind pressure will fully offset the weight of snow.

Separate Diagram for Wind.

Of course taking the wind as a vertical dead load involves two errors: first, the wind is never on the entire roof, as it can manifestly act on one side only; secondly, the wind does not act vertically. Where wind pressure is calculated separately it is assumed to act at right angles to the surface of the roof and on one side only. In large trusses this should always be done, as it will frequently be found that stresses in certain members will be reversed.

That is, members, which under a dead, vertical load show only tension or compression in the strain diagram may (with wind taken

Reversal of Strains.

normal to roof and on one side only) be reversed and show, respectively, compression or tension.

Iron trusses, over eighty feet long, need some arrangement to allow for expansion and contraction. In roof trusses this is provided by anchoring down one end and leaving the other end free to move (horizontally) by placing it on rollers.

It will readily be seen that where there are rollers the effect on the truss will be very different, according to which side the wind is blowing from. In such trusses, therefore, it will be necessary to make three strain diagrams, one for vertical dead load (including snow but no wind); one for wind only on right side; one for wind only on left side.

The truss must then be designed to withstand the strains due to the dead load only; and enough added, where necessary, to withstand the additional or *different* strains due to either pressure. Where both strains are of the same nature they should be added together; where they are of opposite natures they will, of course, offset each other, but the member should be strong enough or stiff enough to withstand either *separately*.

Wind Pressure Depends on Inclination.

As the wind acts horizontally, it will on striking a roof of course cause a different pressure at right angles to the inclination of roof, than is its pressure against a vertical surface. This pressure will therefore vary with the inclination of the roof. To determine it, it is assumed that the greatest wind pressure per square foot of roof surface will never exceed forty pounds. For steep roofs with an inclination of 60° to 90° with the horizon, this is the pressure assumed.

For roofs forming smaller angles with the horizon a complicated trigonometrical formula is used. Its results are as follows:

TABLE XLIV.

TABLE OF WIND PRESSURES ON ROOFS.

Angle of Inclination of Rafters with Horizon.	Pressure or Load, in pounds, per square foot of Roof Surface.
10°	9⅜
15°	14
20°	18½
25°	22½
30°	26½
35°	30
40°	33½
45°	36
50°	38
55°	39½
60° to 90°	40

Approximate Rule for Wind. It will be noticed that approximately the pressures in pounds are about eighty per cent or four-fifths of the number of degrees of the angle of inclination.

Allowance for Snow. Where the wind is taken separately the allowance for snow in the strain diagram for dead loads should be fifteen pounds per square foot of roof surface.

Having once ascertained the amount of load on each joint, the strains on the different members of the truss are found by the general methods given at the end of Chapter I. (Pages 69 to 74, Vol. I.)

Particular attention is again called to the method of notation, and to the necessity of reading off the pieces in their proper order, and of reading around each joint in the same direction. The writer always uses the direction in which the hands of a watch would travel around each joint.

In calculating the strains each joint can be analyzed by a separate strain diagram, or all of the strain diagrams can be combined into one. As the latter method is much more convenient and less liable to error, it is the one always adopted.

Drawing of strain diagram. In laying out the strain diagram we begin with the joint with the least number of members and this usually is at one of the reactions, where we have only one strut and one tie. Having found these strains we pass to one of the joints at

their other ends, and so on. The reason for doing this is that it will be found impossible to draw the lines in the strain diagram representing any joint where there are more than two unknown strains. By beginning, therefore, with a joint of two members only, the strains on these can be found. We then can pass to joints of three members, containing at least one of the former strains and so on. Figures 213 and following ones give a large number of roof designs with their corresponding strain diagrams. We will analyze one or two of these and the student can puzzle out the rest.

Only a few will offer any particular difficulty, and these will be taken up and explained later.

Dotted lines in tension.

In all the figures dotted lines mean that the dotted member is in tension and full lines that the member is in compression. The numbers in Figures 213 to 228 give the amount of strain, in pounds, on each member due to one pound of load at each joint for roofs with inclination angles as shown in figures.

All that is necessary therefore, where roofs are designed similar to any of these figures, and with same inclinations and angles, to

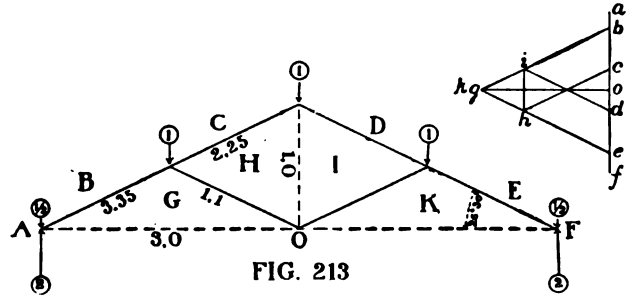


FIG. 213

ascertain the amount of load on the joints and then multiply the number or given strain on each member by the amount of load on each joint. This will give the actual amount of strain on each member.

For instance, we will say we have designed a roof truss similar to Figure 213 with the principal rafter at an inclination of 26° 30′; we will say the trusses are 10 feet apart and 48 feet span, and weight of roof including snow and wind 50 pounds per square foot

measured on the slant. By scaling the rafters we find they measure 27 feet each in length, therefore load on each joint

$$= \tfrac{27}{2}.10.50 = 6750 \text{ pounds.}$$

We now refer to Figure 213 and have the strains, as follows:

Compression on rafter $B\,G = 3{,}35.6750 = +\,22612$ pounds.
Compression on rafter $C\,H = 2{,}25.6750 = +\,15187$ pounds.
Compression on strut $G\,H = 1{,}1.6750 = +\,7425$ pounds.
Tension on tie $G\,O = 3{,}0.6750 = -\,20250$ pounds.
Tension on tie $H\,I = 1{,}0.6750 = -\,6750$ pounds.

Had we drawn the strain diagram and made

$$a\,b = e\,f = \tfrac{6750}{2} = 3375 \text{ pounds at any scale,}$$

and at same scale made $b\,c = c\,d = d\,e = 6750$ pounds, we should find that at the same scale the respective lines would measure:

$b\,g = 22700$ pounds.
$c\,h = 15200$ pounds.
$g\,h = 7400$ pounds.
$g\,o = 20300$ pounds.
$h\,i = 6750$ pounds.

and to ascertain whether these strains were compression or tension we should follow the direction of each line at each joint and see whether it thrusts against or pulls away from the joint.

To draw the strain diagram we first select a convenient scale, by which we will measure all the loads and strains. We now draw our load line, making in (Figure 213) $a\,b =$ the load on $A\,B$ the foot of main rafter, we then make $b\,c =$ the load on joint $B\,C$, the next one on main rafter, $c\,d =$ the load on apex and so on; as we know the reactions will each be just one-half the load, we locate o half way between f and a, in other words we make $f\,o =$ reaction $F\,O$ and $o\,a =$ reaction $O\,A$.

Where to begin.

To get the strains we begin at the joint $A\,B\,G\,O\,A$ at foot of main rafter, for here there are only two unknown strains, namely, the compression on $B\,G$ and the tension on $G\,O$.

In strain diagram draw $b\,g$ parallel $B\,G$; and $g\,o$ parallel $G\,O$ till they intersect at g; then will $b\,g$ be the amount of thrust on the joint or the compression in $B\,G$, and $g\,o$ will be the amount of pull on the joint or tension in $G\,O$. To make sure of these we read off the lines following the proper succession, namely, $A\,B$, $B\,G$, $G\,O$, $O\,A$; referring now to strain diagram we read $a\,b$, this is down or a

vertical load; next *b g*, this is down or towards the joint, therefore compression; next *g o*, this is to the right or pulling away from the joint, therefore tension; and finally *o a* (which brings us back to the point of starting *a*) and being upward is, of course, the direction of the reaction. As our strain diagram is a closed figure *a b g o a*, we know the joint is in equilibrium. Had we failed to get back to the

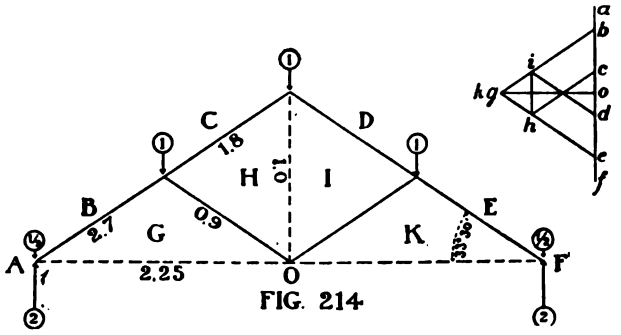


FIG. 214.

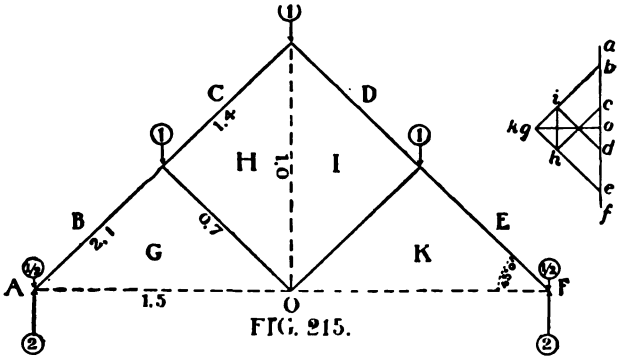


FIG. 215.

point of starting *a*, we should have known there was some missing member to the truss at this joint.

Superfluous members.

On the other hand, if we had had two letters coming onto the same point — (which would be the case with letters *F* and *G* were we to draw strain diagram for Figure 212) — we should know that the member or line between these two letters was superfluous so far as aiding the general truss is concerned. We now (in Figure 213) pass to the next joint with

only two unknown strains. This is evidently the joint at half the height of rafter, we have just found the amount of compression on GB — (note that we now read GB and not BG as before, for around this new joint the hands of a watch would travel in the direction GB) — and the only unknown strains are the compressions on CH and on HG. In strain diagram we draw ch

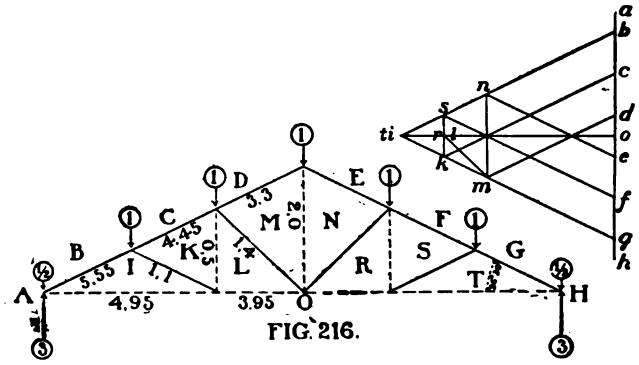


FIG. 216.

parallel CH and hg parallel HG till they intersect; ch will then be the amount of compression on CH and hg the amount of compression on HG. We now read off the pieces in succession BC,

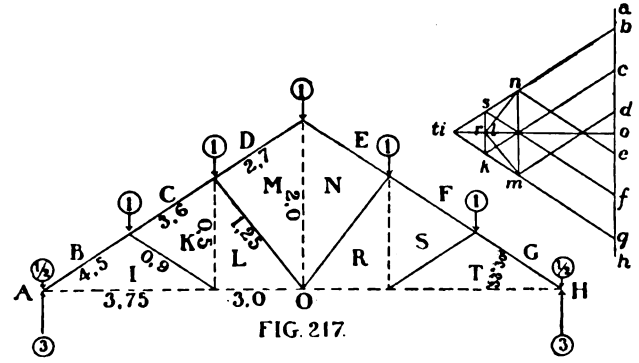


FIG. 217.

CH, HG and GB; and in strain diagram; bc, downwards, therefore a vertical load; ch downwards towards joint, therefore

compression; $h\,g$ to the left, towards joint, therefore compression; and $g\,b$ upwards towards the joint, therefore compression; as our figure is a closed one, we having arrived back at the point of start b, the joint is in equilibrium. We next examine similarly the joint at apex and at centre of horizontal tie. The joints to the right will be

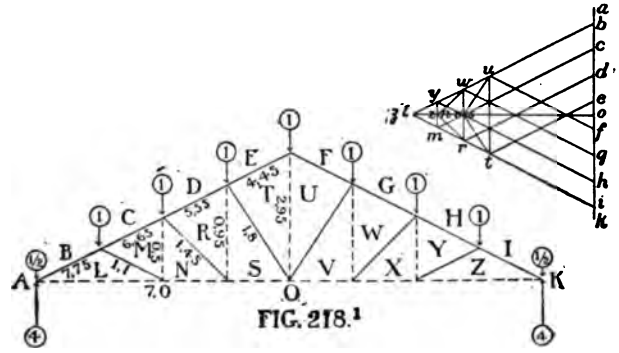


FIG. 218.[1]

similar to the corresponding left-hand joints, as the truss is uniformly loaded. Figures 214 and 215 are similar to 213, the only difference being in the increased pitch of the rafter.

When we analyze Figure 216 we first take the joint $A\,B$ at foot

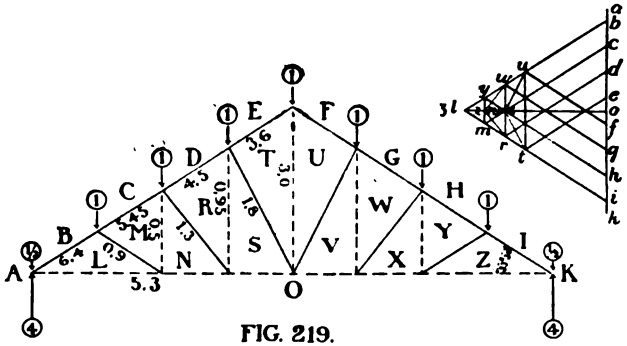


FIG. 219.

of rafter; next the joint $B\,C$ immediately above; next the first joint along tie-rod; next the second joint along rafter and so on. Figures 217, 218[1] and 219 are similar. Figures 220, 221 and 222 will

[1] Angle of rafter (or angle Z) in Figure 218 is 26° 30′.

present no difficulty; nor will Figures 226, 227 and 228, but in the latter three we find that we cannot follow the above rule of passing from a joint along rafter to one on tie-rod; for we begin at joint $A\,B$

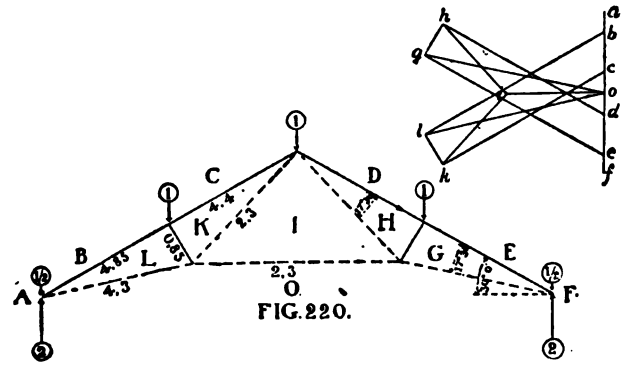


FIG. 220.

(Figure 226) and find the strains $B\,S$ and $S\,O$; we next pass to joint $B\,C$ and find the two unknown strains $C\,R$ and $R\,S$; were we now to pass to the joint at the tie-rod, we should find it impossible to continue, for there would be three unknown strains, namely, $R\,N$,

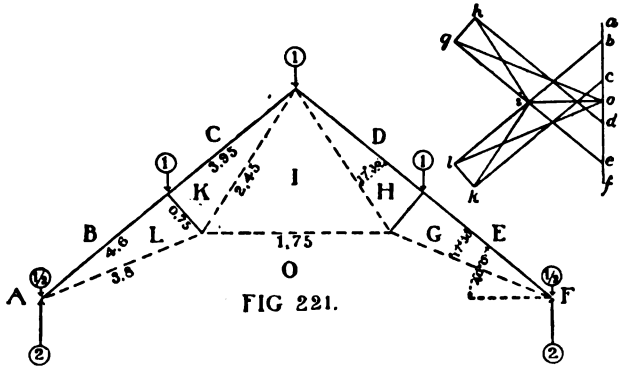


FIG 221.

$N\,M$ and $M\,O$; but if we pass first to joint $C\,D$ we find the two unknown strains $D\,N$ and $N\,R$, and then going back to the joint at tie-rod, there remain only two unknown strains $N\,M$ and $M\,O$ which we can now find.

Figure 223, we will find, presents a difficulty at the joint $C\ D$ and this truss, in effect, cannot be analyzed by the same method as the others.

We begin at joint $A\ B$ and find the unknown strains $B\ L$ and $L\ O$; we pass to the joint $B\ C$ and find the strains $C\ M$ and $M\ L$; we now pass to first joint along tie-rod and find the strains $M\ N$ and $N\ O$; we now pass to joint $C\ D$ but find three unknown strains $D\ S$, $S\ R$ and $R\ N$; we try the second joint along tie-rod, but this, too, has three unknown strains $N\ R$, $R\ U$ and $U\ O$; so have all the other joints, we are, therefore, completely stopped. We reason, however, that

Analysis of Figure 223.

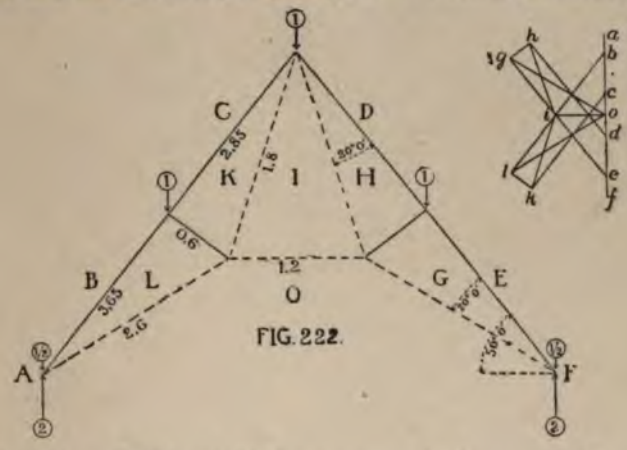


FIG. 222.

the duties of $L\ M$ and $M\ N$, are apparently the same as those of $T\ S$ and $S\ R$, namely, to truss transverely the long (half) lengths of rafter. As the loads are the same on all joints we will assume that

$$S\ R = M\ N \text{ and } T\ S = L\ M.$$

We can now continue our work, for at joint $C\ D$ there only remain two unknown strains $D\ S$ and $R\ N$. Passing to the second joint along tie we can continue, for, $N\ R$ being now known, there only remain the two unknown strains $R\ U$ and $U\ O$.

This method of reasoning is correct, where loads are uniform and the rafter divided into four equal panels; were this not the case, we should have to find the strain on $R\ N$ first, and by one of the methods shown in Figures 223*a* and 223*b*.

In the former we assume (temporarily) that the intermediate panels or joints do not exist.

We take care to reproportion our loads on the joints, as properly

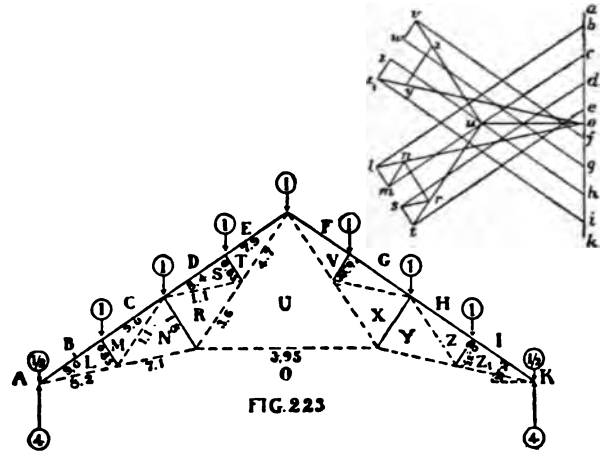


FIG. 223

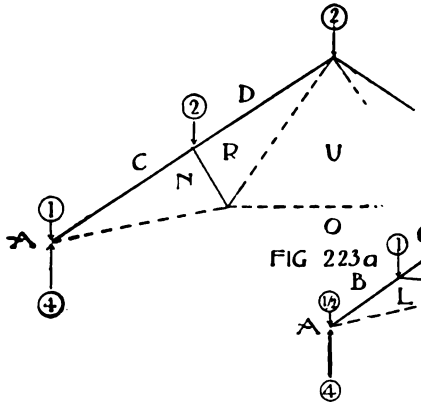


FIG 223a

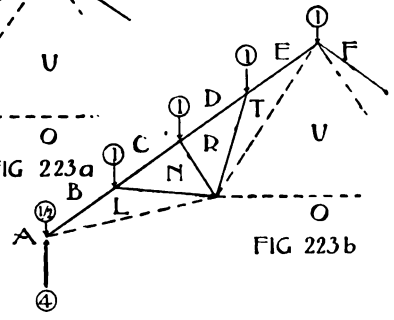


FIG 223b

there are now only two panels to main rafter, where before there were four.

We now find the strain on $R\ N$ without any difficulty by drawing a separate strain diagram similar to the one shown in Figure 220 and

having found the strain on *R N* we proceed with our original strain diagram annexed to Figure 223.

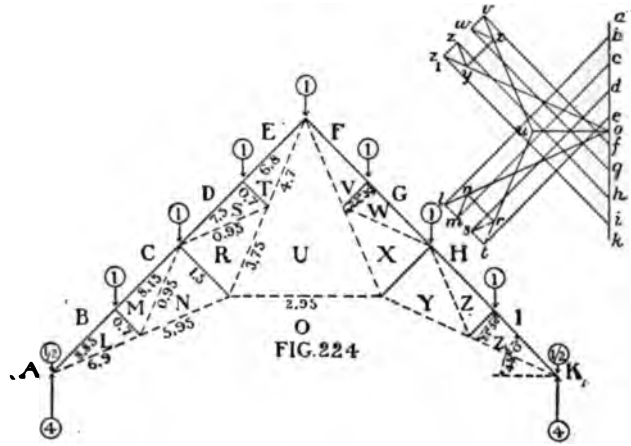


FIG. 224

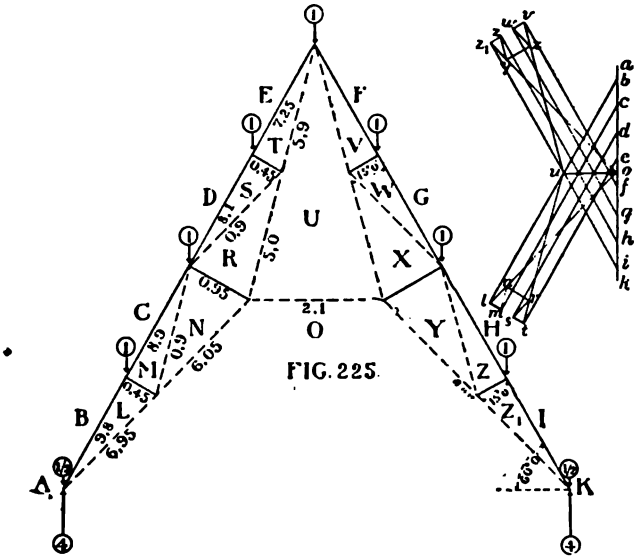


FIG. 225

Or, we will assume (temporarily) that Figure 223 is altered to

have three struts as shown in Figure 223*b*, taking care to keep *R N* the same, and the divisions of rafter the same, the loads on joints will therefore not be changed. We now find *R N* without difficulty by drawing a separate strain diagram on the principle of that shown

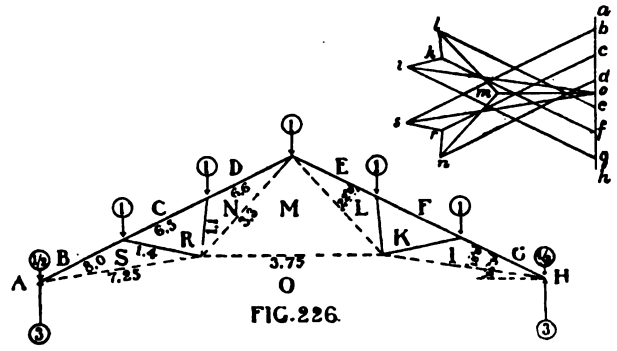


FIG. 226.

in Figure 226; and having found *R N* proceed with our original strain diagram without difficulty.

Figures 224 and 225 are similar to Figure 223, excepting the inclination of main rafters, and angles between rafters and tie-rods.

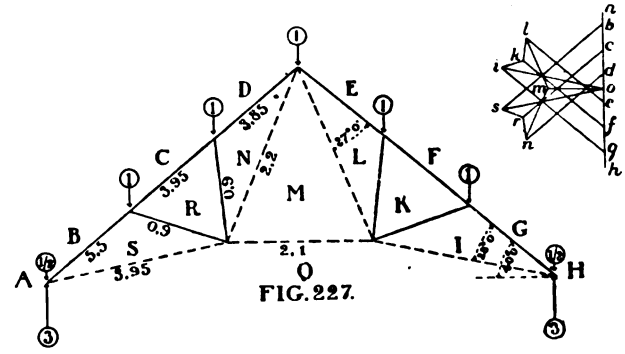


FIG. 227.

Figures 213 to 219 are best adapted to wood, or wood and iron construction; while Figures 220 to 228 are best adapted to iron construction, they being on the principle of a pair of inclined trussed

beams tied together at one point and there taking up the horizontal thrust due to the inclination. Sometimes, if more convenient for

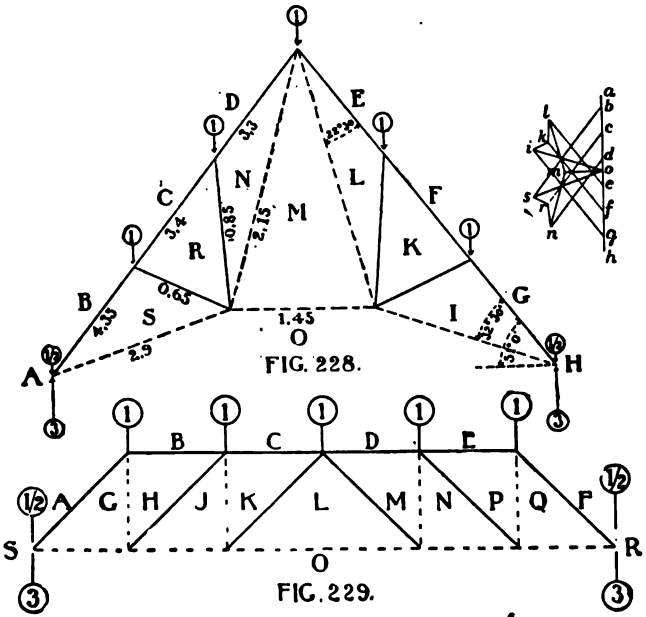


FIG. 228.

FIG. 229.

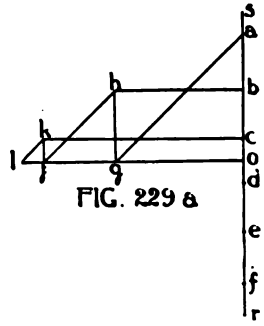


FIG. 229 a.

securing roof beams, etc., the principals are made of wood, the balance of truss usually being of iron.

There are, of course, many different truss designs, based on those

given in Figures 213 to 228 and they can be similarly analyzed.

Howe Truss. Figure 229 shows a "Howe" truss with six bays, it will present no difficulty in analyzing.

The central vertical member is not needed unless the weights are placed along the bottom chord, in which case it will be needed to

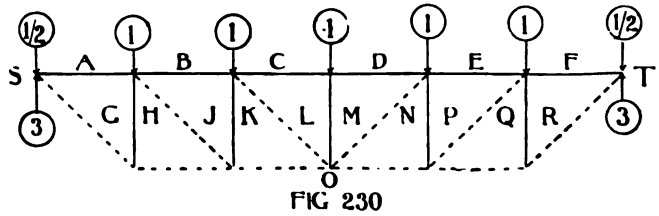


FIG 230

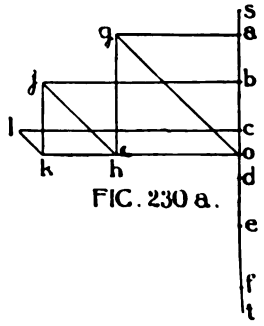


FIG. 230 a.

transfer the load to the top, and all the other verticals will be increased over the strains shown in diagrams by the amount of their respective loads on bottom chord.

This truss is frequently drawn and used upside down from that shown in Figure 229, in which case all the strains will be reversed (see Figure 230), those in compression in Figure 229 becoming tension in Figure 230; and those in tension in Figure 229 becoming compression in Figure 230. It will also be found that the central vertical member is needed, if the loads are placed along the top chord. If the loads (in the reversed truss, Figure 231) are placed along the bottom chord the central vertical will not be needed, and as the loads will be taken up to the top chord by means of the slanting ties, it is better in this case to make a strain diagram with the load

arrows in their right place and as shown in Figure 231. The end loads of this and subsequent trusses have been omitted, as they do

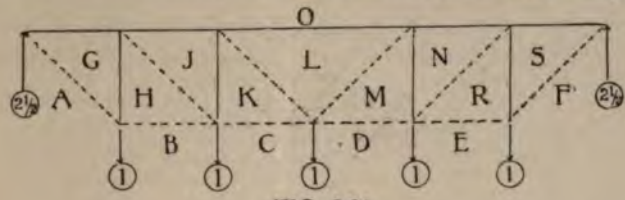


FIG. 231

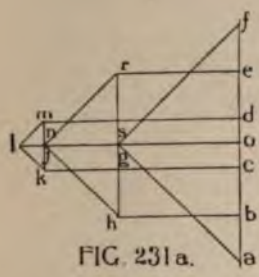


FIG. 231a.

not affect the strain diagrams, and unnecessarily increase the number of letters used. It will be noticed that the entire strain diagram and each part, is the reverse of what it was in Figure 229. There is, however, no difficulty in analyzing this truss.

In Figures 232 and 233 we have two examples of the "Warren" truss.[1] Here, too, it will be seen that the reversing of the truss reverses all the strains. If the loads were along the bottom chords we should have to draw the arrows in their proper places. Figure 235 gives an example. In Figure 234 we have an example of a "lattice" truss. This cannot be analyzed unless we divide it into two reversed Warren trusses as shown in Figures 232 and 233. We analyze each of these separately, and then imagine them laid over each other, adding together the separate strains, where they cover.

Warren Truss.

Lattice Truss.

If there were vertical members in Figure 234, we should analyze it by dividing it into two reversed Howe trusses, like those in Figures 229 and 230. In this case our widths of panels would be the same as in the original truss, and we should use all the verticals in both trusses, the loads therefore to be assumed on each of the joints of the divided or part trusses should be *only one-half of the original loads.* The "Whipple" truss is on the Howe truss principle with verticals bisecting the diagonals. This truss can be analyzed same as the Warren latticed

Whipple Truss.

[1] The true "Warren" truss usually has the diagonals drawn at 60° inclination.

truss, by dividing it into two halves, each having every other vertical and every other diagonal, and consequently a full load on each joint. Arched trusses are usually built up of a series of panels formed on the Howe or Warren or latticed truss principles, the only difference being that their top and bottom chords instead of being horizontal,

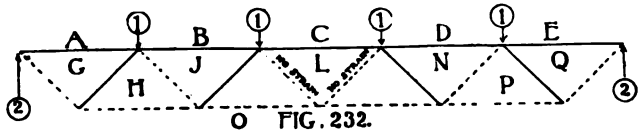


FIG. 232.

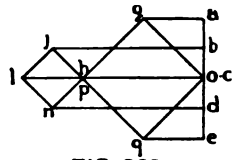


FIG. 232 a.

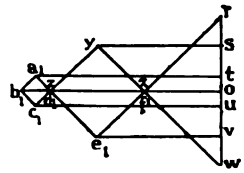


FIG. 233 a.

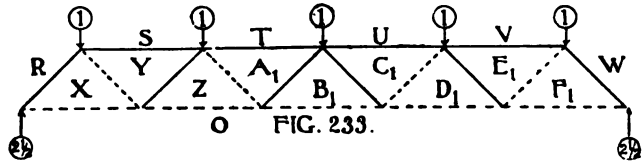


FIG. 233.

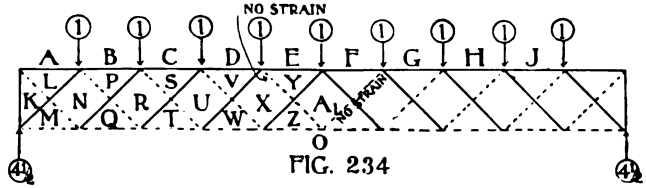


FIG. 234

are made up (or assumed to be made up) of a series of straight lines at different inclinations. They are analyzed without difficulty, where they do not have horizontal tie-rods or abutments to take up their horizontal thrusts.

Arched Trusses.

Figure 236 is an example of an arched truss built on the Warren principle. It will be noticed that the struts $B\,F$ and $F\,O$ are shown as if coming to a point. If this were done in practice the truss in all probability would not have

sufficient bearing. We should, therefore, enlarge the foot of truss, by means of heavy plates and angles, but our calculation will have to

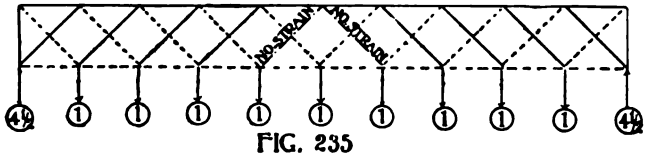


FIG. 235

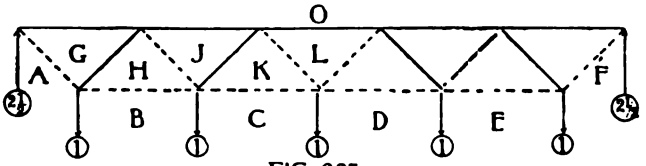


FIG. 235 a

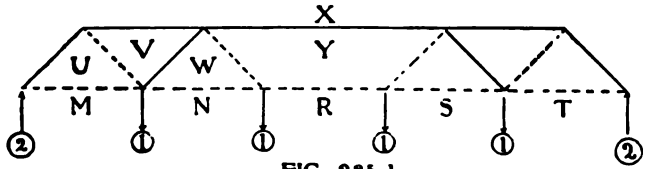


FIG. 235 b.

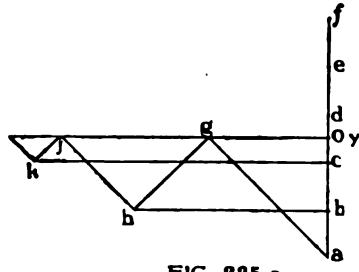


FIG. 235 a_1.

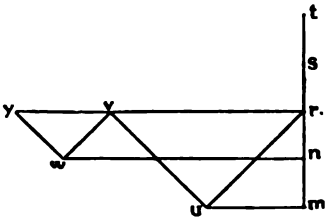


FIG. 235 b_1.

be made as drawn, or we will find the analysis of the truss impossible

Figure 237 shows a latticed arched truss, built upon the Howe principle. This we separate into two reversed Howe trusses, Figures

238 and 239 and find no difficulty in analyzing the strains. We must remember, however, to use only half loads at each joint.

In separating the truss we note that in Figure 239, we have the additional tension member *J I* (see Figure 229); and in Figure 238 the members *K J* and *I H* are in tension instead of in compression as they were in Figure 230. These changes are due to the fact that the upper and lower chords are broken instead of straight lines.

Had the truss been a Warren latticed truss, we should in separating find that we had only half the number of joints and hence would use the full load on each.

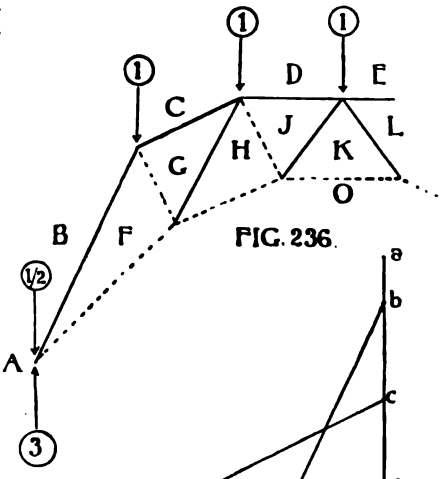


FIG. 236.

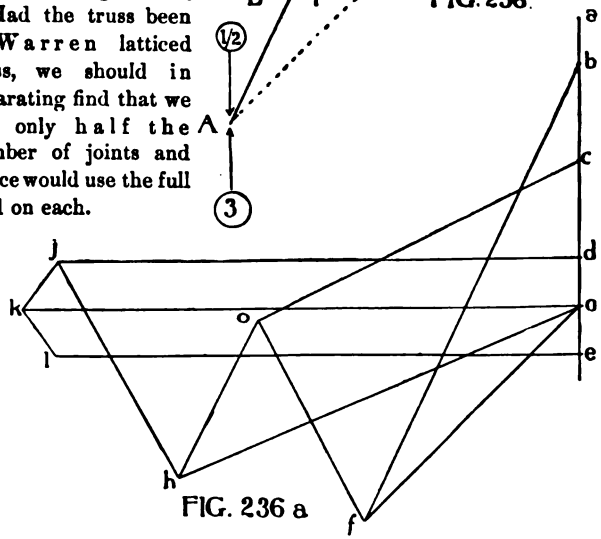


FIG. 236 a

Where there is a horizontal tie or abutment to take up the horizontal thrust of an arched truss, we must find the amount of this thrust by the rules given at the end of Chapter I and at the beginning of Chapter V (both in Vol. I). Having found this we draw an arrow at the foot of the truss in the proper direction to represent this tie (if a rod) or thrust (if an abutment) and then proceed to analyze the truss without difficulty.

Scissors Truss.

In Figure 240 we have what is known as a " scissors " truss. It can be built in either wood, or iron, or a combination of both. For small spans it is a cheap truss where a vaulted ceiling is needed. If we attempt to analyze this truss in one strain diagram, as is done in Figure 243, we first take the lower joint $A\ B$ and find the two unknown strains $B\ G$ and $G\ O$.

At the next joint $B\ C$ there remain three unknown strains, we therefore abandon it for the present and pass to the joint at the

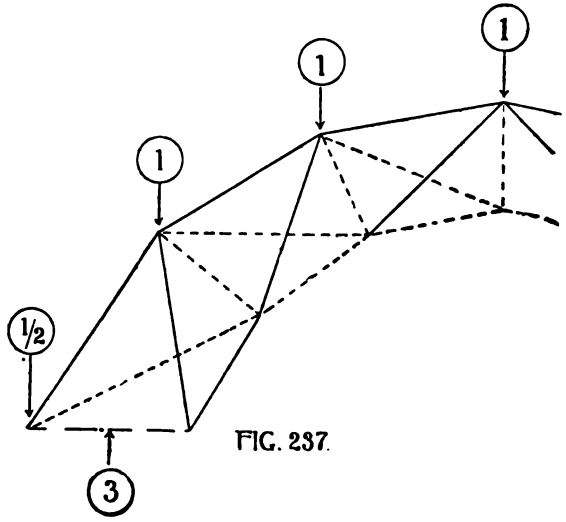


FIG. 237.

apex, here there are but two unknown strains $D\ J$ and $J\ C$ and we find them by drawing in our strain diagram Figure 243 the lines $d\ j$ and $j\ c$ parallel to them.

Having now found the point j in the strain diagram there is no difficulty with the rest.

This truss is really a combination of two trusses; we might build it by dividing it horizontally along the line $J\ H$ as shown in Figure 240. We should then have a bottom truss with two inclined struts $B\ G$ and $E\ K$, and one horizontal strut $J\ H$ and two ties $G\ O$ and $K\ O$, the loads on the joints $B\ J$ and $J\ E$ will each be increased by half of the apex load. Over this truss we should have another truss, with two inclined struts $J\ C$ and $D\ J$ and a horizontal tie

J H. This truss transfers its load to the lower truss. Now the remarkable thing that we have discovered is that the member *J H* is at one and the same time both in compression and tension. If we analyze the top truss separately (Figure 241) and also the bottom truss (Figure 242) we find that the compression greatly exceeds the

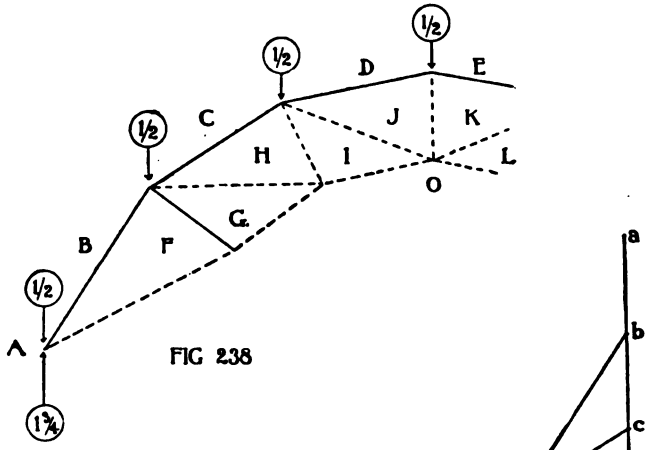


FIG 238

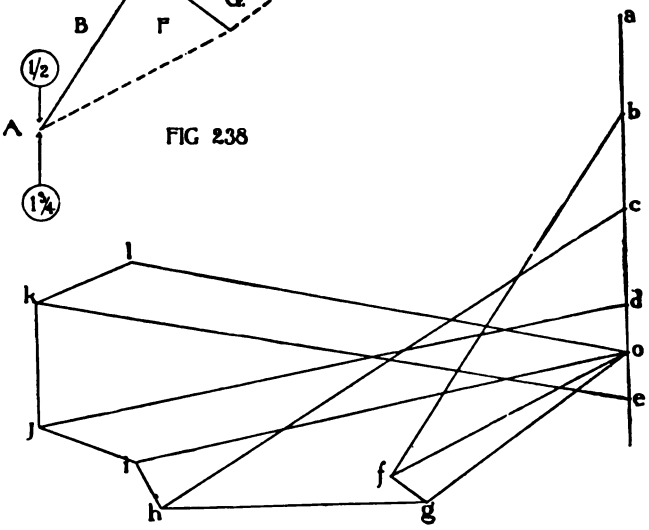


FIG. 238 a.

tension; the member, however, should be designed to resist both, having straps at the ends, sufficient to take up the tension. The actual stress in the member itself will, of course, be the difference between the two, and by referring to our single strain diagram (Figure 243) we see this clearly, for *h j* is the difference between *h o* the total

compression and *j o* the total tension. Usually there is a bolt at the joint *H O* connecting the two ties. This is done, as, when the wind

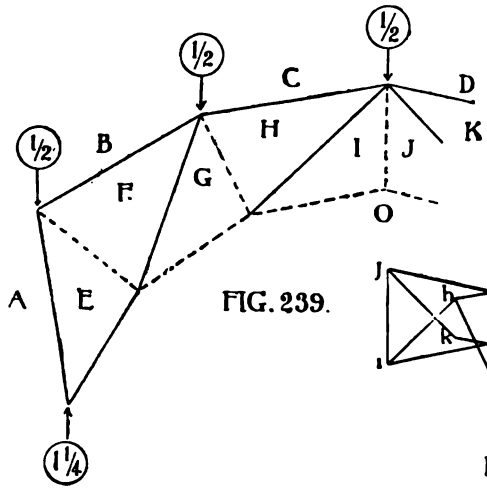


FIG. 239.

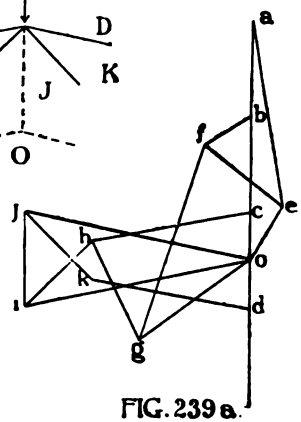


FIG. 239 a.

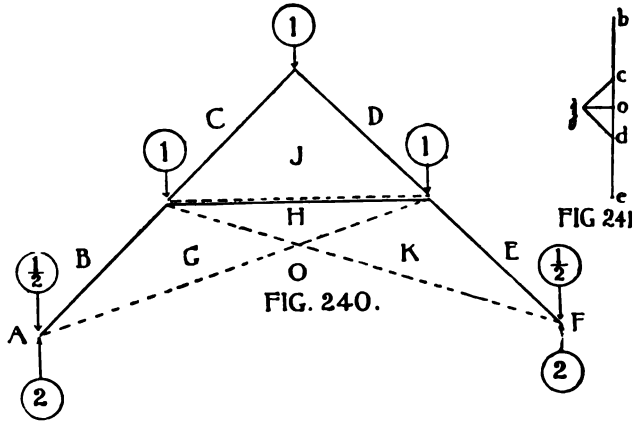


FIG. 240.

FIG 241

blows from one side only, the tie pointing to that side becomes a strut and is stiffened by being reinforced at this joint.

Figure 244 represents what is known in Gothic architecture as a "hammer-beam" truss. We can analyze this truss in two ways, first (Figure 244) assuming that the wall takes up the thrust of the truss, due to the absence of any horizontal tie; or second (Figure 246) we can consider the truss as composed of two inclined trussed rafters united at their upper joints and so taking up their own thrust.

Hammer-beam Truss.

The latter method will require a very much heavier truss.

In the latter case the semicircular member in the central panel becomes a tie, but the two lower quarter-circle members do not act at all, while in the first assumption the reverse is the case, the two lower quarter circle members acting as struts, while the semicircular member has no duty. In either case, however, these members come

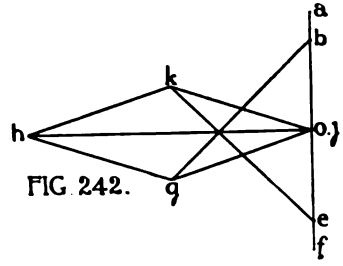


FIG. 242.

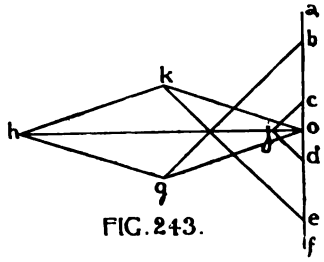


FIG. 243.

into play under wind-pressure calculated from one side only; it should be remarked here that when drawing the strain diagram all curved members must be assumed to be straight — (as shown at $L\,O$ in Figure 244 and at $Y\,O$ and $T\,O$ in Figure 246). Of course, the stress to resist the strain thus found will be greatly increased when the curved member is called upon to do the same duty as a straight member. The increase in stress is equal to that produced on a beam of the length of the curved member with a *bending-moment* at its centre equal to the original strain multiplied by the (longest) versed sine of the circle, or:

Increased stress in curved members.

$$m = s.x \qquad (129)$$

Where m = the (cross) bending-moment, in pounds-inch, existing in a curved strut or tie at its centre due to longitudinal strain.

Where s = the calculated longitudinal strain that would come on the strut or tie if it were straight, in pounds, per square inch.

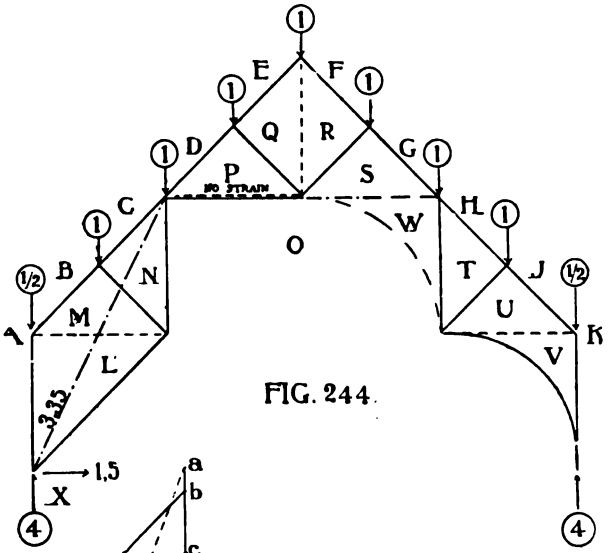


FIG. 244.

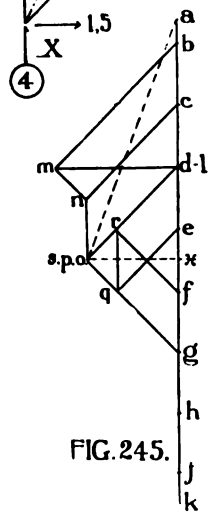


FIG. 245.

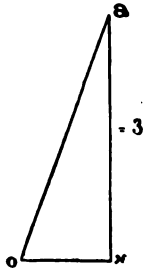


FIG. 247.

Where x = the length, in inches, of the longest versed sine of the curve (at the centre.)

If the largest versed sine is not at the centre, the bending-moment will be at the point where the largest versed sine is located. Of course, the curved member must be designed to resist both the

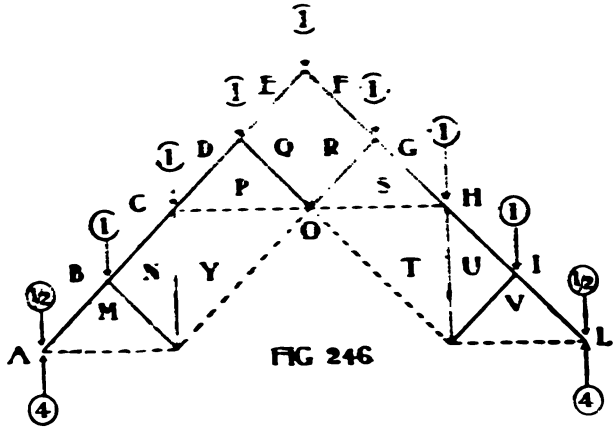


FIG 246

longitudinal strain and the additional strain due to the bending-moment.

Example I.

Example of curved strut.

A Georgia pine strut is 10 *feet long (measured on a straight line) but is curved, the versed sine at centre being* 20 *inches. It resists a compression strain of* 15000 *pounds. What size strut is required?*

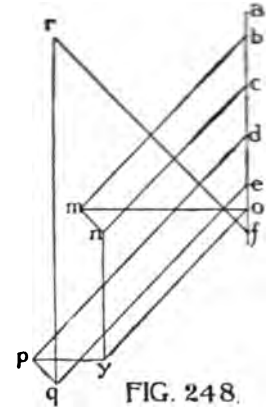


FIG. 248.

The bending-moment at centre will be Formula (129)

$$m = 15000.20 = 300000 \text{ pounds-inch.}$$

The safe modulus of rupture $\left(\frac{k}{f}\right)$ for Georgia pine (Table IV) is 1200 pounds. Inserting these values in Formula (18) we have the required moment of resistance to this bending-moment:

$$r = \frac{300000}{1200} = 250.$$

From Table I, Section No. 2, we have for a rectangular cross-section

$$r = \frac{b.d^2}{6} \text{ therefore}$$

$$\frac{b.d^2}{6} = 250, \text{ and}$$

$$b.d^2 = 1500$$

If now we make $d = 12$ inches, we should require a width

$$b = \frac{1500}{144} = 10{,}4 \text{ inches.}$$

It should be noted that d must always be assumed in the direction of the versed sine, that is resisting the bending-moment.

We must now add sufficient material to resist the longitudinal compression. As the strut will evidently be nearly square, 12 inches will be our least diameter, inserting therefore values in Formula (3) we should have (assuming the ends to be secured by bolts only):

$$15000 = \frac{a.750}{1 + \frac{120^2.0{,}00067}{12}} \text{ or,}$$

$$a = 36 \text{ square inches.}$$

Now as $a = b.d$ and d having been fixed at 12 inches we should have,

$$b = \tfrac{36}{12} = 3 \text{ inches.}$$

Adding this to the above we should require a strut,

12 inches $\times$ (10,4 + 3) = 12 $\times$ 13,4

In practice we should probably make it 12 inches $\times$ 12 inches.

Example II.

Example of curved tie. *The lower chord of a wrought-iron arched truss is made of a channel iron. At one panel the length between bearings is 5 feet; the tension 36000 pounds, the versed sine of the curve 6 inches. What size channel is required?*

The bending-moment m will be

$$m = 36000.6 = 216000 \text{ pounds-inch.}$$

Inserting values in Formula (18) we have required moment of resistance

$$r = \frac{216000}{12000} = 18$$

The additional area required to resist the direct tension will be

$$a = \frac{36000}{12000} = 3 \text{ square inches.}$$

We now consult Table XXI. We take the neutral axis normal to

web, as this will, of course, be the position of channel in the truss and in resisting the bending-moment.

We select for a trial the 10½ inch — 105 pounds per yard channel; its area a is 10.5 square inches, and its moment of resistance $r = 24.64$. Now of the area 3 square inches resists tension leaving us 7.5 square inches or $\frac{7.5}{10.5} = \frac{5}{7}$ of the whole amount to resist cross-bending. The amount of r available to resist cross-bending will, therefore, be $= \frac{5}{7} r = \frac{5}{7} 24.64 = 17.6$.

This is not quite equal to the required r (18) but is near enough for all practicable purposes, we should, therefore, use a 10½ inch — 105 pounds per yard channel.

We will now return to our hammer-beam truss. Assuming then that the wall is capable of resisting the thrust we should analyze our truss somewhat similarly to the way we did with the scissors truss.

Truss with abutment.

On the left side of truss (Figure 244) we omit the circle corresponding to $O\,W$ entirely, and draw the one corresponding to $O\,V$ straight. We now can assume that we can divide the truss horizontally at $P\,O$ which will give us a truss above $P\,O$ similar to Figure 213 and a truss below it consisting of an inclined trussed strut running from bottom joint X to joint $C\,D$ with a horizontal strut $P\,O$ and an outward thrust $O\,X$ to be resisted at its foot. To obtain this outward horizontal thrust $O\,X$ we will temporarily consider the inclined trussed strut as a straight line running from joint X to $C\,D$. The load will evidently be three-quarters of the load coming on the entire rafter. If then in Figure 247 we make $a\,x$ = this load or = 3 in our case, and draw $x\,o$ horizontally and $o\,a$ parallel to the straight line (representing the inclined strut in Figure 244) we shall have the amount of the horizontal thrust $x\,o$. We can now make separate strain diagrams for the upper and lower trusses, in which case we will find that for the upper truss there exists a tension just equal to $x\,o$ in the member $P\,O$ and for the lower truss there exists a compression just equal to $x\,o$ in the member $P\,O$. In other words there is no stress in $P\,O$. This is confirmed by drawing the combination diagram (Figure 248) where p and o fall on the same point.

That these strains just equal each other is due to the design of the truss and its uniform loading. Were the design of the parts less symmetrical, or the load not symmetrical, or the wind on one side only, the two strains would not be found to be equal.

Truss without abutment.

In Figure 246 where we consider the wall as incapable of taking up the thrust, and the truss has to take care of it itself, it will be found that this horizontal member (now called $P\ Y$) is in tension.

The drawing of the strain diagram (Figure 248) presents no difficulty, but it will be seen that nearly all the strains on the truss are very much larger.

The actual thrust of the truss Figure 244 on the wall will be parallel to and equal to $a\ o$ of Figure 245. To resist it there must be weight enough to deflect this line sufficiently not to overturn the wall. This subject was thoroughly treated in Chapter V, Vol. I, under the heading "Arch with Abutment."

Wind load-line normal to roof.

The analysis of strains due to wind pressure, when taken separately on one side of the roof only, and at right angles to same, is treated the same as for dead loads, excepting that the load line is no longer vertical, and is drawn therefore parallel to the wind arrows (that is, at right angles to the main rafter on which wind blows). Where the roof is a broken one the wind load line is broken also, its different parts being proportioned to the amounts and parallel to the directions of the wind on the different parts.

Wind reactions unequal.

It will be readily seen that the left and right reactions due to the wind cannot be equal, and the calculation of these reactions offers the only difficulty.

Let us take a truss similar to the one shown in Figure 240 and overlooking the steady or dead load assume that the wind is blowing from the right hand side. Its effect will be as shown in Figure 249.

The pressure on the apex or $A\ B$ will be equal to half the length of rafter $B\ G$ multiplied by the product of the distance between trusses and the wind pressure per square foot, as given in Table XLIV. It is indicated by the arrow $A\ B$. The pressure in the middle joint — indicated by arrow $B\ C$ — will be this same (latter) product multiplied by the sum of half the length of $B\ G$ plus half the length of $E\ C$. The pressure at the foot — or arrow $C\ D$ — will be the same product multiplied by half the length of $E\ C$. The reactions at p and q which resist this wind pressure will evidently be in the opposite direction to the wind, or slanting, as shown. If, therefore, the truss and dead load is not heavy enough to sufficiently deflect these reactions, there would need to be bolts or buttresses at

p and q to keep the truss from sliding-off the wall. This will be explained at the end of the chapter in speaking of spires.

We now have the amount and direction of wind pressures and

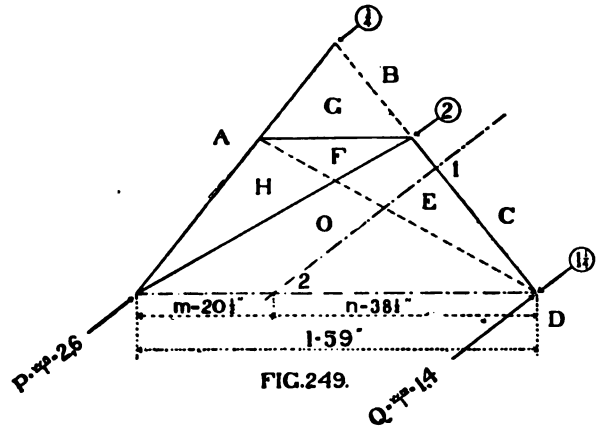


FIG. 249.

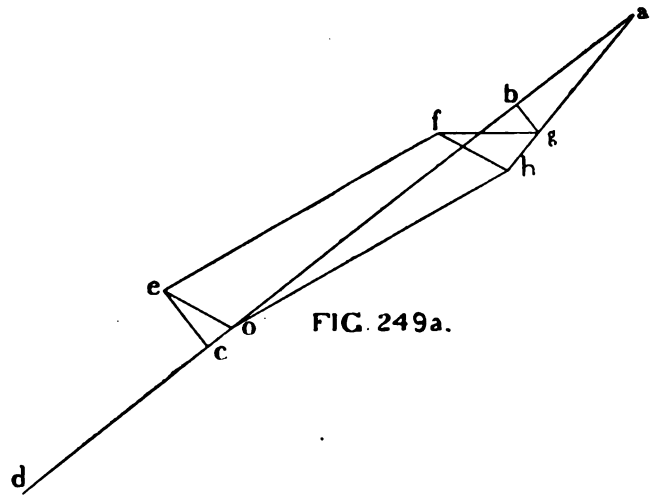


FIG. 249a.

direction of the reactions, but we do not know the amounts of the reactions, as they evidently cannot be equal.

If we imagine the neutral axis of the entire wind pressure, that is the entire wind pressure concentrated at the point 1 at the centre of the length of rafter, the effect, so far as reactions are concerned will be the same.

We now draw the horizontal line $P\,Q$ connecting the feet of rafters, prolong the centralized wind pressure (or neutral axis) arrow 1 till it intersects $P\,Q$ at 2, then will the reactions P and Q be inversely as the divisions of line $P\,Q$; that is, if $P\,Q = l$ and $P\,2 = m$ and $2\,Q = n$ and the whole wind pressure $= w$, we should have

Amount of wind reactions.

$$p = \frac{w.n}{l} \qquad (130)$$

and

$$q = \frac{w.m}{l} \qquad (131)$$

Where $p =$ the amount of (slanting) left reaction, in pounds, due to wind pressure.

Where $q =$ the amount of (slanting) right reaction, in pounds, due to wind pressure.

Where $l =$ the length, in inches, measured horizontally between centres of feet of truss.

Where m and $n =$ respectively, in inches, the lengths into which the horizontal line is divided by the prolongation of the centre line (or neutral axis) of wind pressure, m being nearer p and n nearer q.

The analogy between these formulæ and Formulæ 14 to 17 should be noticed. The analyzing of the strains by means of the diagram Figure 249*a* will present no difficulty. We draw $a\,b$ parallel and equal the pressure $A\,B$ at the apex, $b\,c = B\,C$ pressure at central joint and $c\,d = C\,D$ pressure at foot. Along this load line $a\,d$ we now lay off in opposite directions the reactions, namely,

$d\,o = D\,O$ or reaction q

and

$o\,a = O\,A$ or reaction p.

We first find the strains on the foot joint $O\,A$ by drawing $a\,h\,o\,a$, then skip to the apex joint and draw $a\,b\,g\,a$; the rest presents no difficulty.

It will be noticed that the effect of the wind is to reverse the strains on two pieces.

The upper part of the rafter on the side on which the wind is

blowing is now in tension, whereas in Figure 240 it was in compression; further the tie *H O* now becomes a thrust member opposing the wind, where before it was in tension.

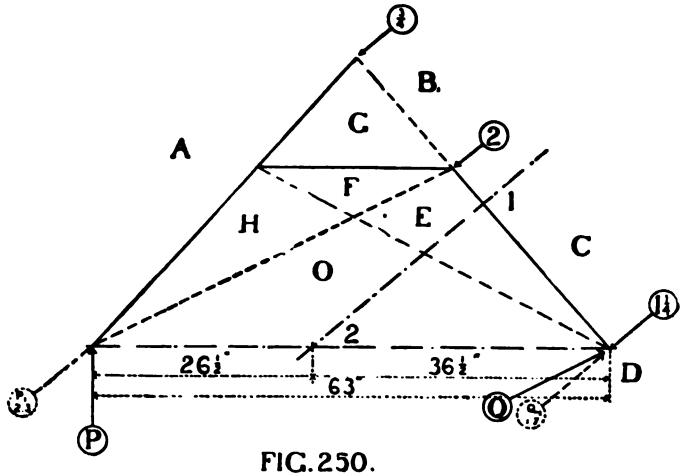


FIG. 250.

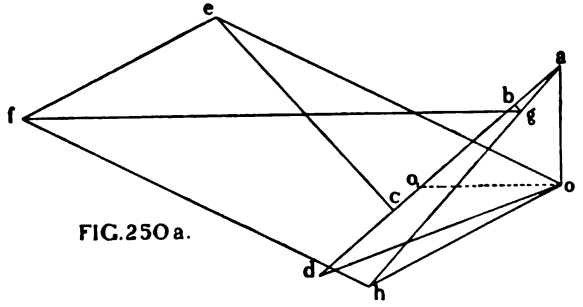


FIG. 250a.

Had we drawn the wind on the left side of truss the calculation would have been similar. The direction of *p* and *q* would have been from right upwards to the left; *p* would have been the larger reaction and we should have found the strains same as in Figure 240, except on *A G* which would become tension and on *H F* (or *O E*) which would now become compression. It is not necessary to make

more than one wind diagram, therefore, where both feet of the trusses are bolted down.

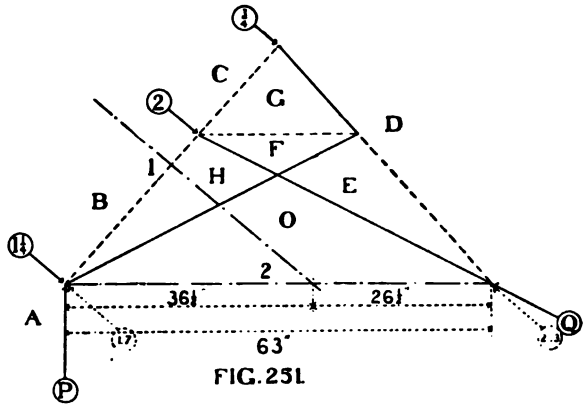


FIG. 251.

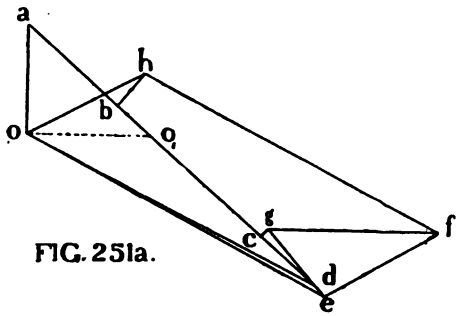


FIG. 251a.

Truss with rollers.

If, however, one foot only were bolted down and the other foot were placed on rollers, to allow for expansion and contraction, we should have to make two diagrams, with the wind from left and right respectively, as it makes a decided difference to the strains as shown in Figures 250 and 251. In Figure 250 the wind is from the right with rollers under the left or opposite foot of truss; in Figure 251 the wind is from the left and the rollers are under the left (or same) foot of truss. It will be readily seen that in both cases the (left) roller foot will not oppose any tendency to slide due to the wind, the right

foot must therefore be bolted down to resist this tendency in both cases. We can readily see therefore that in Figure 251, the member HF or OE must be a brace to keep the truss from sliding towards the right foot, whereas in Figure 250, it must be a tie to hold back the truss from sliding away from the right foot. The strain diagrams will show this to be the case. To obtain the reactions we proceed as before, so far as the central wind pressure ($\frac{1}{2}$) and lines P 2 and 2 Q are concerned, but it is evident that the reaction P will be vertical, while the reaction Q will no longer be in direct opposition to the wind as shown by dotted line, as it is deflected from this line, owing to the horizontal (sliding) strain to be taken up from the other foot.

Wind opposite rollers.

We therefore draw in Figure 250*a* the load line $a\ b\ c\ d$ as before, also the reactions $d\ o_1 = q$ and $o_1\ a = p$. Through a draw the vertical projection $a\ o$ of $a\ o_1$ — (that is draw $o_1\ o$ horizontally and $a\ o$ vertically) — then will $o\ a$ be the amount of vertical reaction p and $d\ o$ the direction and amount of vertical reaction q. The rest of the strain diagram is easily drawn, remembering to take the foot joint first and then to skip to the apex joint.

Wind on roller side.

To analyze Figure 251, it must be treated in exactly the same way and it offers no difficulty. We notice that Figure 250 has strains similar to those in Figure 240 excepting the upper part of rafter on the wind side, which is now in tension. In Figure 251, however, things are greatly changed. The upper end of rafter becomes a tie, both ties become struts and the short strut $G\ F$ (or $J\ H$ in Figure 240) now becomes a tie.

Wind and load in one diagram.

Wind strains and steady loads can be calculated from one strain in diagram, as shown in Figure 252 and Figure 252*a*. This is a combination of the conditions obtaining in Figures 240 and 249 and the strain diagram of their respective strain diagrams, Figures 243 and 249*a*. By simply following around the arrows, as there shown (first obtaining the amounts of $A\ B$ and $K\ O$) the diagram offers no difficulty whatever. If an actual example were figured out, first separately, as shown in Figures 240 and 249, and then in combination, as shown in Figure 252, the result would be the same, remembering to make all compressive strains positive and all tension strains negative and to obtain the arithmetical result of the consequent additions or subtractions. A practical example will be given in the next chapter.

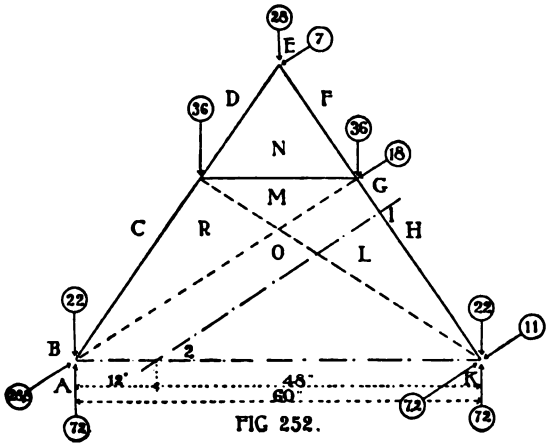


FIG 252.

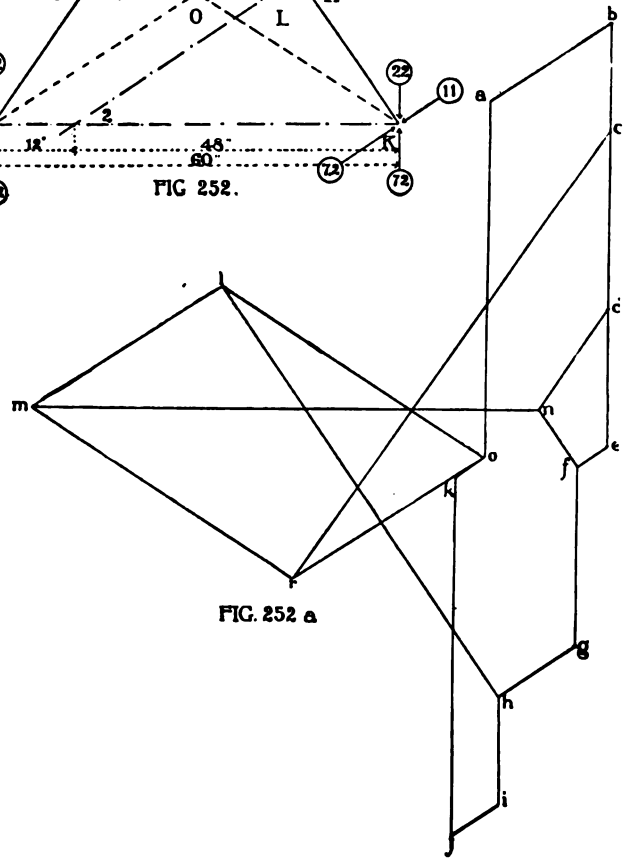


FIG. 252 a

Where the roof rafter or top chord of a truss is not in one straight line, but is made-up of a series of differently inclined lines, as in a

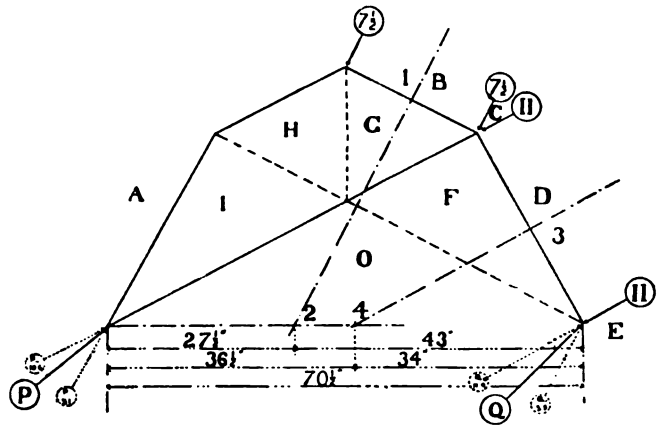


FIG.253

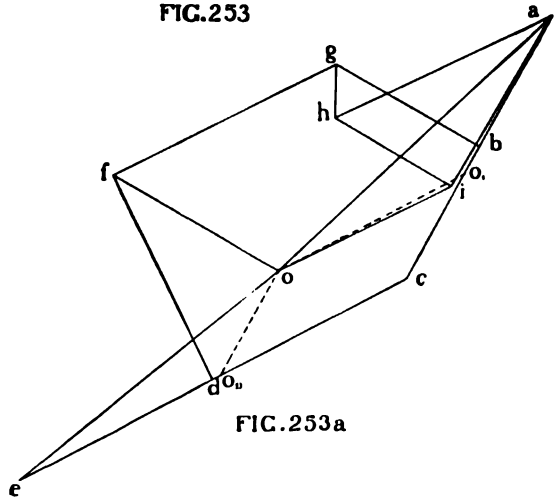


FIG.253a

broken roof, or in a circular or arched truss, the wind pressure will, of course, be at different angles too, each at right angles to its respective surface. We can in such a case, work out separately the reaction due to the wind pressure on each surface, and then obtain the resultant

Wind on broken roof.

(reaction) of all these lesser ones, as explained on p. 71, Vol. I, or we can combine them all into one diagram. In Figure 253, we have a mansard roof with wind pressure from the right. We draw the neutral axis (or central lines) of wind pressure on each rafter prolong them to their intersections with the horizontal and can thus figure out the respective reactions due to each. In the strain diagram Figure 253*a* we now draw $c\,e$ parallel and equal to total wind pressure (3) on rafter $F\,D$; and $a\,c$ parallel and equal to total wind pressure (1) on rafter $B\,G$. Draw $e\,a$ and it will be the direction and amount (sum of) the actual reactions. Now to get each reaction separately make $e\,o_{\text{II}} =$ to reaction q of wind pressure (3) on $F\,D$; and $c\,o_{\text{I}} =$ to reaction q of wind pressure on $G\,B$; of course $o_{\text{II}}\,c$ and $o_{\text{I}}\,a$ will equal the reactions p due to the respective wind forces. Or we might divide $e\,c$ and $c\,a$ so that

$$e\,o_{\text{II}} : o_{\text{II}}\,c = P\,4 : 4\,Q$$

and

$$c\,o_{\text{I}} : o_{\text{I}}\,a = P\,2 : 2\,Q$$

We now draw $o_{\text{II}}\,o$ parallel $c\,a$ and if we have drawn correctly $o\,o_{\text{I}}$ must be parallel $e\,c$. Having found o the rest of the diagram presents no difficulty. We make $a\,b = A\,B$ or pressure at apex; $b\,c = B\,C$ pressure at joint C due to wind on $G\,B$; $c\,d = C\,D$ pressure on same joint C due to wind on $F\,D$ and finally $d\,e = D\,E$ pressure at foot and proceed with the other lines.

Were we to make a strain diagram for a steady load on all joints we should find similar strains on all members except $I\,H$. This is a compression member of the truss, but becomes subjected to tension when the wind blows from the right.

Similarly $G\,F$ would become tension if the wind were from the left.

Wind tendency to overturn roofs.

We will consider the tendency of wind to overturn roofs, and this can best be done by calculating one or two practical examples of steeples. Before doing this, however, the student should be warned to *always* arrange for *wind braces*, that is, diagonal ties between the trusses and in a plane at right angles to the trusses. The object of these, is to make all the trusses (that is the entire roof), act as one mass and thus keep the wind from blowing over each truss individually, and thus collapsing the roof. The arrangement of these ties varies with circumstances. They are usually placed immediately under the roof surface, that is, from foot of

Wind braces.

one rafter to apex of next rafter on each side. If the rafters are long they are placed diagonally in the individual panels. Sometimes a longitudinal truss is made the entire length of ridge, by wind bracing from both sides of each apex to centre of each neighboring main tie.

Example III.

A wooden, slate-covered steeple, 27′ 6″ high, covers a 24′ square brick tower. The walls are 16″ thick at the top and the steeple is anchored down four feet into the walls. Is this sufficient to keep it from blowing over?

Wooden steeple.

It makes no difference just how the anchoring is done, it can be either by means of iron anchors bolting the plate down to the masonry below, or by means of wooden trestle work built inside and against the walls, which will also force the steeple to lift the walls from their bearings, before the steeple can topple over.

The wind pressure on one side of the roof will be the area of this side multiplied by 40 pounds per square foot (see Table XLIV) and it will act or can be considered as centralized at the centre of gravity of the side, which, being a triangle would be at one-third its height, or at D in Figure 254. The length of rafter will be 30′, therefore area of one side $= 30.12 = 360$ square feet, and wind pressure $F D = 360.40 = 14400$ or say 15000 pounds total wind pressure which we consider as centralized at D and normal to rafter. Resisting this, we have the dead load, that is the weight of steeple and of the masonry as far as steeple will have to lift it.

Taking the weight of steeple at 20 pounds per square foot (making, of course, no allowance for snow) and weight of brickwork at 112 pounds we have weight of (four sides of) steeple,

$$= 4.360.20 = 28800 \text{ pounds.}$$

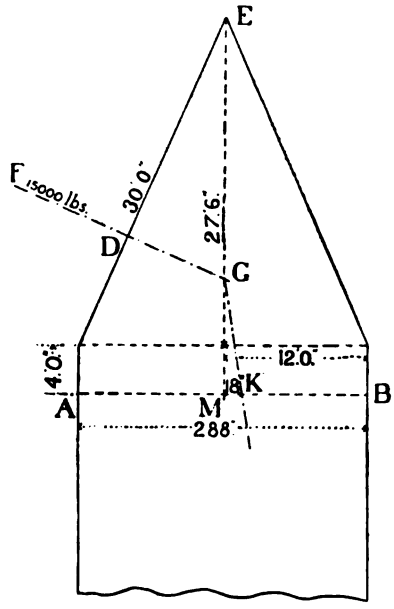
Weight of masonry

$$= 4.(24 - 1\tfrac{1}{2}).4.112 = 54208 \text{ pounds,}$$

or total dead (vertical) load $= 83000$ pounds.

We now draw $c\,a$ parallel and $= 15000$ pounds, the wind pressure; and $a\,b$ vertically and $= 83000$ pounds, the dead load. Draw $c\,b$ and from intersection G of $F D$ with the main vertical neutral axis of the dead load draw $G K$ parallel $c\,b$ till it intersects the horizontal

joint line $A\ B$, which we draw four feet below the tops of walls or at the anchor level. We now use Formulæ (44) and (45) to obtain the extreme edge strains at A and B.



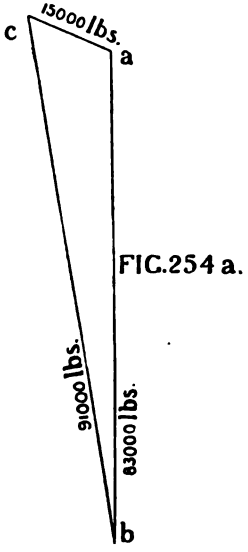


FIG.254 a.

FIG.254.

We have these values:

p = the pressure = $c\ b$ = 91000 pounds.

$x = M\ K = 18''$ (by measurement)

$d = A\ B = 24.12 = 288''$

a = area of wall at $A\ B = 4.16''.(288 - 16)$

$= 17408$ square inches.

We have then for pressure at nearer edge B

$$= \frac{91000}{17408} + 6.\frac{18.91000}{17408.288}$$

$= +7{,}21$ pounds compressive pressure, per square inch; and at A

$$= \frac{91000}{17408} - 6.\frac{18.91000}{17408.288}$$

$= +3{,}25$ pounds compressive pressure, per square inch.

If the latter value had been a negative one, we should have had to rely on the quality of the mortar not to tear apart at *A* and thus allow the steeple to fall. It would be better, however, in such a case to carry the anchoring process further down, and thus gain more dead vertical load to resist the wind pressure.

Example IV.

A square stone steeple has 12″ stone sides at the top; 19 feet from the top vertically, the length of side is 12 feet. Is the steeple safe against wind pressure at this point?

Stone steeple.

This example is intended to show that we can examine any point of steeple similarly to the manner of examining the base.

In Figure 255, *A D* measures 12 feet = 144″; *M C* = 19 feet and *C D* scales 20 feet, hence area of each side = 6.20 = 120 square feet, and weight of each side approximately = 120.150 = 18000 pounds and weight of four sides or vertical load = 4.18000 = 72000 pounds.

The wind pressure will be

$$= 120.40 = 4800$$ pounds and will be centralized at *B* or one-third the height of *A C*.

We draw *B G* normal to *A C* till it intersects *C M* at *G*.

Make *a b* = 72000 pounds and vertical, draw *c a* = 4800 pounds and parallel *B G* and draw *a b* which scales 74000 pounds. Draw *G K* parallel *c b* and we find *K* is 4″ distant from centre of joint *M*. The area of joint is = 4.11.144 = 6336 square inches. We have, therefore, pressure at nearer edge of joint *D*,

$$= \frac{74000}{6336} + 6.\frac{4.74000}{6336.144}$$

$$= + 13.65$$ pounds, per square inch.

and at edge *A*

$$= \frac{74000}{6336} - 6.\frac{4.74000}{6336.144}$$

$$= + 9.75$$ pounds per square inch.

In a similar manner we might examine any other joint of the steeple.

It will be found that at the very top or near it the greatest danger exists. The finial frequently exposes a large surface to the wind and almost at right angles to the vertical dead load, deflecting this

line much more in proportion. Then, too, the mortar is so much exposed that it cannot be relied on. For this reason there is placed usually a vertical iron tie-rod from the finial to some point below, frequently even below the springing line of the steeple. This arrangement is all right, provided the rod is properly put in. The writer has seen ponderous rods 2″ diameter or more and perhaps, fifty or sixty feet long, intended as tie-rods, that had become so loosened by contraction and expansion that they could be easily

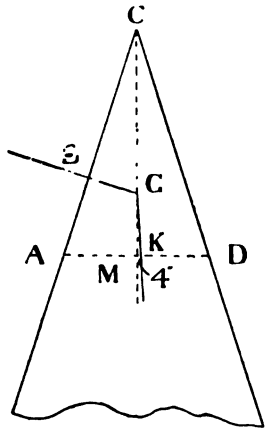


FIG. 255.

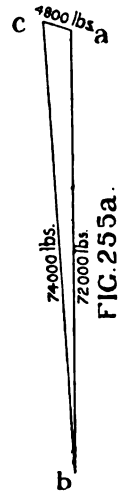
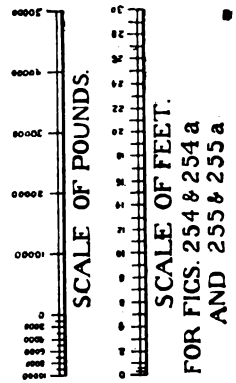


FIG. 255a.



swayed back and forth by the hand. Hence their only service was their own dead weight.

Vertical tie-rods in steeples.

The way to put in such rods is to leave the lower end free to move vertically, that is up and down, but secure it against lateral movement and then to attach to the lower end a heavy weight, proportioned according to circumstances.

Wind allowances for different surfaces.

In figuring the allowance for wind it is customary to take only one-half the usual allowance for circular or surfaces slanting to the direction of wind. This is done because they offer less resistance than a flatly opposed surface, allowing the wind to slip by more readily. In a circular steeple we should take as our area the half circumference of base multiplied by the length on the rafter line and

multiply this by only one-half the pressure for wind as given in Table XLIV. This usually would be 20 pounds per square foot.

For an octagonal roof we should assume full pressure on the central surface and only half pressure on the two side surfaces. Where the octagon is a regular one, this would amount practically to assuming double pressure on one surface only.

There is no danger of a steeple blowing over diagonally, as the sides would in such a case present slanting surfaces to the wind, allowing it to slide off readily; and besides the base line, being the diagonal of the square, would be so much longer.

On vertical (wall surfaces) the writer usually allows only for a maximum wind pressure of 30 pounds per square foot, for, as a rule, they (or part of them) are low down near the ground and therefore not exposed to the full force of the wind.

CHAPTER XII.

WOODEN AND IRON TRUSSES.

IN Figure 256 we have the design of one-half of an ordinary king-post truss, with three bays to each principal rafter. Figure 257 gives the strain diagram and Figure 258 the design of the truss in detail. The length of principal rafter is 25 feet 6 inches, therefore

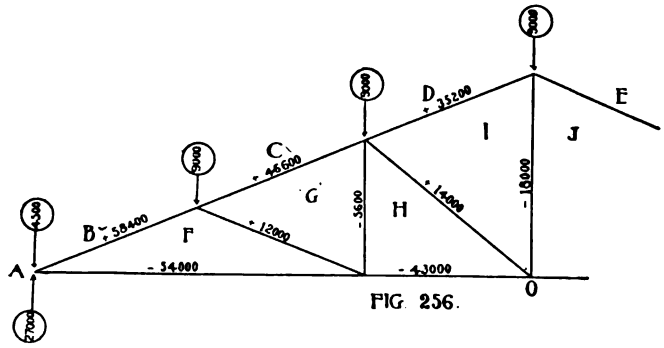


FIG. 256.

length of each bay 8 feet 6 inches. The trusses in the building from which this example has been taken had to be spaced far apart — 19 feet from centres — on account of ceiling and skylights; this, however, is not an economical arrangement, as it increases not only the sizes of the truss members, but of all the purlins as well. The weight assumed was 55 pounds per square foot of roof surface, being made up as follows:

Large spaces extravagant.

Slate,	7 pounds.
Plaster,	8 pounds.
Boards and construction,	10 pounds.
Wind and snow,	30 pounds.
Total	55 pounds.

The slate was the ordinary roofing slate; if the slate used had been called for of even thickness and ¼ inch thick the allowance would have been 10 pounds.

The weight of "Boards and construction" was estimated and not checked off, in this case. In important constructions and particularly in ironwork, this should be carefully weighed up.

The load on each joint of our truss was therefore:

$19.8\frac{1}{2}.55 = 8882$ pounds,

or say 9000 pounds on each joint, excepting, of course, the joints at reactions which carry only half-loads.

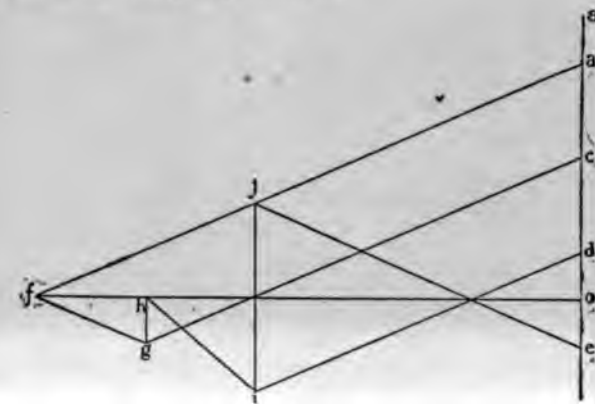


FIG. 257.

The loads on the truss being symmetrical, each reaction will be one-half the total load or 27000 pounds.

There is no difficulty in drawing the strain diagram, or in finding the stresses in each member, the latter are marked on Figure 256, the positive + sign denoting compression, and the negative — sign tension.

Detailing wooden truss.

We now proceed to detail the truss. The principal rafter will evidently, though not necessarily, be made in one piece from foot to apex, the largest strain — at the bottom panel — will therefore determine its size. This is 58400 pounds compression. The rafter is evidently a series of columns each 8 feet 6 inches long, or comparatively short columns; for the struts brace the rafter against yielding downward;

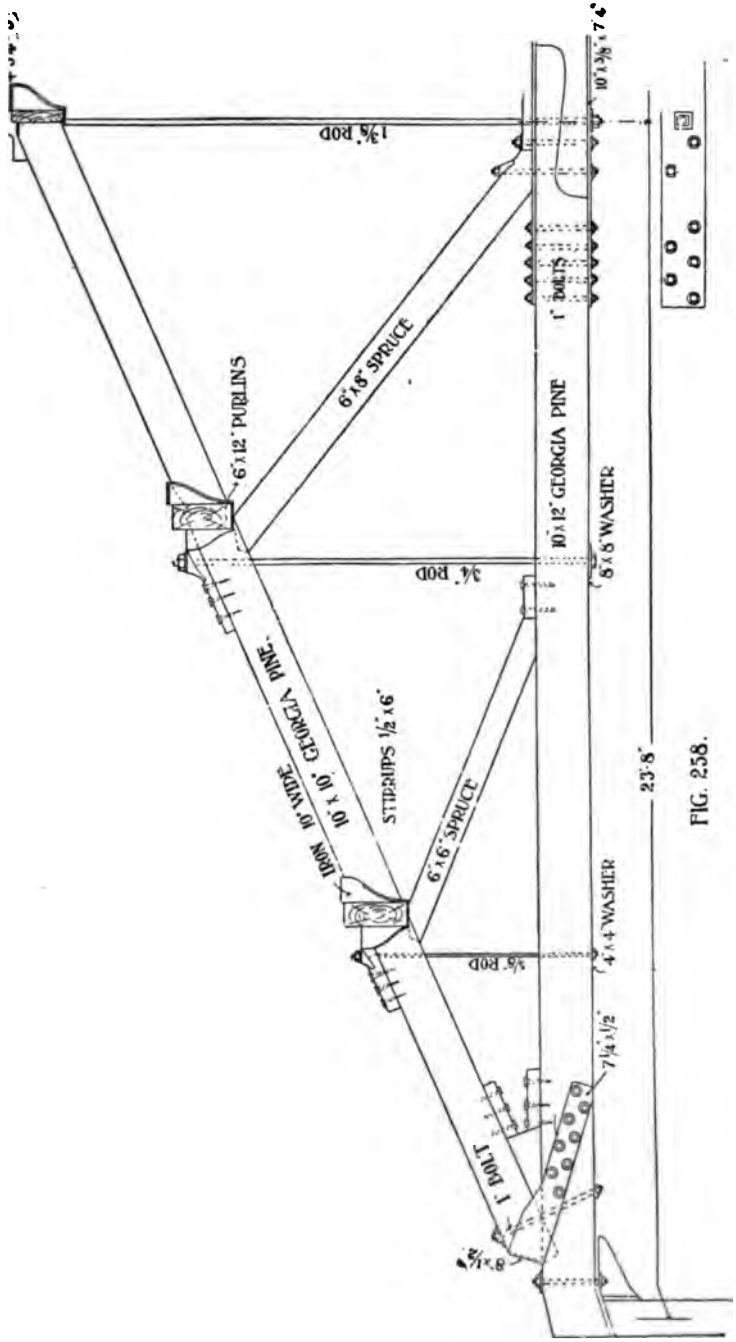

1¾" ROD
6"x12" PURLINS
6"x8" SPRUCE
10"x12" GEORGIA PINE
1" BOLTS
10"x¾"x7'4"
8"x8" WASHER
¾" ROD
10"x10" GEORGIA PINE
STIRRUPS ½"x6"
6"x6" SPRUCE
4"x4" WASHER
⅞" ROD
7¼"x½"
1" BOLT
23'8"

FIG. 258.

the loads and tie-rods keep it from yielding upwards; and the purlins keep it from yielding sideways. The safe compression per square

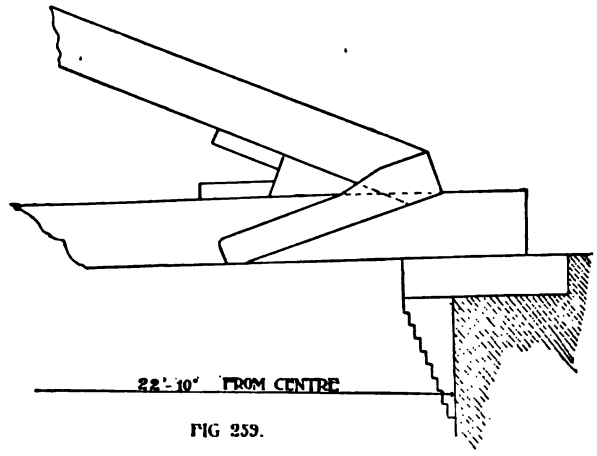


FIG 259.

inch of Georgia pine (see Table IV) is 750 pounds. Without bothering with the complex Formula (3) for columns, we will assume that we can safely use

Principal rafter.

$$\left(\frac{c}{f}\right) = 700 \text{ pounds in our case,}$$

we require then in the main rafter an area of cross-section

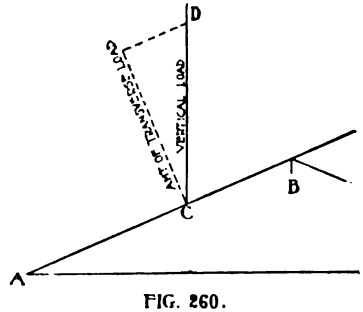


FIG. 260.

$$a = \frac{58400}{700} = 83{,}4 \text{ square inches,}$$

or say a timber $8\frac{1}{2}$ inches x 10 inches. This we increase to 10 inches x 10 inches to allow for cutting away at bolt-holes.

If the purlins had not been placed directly over the joints we should have to increase this size to provide for the transverse strain in each panel. In such a case we should have a beam 8 feet 6 inches long, supported at both ends. The transverse load on the beam would not be the full

vertical load, but only its resultant normal to the rafter, as shown in Figure 260.

Vertical load on slanting beam.

If $C\,D$ were drawn at any scale equal to the vertical transverse load on rafter $A\,B$, draw $E\,D$ parallel rafter, and $E\,C$ normal to same, then will $E\,C$ measured at same scale as $D\,C$ represent the transverse load on the rafter $A\,B$, the latter being considered as a beam of length $A\,B$ supported at A and B.

Returning to our truss; if it were not supported sideways by purlins we should have to greatly increase the size of rafter, to guard against lateral flexure, see Formula (5).

Tie-beam.

The stress in tie-beam is 54000 pounds tension, the safe $\left(\frac{t}{f}\right)$ for Georgia pine (Table IV) is 1200 pounds, therefore required area

$$a = \frac{54000}{1200} = 45 \text{ square inches.}$$

In a wooden truss the principal members are naturally of same thickness, therefore we should require $4\frac{1}{2}$ inches x 10 inches to resist tension. To this must be added the necessary area for bolt-holes, transverse strain due to ceiling hanging to tie-beam, etc. In our case the beam was pieced, as shown at centre, and it was therefore rather heavily increased and made 10 inches x 12 inches.

Struts.

The struts were of spruce; their size will, as a rule, be determined by the area of their bearing against the principals — so as not to indent these — rather than by their length as columns, though both should be tried.

By making our larger strut 6 inches x 8 inches we get a bearing against the rafter of about 6 inches x 10 inches, or say 60 square inches, this would make a compressive stress *across the grain* on the Georgia pine rafter of

$$\frac{14000}{60} = 233 \text{ pounds per square inch.}$$

Table IV gives 200 pounds as safe across the grain on Georgia pine, but we can pass the above as safe. It should be remembered that

Nature of compression large blocks.

the action of crushing stress in short blocks greatly resembles that of an explosion. A short block or cube of stone, if compressed, will fly off in the four directions normal to the four free sides, with sudden and great force, as if there had been an internal explosion. If the block is large

the central fibres will tend to explode outwardly, while, of course, the fibres nearer the edge will tend to explode inwardly and as a result the central fibres, meeting with the opposition of the external fibres, will resist more compression than they would if they were free. This is confirmed by actual experiment, where it is found that large blocks of the same material will resist crushing proportionately to a very much greater extent than will the small blocks.

Where, therefore, we are proportioning the sizes of struts — or later the sizes of washers — for their bearing area, *across* the wood, we will bear the above in mind, and if the safe values for crushing across the grain, as given in Table IV, require unusual dimensions, we will increase the amount of the safe value per square inch, as our judgment may dictate.

Now trying the strut as a column of spruce 6 inches x 8 inches, or 48 inches area, and 9 feet, or 108 inches long, we should have from Formula (3) the safe load:

$$w = \frac{48.650}{1 + \frac{108^2 . 0{,}00033}{\frac{6^2}{12}}}$$

$$= 13700 \text{ pounds},$$

which is near enough to be safe.

In calculating the other strut we should find that as a column 6 inches x 6 inches and 6 feet 3 inches long it can safely carry 14400 pounds, the actual strain being 12000 pounds. For pressure across the grain against the rafter and main tie-beam we have an area of about 6 inches x 8 inches against each or 48 square inches, therefore actual compression per square inch across the grain = 250 pounds, which they can safely stand, as these parts are of Georgia pine and the crushing area quite large.

Tie-rods.

We next proportion the sizes of tie-rods, which will be of wrought-iron. The safe tensional stress of wrought-iron being 12000 pounds per square inch we require areas as follows:

For the principal rod,

$$= \frac{18000}{12000} = 1\tfrac{1}{2} \text{ square inches.}$$

For the side rods,

$$= \frac{5600}{12000} = \tfrac{7}{15} \text{ or say } \tfrac{1}{2} \text{ square inch.}$$

The rods being circular the principal rod would need to be $1\frac{3}{8}$ inch diameter and the side rods $\frac{1}{16}\frac{3}{8}$ inch diameter or say $\frac{3}{4}$ inch.

We also place a small $\frac{5}{8}$ inch diameter rod in each end panel to keep the tie-beam from sagging. The rods will all have to have their ends upset, or else they will not have enough sectional area between the threads. In practice, however, it would be cheaper, where the rods are so small and short, to enlarge their diameter and pay for the extra material, rather than to save this and have to pay for expensive blacksmith's work.

Upset ends.

Screw ends should only be upset where the material saved fully counterbalances the labor.

We next proportion the washers, they bear against Georgia pine and across its grain and should, therefore, be proportioned at about 200 pounds per square inch.

Washers.

For the principal rod we will need, therefore, a bearing area for each washer of

$$\frac{14000}{200} = 70 \text{ square inches,}$$

or say 8 inches x 8 inches.

For the side rods we will need

$$\frac{12000}{200} = 60 \text{ square inches,}$$

or about the same, we will therefore to save expense make them all alike.

The lower washers are made to bear horizontally against both head (or nut) and tie-beam, but the upper washers have to be modelled to bear against the slanting side of rafter, and horizontally against nut. It will also be noticed that the lower end of the washer is "toed-in" to the rafter to keep the washer from sliding.

The wooden blocks which are screwed onto the rafter at each washer were made to allow for cutting away, to provide horizontal surfaces on which to rest the bridle irons which carried the purlins.

It will be noticed that the principal tie-beam is pieced at the centre. The cut or halving was made slanting, so as to force each half to bear on and pull against the other half; and all sharp edges, where sudden increase in strains would take place were avoided by rounding off, as shown. Wrought-iron plates were placed over and below the cut and sufficient bolts placed each side of the cut, not to crush the wood.

Halving long timbers.

We now must still design the foot-joints.

Bearing area. The bearing area required must first be found; we can safely put 200 pounds per square inch on Georgia pine, across the grain, and our load at each reaction being 27000 pounds, we need

$$\frac{27000}{200} = 135 \text{ square inches of bearing area, and as the timber}$$

is 10 inches wide, it would have to rest on its bearing for a length of 13½ inches.

In this case, however, we made it 24 inches, because the principal rafter bearing on the tie-beam so far from the support, it would have been apt to bend it. The left end rested on an iron column and was bolted to it as shown in Figure 258. In Figure 259 is shown the right end, which rested on and was bolted to a wall. The latter was corbelled out under the truss and capped with blue stone.

Bolt at foot. We next provide at each foot an inclined bolt, to hold the rafter down to the tie-beam and keep it from jumping out of the strap, which it might do, if subjected to heavy transverse strains. We next "toe-in" the rafter to the tie-beam as shown, taking care to get enough bearing area and area ahead of the toe to keep it from pushing or shearing its way out through the tie-beam, or from being sheared off itself. In our case, however, the toe is made small, the only reliance we place on it being to steady the rafter sideways. The rafter transfers all the load vertically across the tie-beam, we shall therefore need the same length of bearing area on the tie-beam, 13½ inches, as the latter needs on the wall.

We more than get this by means of the hardwood block, driven in tightly with its edge grain against the rafter and tie-beam and held in position by screwed on blocks.

Calculating strap. We next calculate the strap. The strains coming on it are 54000 pounds tension and resisting this 58400 compression. The lesser will, of course, be the one it must resist. The width of strap required not to crush the rafter must first be determined. The strap bears almost along the grain of the rafter, but not quite, we will therefore reduce slightly the safe allowance for compression along the grain as given in Table IV, and allow, say, 670 pounds per square inch, we then require $\frac{54000}{670} = 80$ square inches, or the rafter being 10 inches wide,

a strap 8 inches wide. The thickness of strap should be sufficient not to shear off, we have two shearing areas one at each angle or $\frac{54000}{2} = 27000$ pounds on each. The safe shearing stress on wrought-iron being 8000 pounds per square inch (see Table IV) we need an area at each angle $= \frac{27000}{8000} = 3\frac{3}{8}$ square inches. The strap being 8 inches wide would therefore need to be about $\frac{7}{16}$ inch thick. It was made $\frac{1}{2}$ inch, however, as in this case, it was doubtful whether the ironwork would be of a high character. Now each side of the strap will have to withstand a tensional strain of 27000 pounds. While the safe tension on wrought-iron in Table IV, is 12000 pounds, in this case it was assumed at 9000 pounds only, or the strap needed a sectional area $= \frac{27000}{9000} = 3$ square inches.

The strap being $\frac{1}{2}$ inch thick, would therefore need to be 6 inches wide, to which must be added $1\frac{1}{4}$ inch to allow for bolt holes, making the strap $7\frac{1}{4}$ inches x $\frac{1}{2}$ inch. We now settle the number of bolts, which we will make 1 inch diameter.

The safe shearing on a 1 inch bolt will be $= \frac{1}{11}.8000 = 6300$ pounds. Each bolt has two shearing areas, one at each end and will therefore resist 12600 pounds. The total strain being 54000 pounds, we need $\frac{54000}{12600} = 4{,}3$ or say five bolts to resist shearing.

Number of bolts.

For bearing we have an area 1 inch diameter by $\frac{1}{2}$ inch thick against the strap at each end of each bolt, or just one square inch bearing area against iron to each bolt, which will equal a safe resistance to compression of 12000 pounds per bolt, we need therefore $\frac{54000}{12000} = 4{,}5$ or say five bolts to resist the compression of the strap.

The bolts also bear against the Georgia pine tie-beam and tend to crush it along the grain, the safe resistance of the wood being 750 pounds per square inch. Each bolt bears against 1 inch x 10 inches or 10 square inches of wood, and will safely bear therefore $10.750 = 7500$ pounds. We need therefore, to avoid crushing the wood $\frac{54000}{7500} = 7{,}2$ or say eight bolts, which is the number shown in drawing.

We will now similarly analyze an iron truss. This, the same as the above wooden truss, is not intended so much as a guide in design-

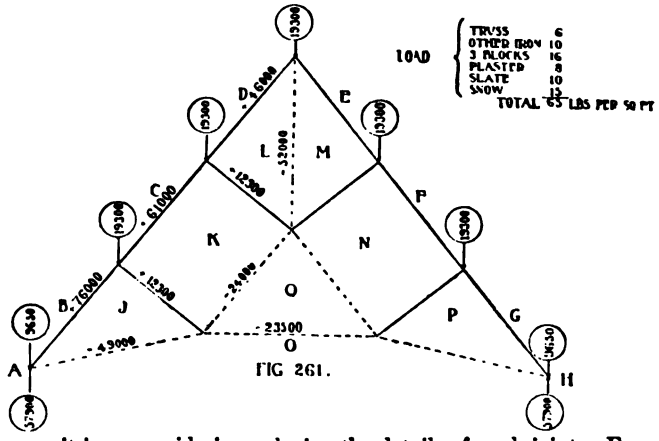


FIG 261.

ing, as it is as a guide in analyzing the details of each joint. Every truss should be designed with reference to its special duty and position, and joints should be designed in each case to be made up as simply as possible.

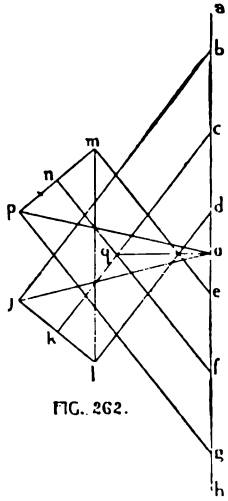


FIG. 262.

Iron truss.

The designer should not hesitate for fear of criticism as to novelty, to make his truss or its joints of any shape that may be most convenient, bearing always in mind, that the simpler the parts and the nearer to standard sizes, the cheaper will be the execution of the work.

Almost any design can be made by the use of wrought-iron or steel, or of cast-iron.

In the two former, care should be taken to analyze carefully the work required of each rivet. Wrought-iron is gradually taking the place of cast-iron, even for shoes and such parts of trusses, as the expense o riveting them up out of different parts, is apt to be cheaper than the cost of the patterns required for castings.

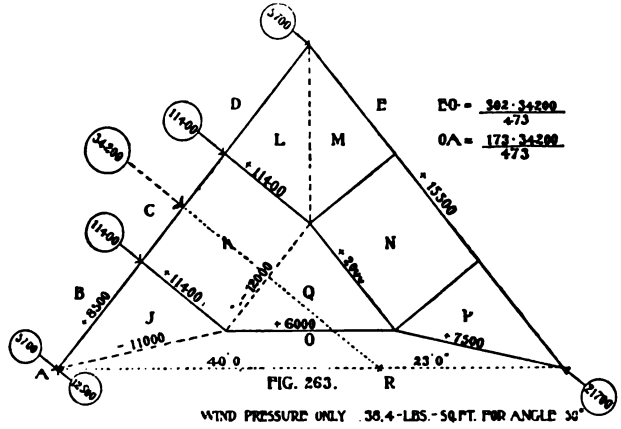
In Figure 261, we have the design of the axial lines of an iron roof truss recently erected in New York City.

Vertical load. The vertical load on this truss was assumed at 65 pounds per square foot and made up as follows:

Weight of truss	= 6 pounds.
Weight of other iron	= 10 pounds.
Weight of 3 inch blocks	= 16 pounds.
Weight of plaster	= 8 pounds.
Weight of slate	= 10 pounds.
Weight of snow	= 15 pounds.
Total (per square foot)	= 65 pounds.

Figure 262 gives the strain diagram for this load; the rafter panels were each 17 feet long, and the trusses placed 17 feet 6 inches from centres, so that the load on each panel was $17.17\frac{1}{2}.65 = 19337$ or say 19300 pounds.

The wind-pressure was calculated separately. In Figure 263 it is supposed to blow from the left. The pressure normal to the roof



surface per square foot for a roof of this inclination (51°) will be, say, 38,4 pounds, (see Table XLIV.)

We should have then on each panel:

$17.17\frac{1}{2}.38{,}4 = 11424$ or say 11400 pounds.

Wind pressure. The total wind-pressure therefore was = 34200 pounds. By prolonging the central axis of the wind

pressure $C R$ till it intersects the horizontal $A H$ at R we obtain the reactions due to wind pressure. As $A R$ measures 40 feet and $R H$ 23 feet, we know that the reaction at H will be

$$= \tfrac{4}{6}\tfrac{0}{3}.34200 = 21700 \text{ pounds},$$

and at A

$$= \tfrac{2}{6}\tfrac{3}{3}.34200 = 12500 \text{ pounds}.$$

STRAIN DIAGRAM FOR WINDLOAD
FIG. 264.

The strain diagram Figure 264, can now be easily constructed. We notice that there are no strains in $M N$ nor in $N P$ due to wind; also that the latter reverses the strains in $Q N$, $Q O$ and $P O$, they all having to resist compression. In Figure 265 are tabulated the strains due to both vertical load and wind pressure, the last column giving the actual result, that is, their sum or difference, as the case may be. In

TABLE OF STRAINS OF ROOF.

Name of piece.	Standing load.	Wind load.	Result.
B. J.	+ 76000	+ 8000	+ 84500
G. K.	+ 61000	+ 8500	+ 69500
D. L.	+ 46000	+ 8500	+ 54500
E. M.	+ 46000	+ 15500	+ 61500
F. N.	+ 61000	+ 15500	+ 76500
G. P.	+ 76000	+ 15500	+ 91500
J. K.	+ 12300	+ 11400	+ 23700
K. L.	+ 12300	+ 11400	+ 23700
L. M.	— 52000	— 15000	— 67000
M. N.	+ 12300		+ 12300
N. P.	— 12300		+ 12300
R. Q.	— 24000	— 12000	— 36000
G. N.	— 23500	+ 2000	— 21500
J. O.	— 49000	— 11000	— 60000
Q. O.	— 23500	+ 6000	— 17500
P. O.	— 49000	+ 7500	— 41500

Fig. 265.

Figures 266 and 267 we reach exactly the same results, by combining both vertical load and wind pressure in one diagram.

In designing the truss, we must remember that were the wind

blowing from the right, the corresponding members of each half of the truss would exchange their respective strains, we must therefore design each such pair of (right and left) members to resist the larger strain.

Detailing the truss.

Figure 268 gives the drawing of truss and detail of joints.

In designing the truss we find that the heaviest strains exist when the wind blows with the exception of the horizontal central tie $Q\,O$ which has its largest tension, when there is no wind; we select accordingly, the heaviest strains for each member and proceed to design the parts.

Take heaviest strains.

For the tie-rods we require areas, as follows:

Size of rods.

Vertical rod $L\,M = \frac{67000}{12000} = 5{,}58$ square inches or a diameter of say $2\frac{3}{8}$ inches.

Inclined rod $K\,Q = \frac{36000}{12000} = 3$ square inches or a diameter of say 2 inches.

Lower inclined rod $J\,O = \frac{60000}{12000} = 5$ square inches or a diameter of say $2\frac{1}{2}$ inches.

Horizontal rod $Q\,O = \frac{23500}{12000} = 2$ square inches or a diameter of say $1\frac{3}{8}$ inches.

By placing sleeve nuts (or turn buckles), as shown, we can tighten up the truss at any time.

We next design the struts to stand 23700 pounds compression each. They are 13 feet 4 inches long or 160 inches long. After several trials we decide to use two 3 inches x 3 inches x $\frac{1}{2}$ inch tees, placed one inch apart, back to back. The weakest way in compression will evidently be with the neutral axis parallel to the web. From Table XXIV, we find their square of radius of gyration for this axis to be $\rho^2 = 0{,}42$ and their area of cross-section $= 2{,}75$ square inches each or $5\frac{1}{2}$ square inches for the two.[1]

Size of struts.

[1] It should be noted that no matter how many tees were placed along the *same axis in the same direction* we should have the same ρ^2: for while each additional one would increase the area, and the moment of inertia i, and the moment of resistance r, the square of the radius of gyration ρ^2, being simply the quotient of the inertia divided by the area, would remain constant, no matter how many tees were used.

We have then for the safe compressive stress in these tees to resist the 23700 pounds compression, (see Formula 3):

$$s = \frac{12000.5\frac{1}{2}}{1+\frac{0,00005.160^2}{0,42}} = 23322 \text{ pounds.}$$

which is near enough. It should be noted that we consider the strut as a column with pin ends. The ends have got separately forged

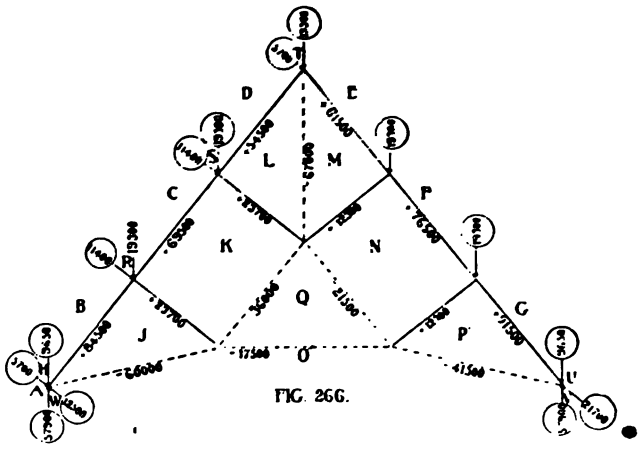


FIG. 266.

pieces, bearing on the pins and riveted between the ends of struts with eight $\frac{5}{8}$ inch rivets.

The rivets were determined, as follows:

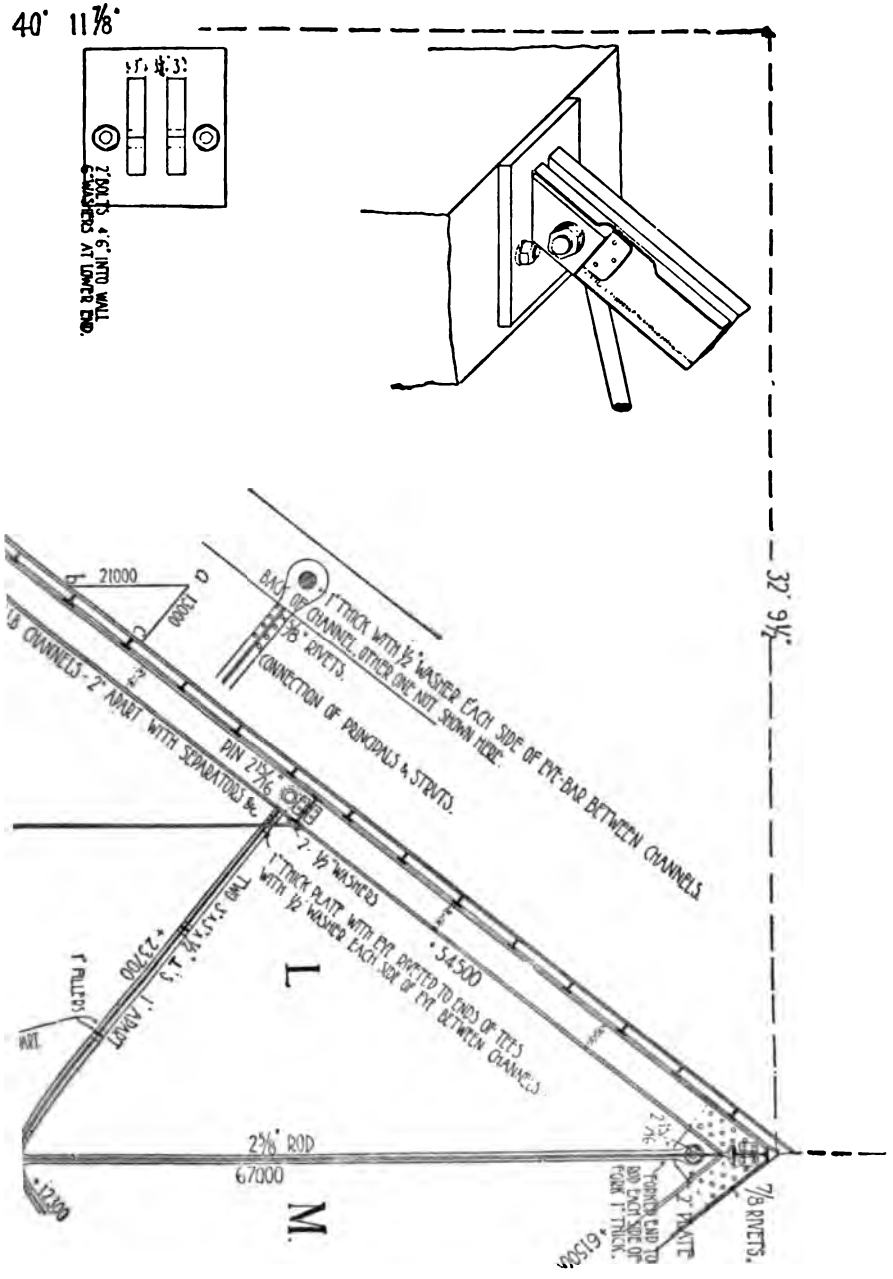
Rivets in strut ends. Bearing area of each rivet $\frac{5}{8}$ inch x 1 inch $= \frac{5}{8}$ square inch or $= \frac{5}{8}.12000 = 7500$ pounds, value per rivet.

Shearing area of each rivet $= 0,3068$ square inches, there being two shearing areas to each rivet, we have 0,6136 square inches resisting shearing, or shearing value of each rivet $= 0,6136.8000 =$ 4908 pounds.

For safe bending-moment on each rivet, we have from Table I, section No. 7 and from Formula(18) transposed:

$$m = \frac{1}{14}.(\frac{5}{16})^3.15000 = \frac{1}{43}.15000 = 360 \text{ pounds-inch.}$$

Remembering to use the larger value $\left(\frac{k}{f}\right) = 15000$ pounds for bending-moment on pins or rivets, we could have read the same


40' 11⅞"
32' 9½"
2 BOLTS 4'6" INTO WALL
2 WASHERS AT LOWER END.
21000
BACK OF CHANNEL, EITHER ONE NOT SHOWN HERE.
⅞" RIVETS.
CONNECTION OF PRINCIPALS & STRUTS.
1" THICK WITH ½" WASHER EACH SIDE OF EYE-BAR BETWEEN CHANNELS
CHANNELS - 2" APART WITH SEPARATORS &c.
PIN 2⅛" DIA.
2 - ½ WASHERS
1" THICK PLATE WITH EYE RIVETTED TO ENDS OF TIES WITH ½" WASHER EACH SIDE OF EYE BETWEEN CHANNELS
.54500
.23700
L
2⅛" ROD
67000
.12400
M.
.60519
7/8 RIVETS.
2" PLATE
FORMED END TO END EACH SIDE OF FORK 1" THICK.

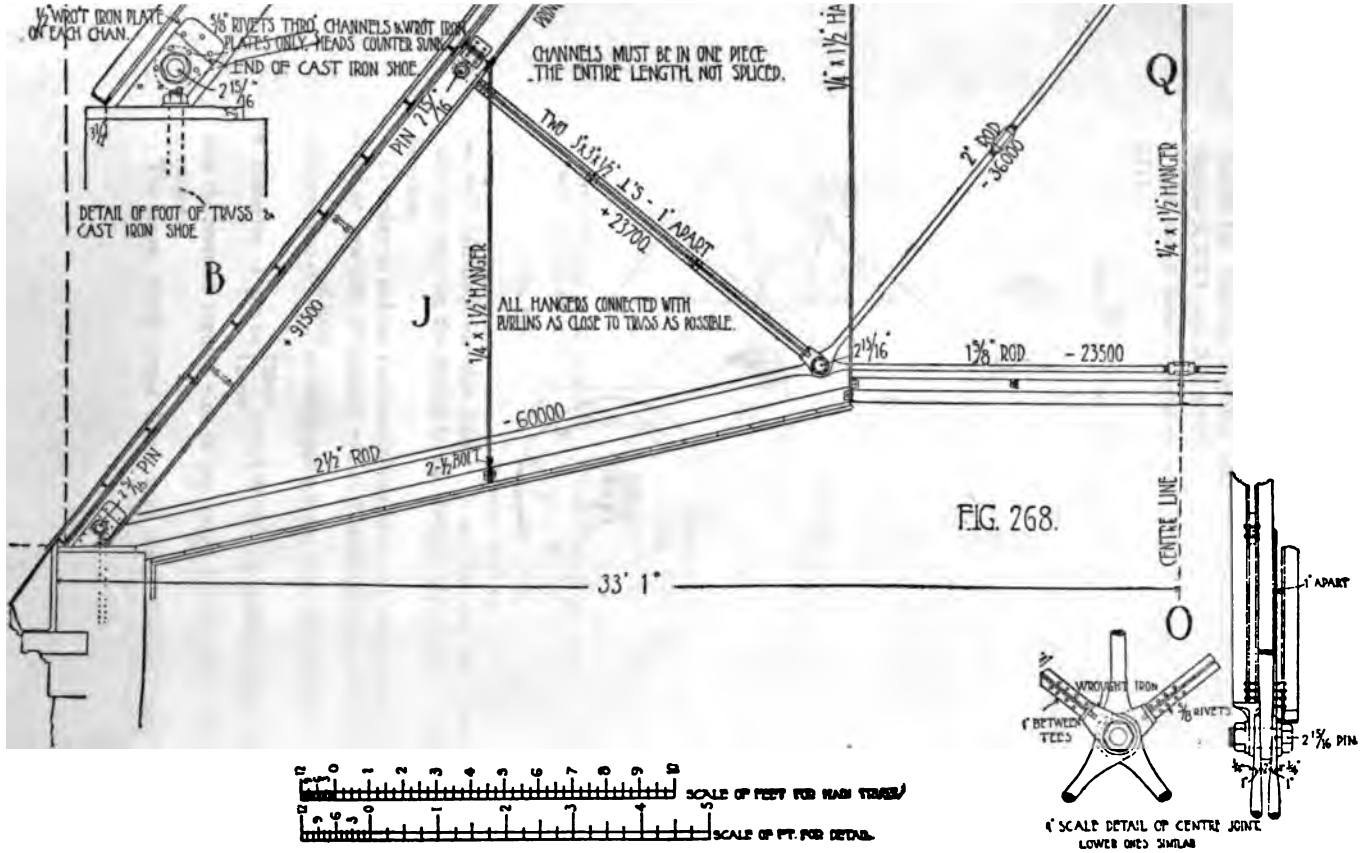

½ WRO'T IRON PLATE ON EACH CHAN.
⅝ RIVETS THRO' CHANNELS & WROT IRON PLATES ONLY, HEADS COUNTER SUNK
END OF CAST IRON SHOE
2 15/16
DETAIL OF FOOT OF TRUSS & CAST IRON SHOE
B
J
Q
O
PIN 7 15/16
CHANNELS MUST BE IN ONE PIECE THE ENTIRE LENGTH, NOT SPLICED.
TWO 5"x3½"x·5 - 1' APART
+ 23700
2" ROD - 36000
¼" x 1½" HANGER
¼" x 1½" HANGER
ALL HANGERS CONNECTED WITH PURLINS AS CLOSE TO TRUSS AS POSSIBLE.
+ 91500
2 15/16"
1⅜" ROD - 23500
- 60000
2½" ROD
2-½" BOLT
2½" PIN
CENTRE LINE
33' 1"
FIG. 268.
1" APART
WROUGHT IRON
⅝ RIVETS
1" BETWEEN TEES
2 15/16 PIN
SCALE OF FEET FOR MAIN TRUSS
SCALE OF FT. FOR DETAIL
1" SCALE DETAIL OF CENTRE JOINT
LOWER ONES SIMILAR

FIG. 268.

results for bearing, shearing and bending moment directly from Tables XXXV and XXXVIII.

The actual bending-moment on all the rivets would be Formula (21):

$$m = \frac{23700.1}{8} = 2962 \text{ pounds-inch.}$$

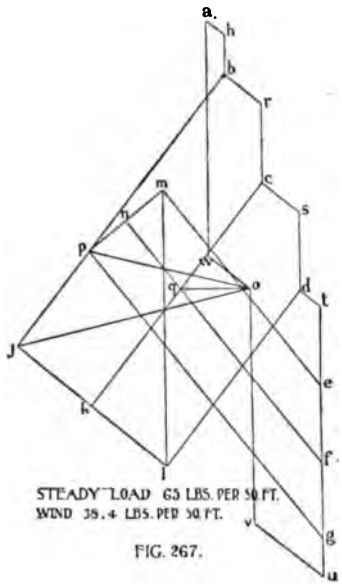


FIG. 267.

We require therefore, to resist bending

$\frac{2962}{360} = 8{,}2$ or say eight rivets.

This being more than required for either bearing or shearing determines the number of rivets to be used.

Calculating the pins. We next decide on size of pin used. It is evident that the joints will be similar as regards arrangement.

The largest rod will be central on pin, each side of it will be another rod and outside of this the strut. We can also readily see that the joint at foot of vertical rod $L\ M$ will be the most severely taxed, and as it is usual to use the same size pin throughout a truss we will calculate for this joint. It is, also, evident that we need calculate only for the strains in the vertical line, as these will be the heaviest.

In order to reduce the bending-moment on the pin, we reduce the head or eye-part of vertical rod to 2 inches thick, this will make a compression of $\frac{67000}{2} = 33500$ pounds per inch thickness and require a diameter of pin $= \frac{33500}{12000} = 2{,}8$ or say $2\frac{1}{16}$ inches diameter, (which was the nearest regular size of pin made by the mill who had the contract).

The same result could have been read directly from Table

XXXVI. For the rod $K\,Q$ we require a width of bearing

$$= \frac{36000}{2\frac{1}{16}.12000} = 1 \text{ inch.}$$

For each strut we require a width of bearing

$$= \frac{23700}{2\frac{1}{16}.12000} = 0{,}67 \text{ or say } \tfrac{3}{4} \text{ inch.}$$

The single shearing area of pin being about $6\frac{3}{4}$ square inches or $= 6\frac{3}{4}.8000 = 54000$ pounds, there can evidently be no danger from that quarter.

We now calculate the bending-moment, we first obtain the vertical resultants of all the strains, by laying off each strain along its respective line of action at a certain scale, and then measuring the length of its projection along the vertical line. The vertical projection of the 36000 pounds-tension is 28000 pounds pulling downwards; the projection of the 21500 pounds-tension is 16500 pounds pulling downwards; the projection of the 23700 pounds-compression is 15000 pounds pushing downwards; the projection of the 12300 pounds-compression is 7500 pounds-compression pushing downwards; the sum of all of these is $28000 + 16500 + 15000 + 7500 = 67000$ pounds downward. Resisting this we have the 67000 pounds of upward pull.

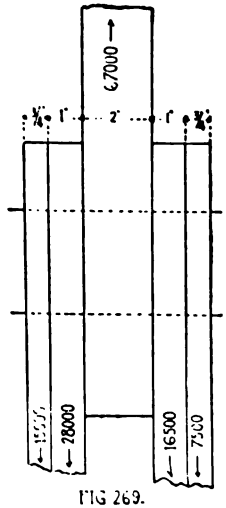


FIG 269.

We now lay out, Figure 269, the pin, and find that we have a double lever arrangement: the fulcrum being the 2 inch wide 67000 pounds strain. To the left of this we have a lever $1\frac{3}{4}$ inch wide loaded with two loads, one 28000 pounds one inch wide and one beyond 15000 pounds $\frac{3}{4}$ inch wide. To the right we have lighter loads, the heaviest bending-moment will therefore be on the left side, or:

$$m = \tfrac{1}{2}.28000 + 1\tfrac{3}{8}.15000$$
$$= 34625 \text{ pounds-inch.}$$

The safe-bending-moment on a $2\frac{1}{16}$ inch pin is:

$$m = \tfrac{1}{14}.(1\tfrac{1}{32})^3.15000 = 37500 \text{ pounds-inch.}$$

The pin is therefore safe. The same result could have been read off directly from Table XXXIX.

Size of Main Rafter.

We next design the main rafter; as this is to be in one length, we design it only for lower panel (91500 pounds-compression) and the other panels will be, of course, too strong. Besides the compression we will have a transverse strain or bending-moment on rafter, per square foot, as follows:

Wind	$=38{,}4$
3 inch blocks,	$=16$
Slate	$=10$
Iron tees (say)	$=5$
Total	$=69{,}4$ pounds,

or allowing for weight of rafter, say, 70 pounds per foot. The rafter lengths being 17 feet and $17\frac{1}{2}$ feet apart, we have total uniform load $u = 17.17\frac{1}{2}.70 = 20825$ or say 21000 pounds. We now draw anywhere along rafter a vertical line $a\,b = 21000$ pounds; then draw $a\,c$ normal to rafter; it scales 13000 pounds, therefore the actual transverse load on rafter is 13000 pounds.

Transverse strain on rafter.

The required moment of resistance to resist this load will be Formula (18)

$$r = \frac{13000.(17.12)}{8.12000} = 27{,}7$$

By reference to Table XXI, we find we require one $12\frac{1}{4}$ inch 80 pounds channels to take care of the transverse strain. We also note that the area of channel is 8 square inches, which is about what we need *additional* for resisting the compression. We will decide then to use two $12\frac{1}{4}$ inches 80 pounds channels.

The square of the radius of gyration is $\varrho^2 = 21{,}10$

Compression on rafter.

The area of the two will be 16 square inches, but as just one-half of their total moment of resistance r is needed to resist transverse strain, we will have only 8 square inches left of the total area to resist compression. The column is 17 feet or say 204 inches long. One end will be a pin end, the other a milled or planed end with fair bearing. We have then for safe compressive load Formula (3):

$$w = \frac{8.12000}{1 + \frac{204^2.0{,}000033}{21{,}1}} = 90300 \text{ pounds},$$

or near enough to pass as safe. If the transverse strain had required

say a moment of resistance of $r = 35$ we should have had left to resist compression of the total area 16 square inches only,

$$= 16 - \left(\frac{35}{2.27,7}\right).16$$

= (about) 6 square inches instead of 8, which (in such a case), would not be enough and would require a heavier channel.

The thickness of our channel web we see from Table XXI, is 0,39 inches, or two webs = 0,78. The safe bearing on pin will therefore be $= 0,78.2\frac{1}{4}\frac{5}{8}.12000 = 28200$ pounds. The web of channel will therefore need thickening at the shoe pin, but not at the two others. At the apex the pin does not bear on channels, but is connected indirectly by a 2-inch thick plate.

Bearing of rafter webs on pins.

At the shoe the channels have planed ends and rest directly on the shoe, the strain therefore against their webs, will be 60000 pounds, due to the rod trying to tear the pin out. The webs take care of 28200 pounds of bearing, leaving $60000 - 28200 = 31800$ pounds, to be transferred by rivets to thickening or reinforce plates. Of this each side takes care of

Reinforce plates to web.

$$\frac{31800}{2} = 15900 \text{ pounds.}$$

The thickness required for each plate, is therefore

$$\frac{15900}{2\frac{1}{4}\frac{5}{8}.12000} = 0,42 \text{ or say } \tfrac{1}{2} \text{ inch each.}$$

The number of rivets required must next be settled. We use $\frac{5}{8}$ inch rivets:

The channel being thinner than the plate determines the bearing value for each, or

$$= 0,39.\tfrac{5}{8}.12000 = 3000 \text{ pounds,}$$

Rivets in reinforce plates.

or

$$\frac{15900}{3000} = 5,3 \text{ or say six rivets required for bearing.}$$

The rivets are in single shear, their area = 0,3068 and their value $= 0,3068.8000 = 2454$ pounds, and we require $\frac{15900}{2454} = 6,5$ or say seven rivets.

For bending-moment each rivet is a single lever held by the half-inch plate, projecting 0,39 inches and uniformly loaded on free end

with its share of the 14100 pounds carried by each channel web, the total bending-moment will therefore be from Formula (25).

$$m = \frac{14100.0{,}39}{2} = 2750 \text{ pounds-inch.}$$

The safe bending-moment on a $\frac{7}{8}$ inch rivet we previously found to be 360 pounds-inch, and require therefore

$$\frac{2750}{360} = 7{,}7 \text{ or say eight rivets.}$$

The disposition of rivets around the pin, however, requires nine rivets.

Pin plates at apex.

We now go to the apex point. We use here a 2-inch plate, its bearing on pin must be all right, as we made the other end of vertical rod 2 inches thick. The upper end of rod we make forked, each side 1 inch thick, and 2 inches between to admit plate.

The plate is so large that there is evidently no danger of the pin shearing out.

For the rivets we can see it will require a large number and therefore decide to use larger or say $\frac{7}{8}$ inch rivets.

Number of rivets.

The bearing value on two webs of each rivet will be $= \frac{7}{8}.2.0{,}39.12000 = 8190$ pounds.

The (double) shearing value of each rivet will be

$$= 2.0{,}6013.8000 = 9620 \text{ pounds.}$$

The safe bending-moment on each rivet will be

$$= \tfrac{1}{14}.(\tfrac{7}{16})^3.15000 = 987 \text{ pounds-inch.}$$

If now we consider that the two rafters have planed ends and butt fairly against each other, taking up the thrust from each, the rivets need only take care of the vertical down pull 67000 pounds, *all* of the rivets taking a share. In that case we should need for bearing $\frac{67000}{9360} = 7{,}2$ rivets, for shearing less, and for total bending-moment, remembering that the channel backs are 2 inches apart and that the rivets are beams supported at both ends and uniformly loaded, from Formula (21)

$$m = \frac{67000.2}{8} = 16750 \text{ pounds-inch.}$$

Therefore number of rivets required to resist bending-moment $= \frac{16750}{987} = 16{,}9$ or say nine each side of joint.

The plate, however, requires ten each side for even distribution.

If the channels do not butt fairly the plate will have to transfer the thrust from one to the other. This thrust will be equal to the *horizontal resultant* of compression on rafter at apex which is 54500 pounds. Its horizontal projection measures 34000 pounds. The rivets each side of joint must take care of this strain and the plate be large enough not to crush under it. We need not calculate any in this case, however, as this strain is just about one-half of the strain for which the total number of rivets were proportioned.

Designing shoe-plate.

We next design the shoe. It is a flat cast-iron plate 2 inches thick, and 28 inches by 24 inches with two flanges each 3 inches thick to receive the pin. As the channels bear directly on the plate, the only strain on the flanges will be due to the pin trying to shear its way out, the strain being 60000 pounds. Resisting this there are two areas to each of the two flanges, each area 3 inches x 3 inches, or a total area $=4.3.3=36$ square inches, the actual stress is therefore

$$\frac{60000}{36}=1667 \text{ pounds per square inch, which is safe.}$$

Bearing area.

Include wind reaction.

The wall must carry the whole load of truss, each reaction is 57900 pounds due to vertical load. To this must be added the vertical resultant (or projection) of the largest wind reaction 21700 pounds. Its vertical projection measures about 14000 pounds making the total vertical reaction 71900 pounds. The area of plate is $28 \times 24 = 672$ square inches, therefore compression per square inch on brick-work

$$=\frac{71900}{672}=108 \text{ pounds, which is safe.}$$

Rockers or rollers.

Had this truss been of larger span, we should have had to place one shoe either on a rocking saddle, or else on rollers. If the latter there should be sufficient rollers, and they should be of large enough diameter not to *indent* the plate, and to roll back and forth freely. It is usual to put upward flanges all around the bottom plate and downward flanges all around the upper plate to hold the rollers in place between them. The flanges should be less than radius of rollers, so as not to meet. The foot of girder must be secured against yielding sideways.

The size of rollers is determined by the following formula:

Formula for rollers.

$$w=l.750.\sqrt{d} \qquad (132)$$

Where $w =$ the safe load, in pounds, on *each* roller.

Where $l =$ the length, in inches, of each roller.

Where $d =$ the diameter in inches, of each roller, if of steel or wrought-iron and rolling between cast-iron plates. If between wrought-iron plates add 25 per cent to w.

Rollers should be used, (under one shoe only,) where trusses *in-doors* are over eighty feet span, or *out-doors* if span is over sixty-five feet.

Example.

A truss of one hundred feet span has a reaction at each end of 95000 *pounds. The shoe-plate is* 20 *inches wide the long way of rollers, and rollers are* 1 *inch in diameter. How many rollers are required?*

Each roller will safely carry from Formula (132)

$$w = 20.750.\sqrt{1} = 15000 \text{ pounds},$$

we shall require therefore

$$\frac{95000}{15000} = 6{,}3 \text{ or say } 7 \text{ rollers.}$$

Of course, where rollers are used some arrangement must be made in the cornice to allow for the movement due to expansion and contraction of the truss; or if the roof is continuous a slip-joint must be provided in the roof itself, the detail of which will depend upon the local circumstances.

CHAPTER XIII.

COLUMNS.

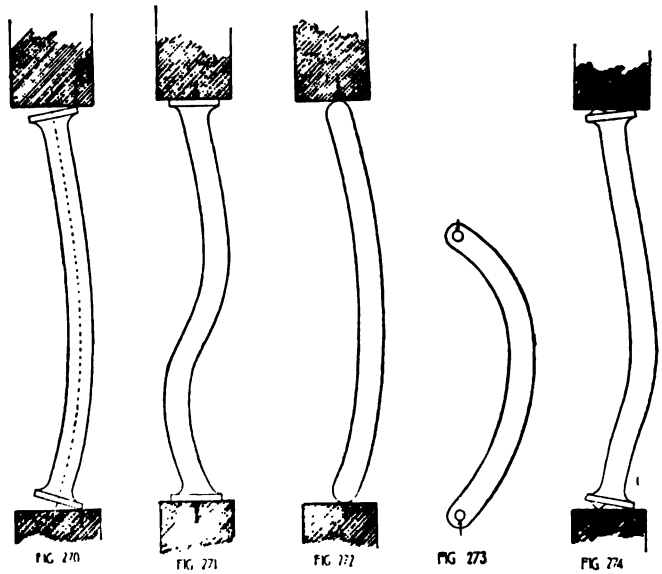


FIG. 270 FIG. 271 FIG. 272 FIG. 273 FIG. 274

THE Formula (3) on page 24 of Vol. I, is, of course, applicable to every kind and shape of column. The square of the radius of gyration used in this formula, can be found by any of the formulæ given in Table I; or if the shape of the column is so unusual that it is not given in Table I, the moment of inertia (i) of the cross-section of the column can be found, and this divided by the area (a) is equal to the square of the radius of gyration, see page 9 of Vol. I.

To find radius of gyration.

In Table II are given the different values of n for cast and wrought iron, steel, wood, stone and brick; also the variations in

this value for "*pin*," that is, rounded or rough bearings at ends, and for "*smooth*," that is, turned, planed or smoothed off bearings at ends. These different values have been arrived at largely by experiments, but the reason why the end bearing affects the strength of column can readily be seen in Figures 270 to 274.

In Figure 270 we have a column with smooth ends between two forces crushing towards each other; as a result the column tends to bend, in this case to the right. The bending of the column will, of course, tip its ends, as these are at right angles to the (dotted) longitudinal axis of the column; as a result the further crushing is taken entirely at the edges of the ends, or at points *A* and *B*, marked with arrows in the figure.

Why smooth end stronger.

FIG. 275

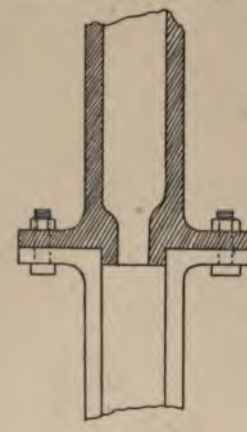


FIG. 276.

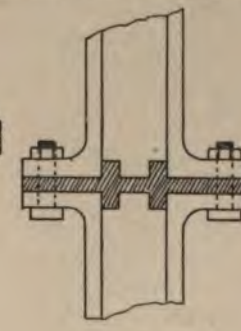


FIG. 276 A.

It will be readily seen that the least pressure at *A* or *B* tends to bring the ends back to the horizontal plane and consequently to straighten the column again. For this reason it is that columns with smooth end bearings give way the same as columns with fixed end bearings, as shown in Figure 271. If on the other hand the ends of the column are rounded, as shown in Figure 272, the effect of the crushing at *A* and *B* is to constantly increase the bending of the column, as the point of contact against the column simply slips around the circular ends, as the column bends more and more. Such

columns break as one curve instead of the triple curve shown in Figure 271. Where there is a pin bearing, as in Figure 273 the effect is, of course, the same as for rounded ends. Rough ends are considered the same as rounded ends, on account of the danger of some roughness or projection on the end of a bearing, which would greatly increase the tendency to bend, as shown at *A* and *B* in Figure 274. For this same reason *wedging* of joints of columns, should *never* be allowed.

Only those columns should be considered as having smooth ends, where the entire bearing end is perfectly smooth, and forms a true and perfect plane at right angles to the longitudinal axis of the column. In iron and steel columns the ends have to be "planed" or "turned" off, both of which are done by machinery. Planing is, as its name implies, a passing back and forth of a sharp metal planer which removes the surface iron, little by little. Turning is the same as planing but the motion of the planer is circular. Turning is done on circular columns, particularly where there are lugs or projections beyond the bearing surfaces, which would be in the way of a straight planer.

Planing and turning.

Cast-iron columns are usually made of circular cross-section and hollow. This is the cheapest cross-section there is for any column, and the metal will do more work per square inch, the thinner (within reason, of course,) the shell is made.

Shapes of cast columns.

Cast-iron columns are also frequently made square in cross-section, and hollow, which does not make a badly proportioned column. All other shapes, however, are bad, and should only be resorted to in unusual circumstances. Such shapes, for instance, are rectangular and hollow, one side (or diameter) being shorter than the other, in this case the neutral axis, when computing the square of the radius of gyration, should be taken parallel to the long side; then there are H or I shapes, T or Z shapes, etc., all bad and weak. Sometimes a column is made square or rectangular but with only three sides. The column will be greatly strengthened, if at intervals there can be cast on the fourth side a connecting bar.

All hollow castings should be drilled, as already explained, to ascertain their thickness, also near their base to allow any water to escape, which might otherwise freeze and burst the casting. Brackets,

and other projections should, by preference, be cast on the column, rather than secured to it by bolts or tap-screws. Of course, they can't be riveted to cast-iron. They should, as far as possible, be of same thickness as shell.

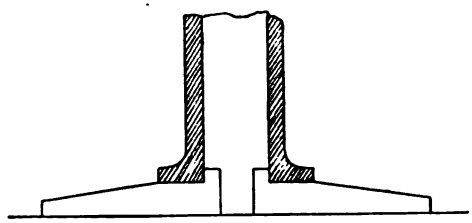


FIG. 277.

Caps and bases.

Where there are heavy capitals, bases or other mouldings or ornamental work that greatly increase the thickness of shell, they had better be made separately and slipped on afterwards, and then secured by tap-screws with countersunk heads.

The bearing should always, if possible, be vertically under and over the shell, not flanged out as shown in Figure 275.

Ends of columns are usually flanged out for bolting together, in which case the angles should be well rounded. It is also well to cast on one of the columns a lug as shown on the upper, etched column in Figure 276.

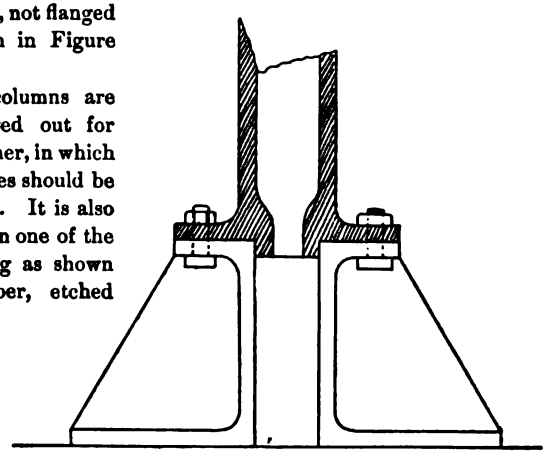


FIG. 278.

To prevent fire spreading up through columns, some building laws require solid plates at all joints; in such cases they should be made

as shown in etched part of Figure 276*a* with upper and lower lugs, and the columns bolted together through the plate.

Bottom plates. Bottom plates, which usually have to spread the weight are made, as shown in Figure 277; as they are often very large, and necessarily therefore quite thick, metal can be saved by gradually reducing the thickness towards the edges, as shown. In such plates a hole should be cast in the centre, to relieve the strain on the plate, when cooling and thereby avoid warping.

No bolts are needed where a bottom plate is used, as shown in Figure 277, as there is no possibility of tipping. Where the spread of base plate has to be very great, flanges as cast on same as shown in section in Figure 278 and in plan in Figure 279. Of course, the flanges can be more numerous. All parts of such a base should be of even thickness. If the casting is unusually large and unwieldy it can be made in two parts, as shown in Figure 279, but in such cases great care should be exercised with the foundation or substructure, to avoid one-half settling away from the other half. This can be done by thick granite stones, or additional iron plates under the main bottom plate.

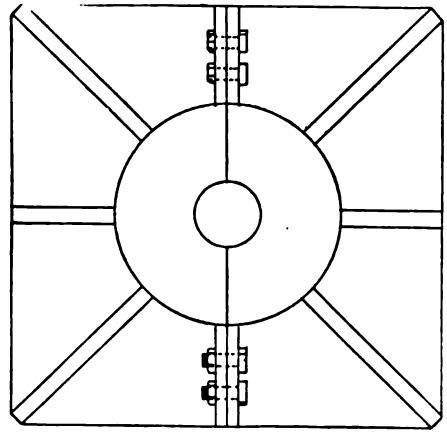


FIG. 279.

The calculation of hollow circular columns is very tedious, particularly as one has to guess at the size and then calculate the strength, often involving several calculations for a single case.

Explanation of Tables XLV to XLVIII. Tables XLV, XLVI, XLVII and XLVIII, have therefore been prepared by the writer to take the place of these calculations, for cast-iron hollow circular columns.

Across the top of the tables, in the horizontal line, are given the lengths in feet of the columns. Down the left side in the vertical line are the *safe* loads in tons for each particular shape. The curved lines each represent a hollow circular section, and at the end of each curve is given first, the diameter of column, second the thickness of shell, third the area of cross-section. The latter enables one to pick out the cheapest section from any of the tables, by selecting any curve below and to the right of the one found — which has a smaller area of cross-section. Sometimes by reference to a subsequent table a still cheaper section can be found.

The tables have been calculated for cast-iron, according to Formula (3); the columns are supposed to have smoothly dressed ends and true bearings. The value used for $\left(\frac{c}{f}\right)$ was 15000 pounds. Any one desiring to use any other value, need only proportion the vertical column of safe loads in tons accordingly; thus, for 12000 pounds we should take $\frac{4}{5}$ of the loads as safe, or we could add $\frac{1}{4}$ to the load to be carried by the column, and find from the tables the diameter, etc., of column necessary to carry the increased load.

Table XLV gives columns from 3″ to 7″ diameter of different thicknesses, and from 5 feet to 12 feet long. Table XLVI gives columns from 8″ to 10″ diameter of different thicknesses, the 8″ and 9″ columns from 8 feet to 15 feet long, the 10″ columns from 10 feet to 20 feet long.

Table XLVII gives columns from 11″ to 13″ diameter, and Table XLVIII from 14″ to 16″ diameter, in both, of different thicknesses and from 10 feet to 20 feet long.

An example will best illustrate the use of table.

Example I.

What is the safe load on a hollow, circular, cast-iron column, with turned ends, 18 *feet long,* 11″ *diameter and* 1½″ *thick?*

Example of use of tables. Columns of 11″ diameter are given in Table XLVII to which we turn, we find the curve marked 11 — 1½ — 44,8 cuts the vertical line 18 about two-fifth way down between the horizontal lines 150 and 155, our column will therefore safely carry 152 tons.

Had we calculated our column by Formula (3) we should have

had from Table I, section No. 8 for

$$\varrho^2 = \frac{5\frac{1}{2}^2}{4} + 4^2 = 11{,}56$$

and for $l^2 = (18.12)^2 = 216^2 = 46656$.

Inserting the values in Formula (3) we have the safe load

$$w = \frac{44{,}8.15000}{1 + \frac{46656.0{,}0003}{11{,}56}} = \frac{672000}{1 + 1{,}2108} = 303962 \text{ pounds},$$

or, 151,98 or say 152 tons same as found from table.

Now, is this the cheapest section for that load and length of column. Its area is 44,8 square inches. We now pass to the next curve below and to the right, it is 13 — 1 — 37,7 or a 13″ diameter column, 1″ thick will be cheaper, as its area is only 37.7 square inches. Then, too, it is stronger, for at 18 feet long it will carry 158 tons cutting the vertical line 18 three-fifth way between 155 and 160. By passing further along, we find also that the 12″ diameter 1¼″ thick column of 42,1 square inches area will be stronger, carrying 161 ½ tons safely at 18 feet of length.

By referring to the next Table XLVIII, we find some much cheaper sections, than any of these. For the 14″ diameter, ⅜″ thick column, has only 31,2 square inches area of cross-section, but we find it carries safely only 143 tons and therefore will not answer. However the 14″ diameter 1″ thick column, has only 40,9 square inches of area and carries 185 tons, and would answer therefore, provided of course, larger diameter is not objectionable. Or, cheapest of all would be the 15″ diameter, ⅜″ thick column which has only 33,6 square inches of area and carries 161 tons. The 16″ diameter, ⅜″ thick column would also be economical having only 35,9 square inches of area and carrying 180 tons at 18 feet of length.

Shapes of wrought-iron columns.

In wrought-iron construction, any number of shapes of columns are used, from plain flat bars, to the most elaborate combinations of the different shapes rolled, most of which are given, with their areas, squares of radius of gyration, etc., in Tables XIX to XXV. In Figures 280 to 287 are given a few combinations, which are frequently used for columns. Besides those shown there are frequently used combinations of I-beams and channels, or of angles and plates formed in the shape of plate and box-girders stood on end.

Figure 280 shows two channels latticed together. A plate might

be used in place of latticing if the channels were placed as shown in Figure 281, but in that case the interior would not be accessible for painting.

Figure 281 is more easily riveted up than 280, but is not quite as strong.

Figure 282 shows an elevation with wrought-iron base, and Figure 283 the side view of base. Architects are apt to use cast bases with

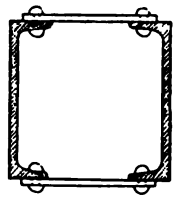


FIG. 280.

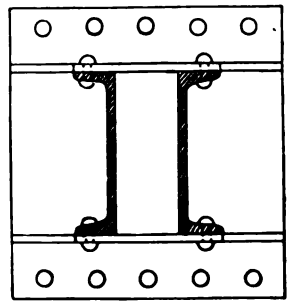


FIG. 281.

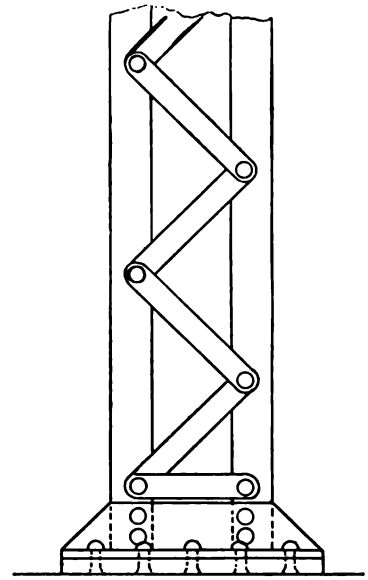


FIG. 282.

wrought-iron work, but as a rule a wrought-iron base can (and should) be designed, which will not only be better adapted to wrought-iron construction, but will be cheaper and stronger, and has the merit that it can be riveted fast to the column or other construction.

Figure 284 shows a column made of four **Z** irons, with central plate. It has the great merit of being a strong column, and though of five parts, it requires only two lines of rivets. This column adapts

itself excellently to building walls, as the horizontal wall girders, and floor girders are easily attached to it, and it readily holds the "filling-in walls" in place, without the use of anchors. Then, too, it can be easily covered with fireproof blocks. The writer recently erected in New York City, a fireproof office-building, ten stories high above the sidewalk, with only twelve inch thick brick walls all the way up, by using these columns combined with horizontal girders in all the walls.

Of course, in such construction, proper precautions must be taken to prevent the building from collapsing under wind pressure. In the case above referred to, in order to avoid cross-partitions, the

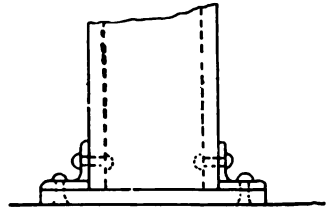


FIG. 283.

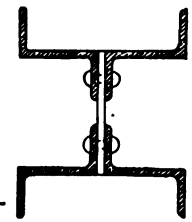


FIG. 284.

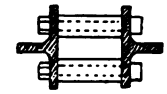


FIG. 285.

wind-bracing was all done in the front and rear walls, and in the floor levels.

Figure 285 gives a combination column of two tees, riveted together with separators.

Figure 286 is made of four angle bars latticed together, a very light, but strong column.

Figure 287 consists also of four angle bars, which adapt themselves more readily to fireproofing, and require less riveting, but the column is not nearly as strong as the previous one. In the last case separators are used in place of lattice bars.

Explanation of Table XLIX.

It would be quite impossible to give any curve tables for wrought-iron construction, but Table XLIX will greatly facilitate the calculation. It will be necessary in each case to find only the ratio of the length of column (in inches) divided by the radius of gyration, or the square of the length, divided by the square of the radius of gyration, and look up the value per square inch of cross-section, according to the

condition of ends of column, and the assumed safe value for $\left(\frac{c}{f}\right)$. The table is calculated respectively for 8000, 10000 and 12000 pounds per square inch values for $\left(\frac{c}{f}\right)$. In buildings use the value 12000 for wrought-iron.

To find the ratio look up the value of the square of the radius of gyration in the tables, or if it is not given, find the moment of inertia according to rules given in Table I on page 10 of Vol. I, and divide by the area, see page 9, Vol. I.

It should be borne in mind that where pieces are doubled or their number increased *along the same neutral axis*, the moment of inertia will be doubled or increased accordingly. But the square of the radius of gyration will remain constant, as it simply represents a

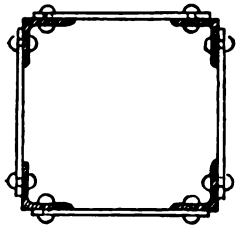


FIG. 286.

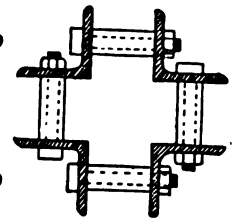


FIG. 287

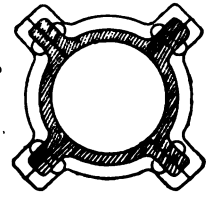


FIG. 288

ratio, and the area and moment of inertia increasing in the same amount, their ratio, which gives the square of the radius of gyration, will, of course, remain constant. In some cases, it will be easier to find the radius of gyration, instead of its square, and in such cases the second column in Table XLIX should, of course, be used.

An example will best illustrate the use of Table.

Example II.

Example of use of Table XLIX. *A flat eye-bar of wrought-iron* $1\frac{1}{2}''$ *x* $6''$ *in a truss, is liable at times to be under compressive stress, what will it safely stand? The bar is* $5'$ $6''$ *long from centre to centre of eyes.*

From Table I, section 2, we have

$$g^2 = \frac{(1\frac{1}{2})^2}{12} = 0{,}19$$

the area will be

$$a = 1\tfrac{1}{2}.6 = 9 \text{ square inches,}$$

and

$$l^2 = 66^2 = 4356$$

The ends being eye-bars, we use, of course, in Table II, the value n for "both ends pin ends," or

$$n = 0{,}00005$$

Inserting the values in Formula (3) we have for the safe compressive strain,

$$w = \frac{9.12000}{1 + \frac{4356.0{,}00005}{0{,}19}} = 50232$$

Now had we used Table XLIX we should have had the ratio

$$\frac{l^2}{\varrho^2} = \frac{4356}{0{,}19} = 22926$$

The nearest value to this in the first column of the Table is 22500 and under the heading "both ends pin ends" for a value of $\left(\frac{c}{f}\right) =$ 12000 pounds, we find 5655 which is the safe load per square inch on our bar, or the total safe strain,[1]

$$w = 9.5655 = 50895 \text{ pounds.}$$

Which closely approximates the above result.

In our case it would have been easier to use the second column of Table XLIX, we should have had

$$\varrho = \sqrt[2]{\frac{(1\frac{1}{2})^2}{12}} = \frac{1\frac{1}{2}}{\sqrt[2]{12}} = \frac{1{,}5}{3{,}464} = 0{,}433$$

and for the length

$$l = 66$$

therefore the ratio

$$\frac{l}{\varrho} = \frac{66}{0{,}433} = 152$$

The nearest value to this in the second column of Table XLIX is 150 which would give the same result as before.

Phœnix Columns. There are several patent wrought-iron columns made, of which the "Phœnix" column is undoubtedly the best.

It is made up of from four to eight segments, riveted together.

[1] (Obtained by multiplying by the area of bar.)

Each segment somewhat resembles a channel with the web bent to a segment of a circle, instead of being straight.

Figure 288 shows one of the smaller columns, made up of four segments. For heavier columns each segment is rolled thicker, as shown in the figure in outline. When it is necessary to have very heavy columns flat pieces are inserted between each flange, as shown in Figure 289. These columns can be readily covered with fire-proof blocks to make a circular finish in buildings, and are largely used both for this reason, and on account of their great strength, (owing to all the metal being near the outer edge).

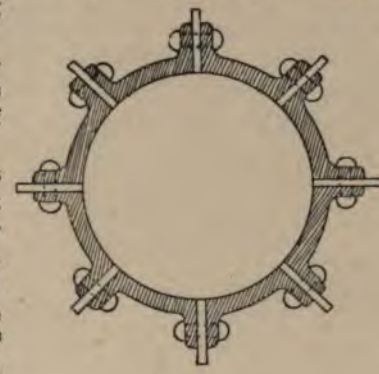


FIG 289.

Table L gives the properties of these columns, as taken from the hand-book of the Phœnix Iron Co.

Column *G* is made in eight sections, column *E* in six sections, all the others in four sections.

Explanation of Table L. In column *A* $\frac{3}{8}''$ rivets are used; in columns B^1 and B^2 $\frac{1}{2}''$ rivets are used; while in the others $\frac{5}{8}''$ rivets are used, until the shell becomes over $\frac{5}{8}''$ thick when $\frac{3}{4}''$ rivets are used.

The thickness given is the thickness of shell; the diameters are the "inside diameter"; "outside diameter," (or inside diameter plus the two thicknesses of shell); and the "diameter over all," that is to outside of flanges. The weight per yard of these columns will, of course, be in pounds, ten times the given area. It will be noticed that the "radius of gyration" is given, and not its square, which will make the use of Table XLIX in connection with these columns much easier.

When calculating the load on a column it should be borne in mind, that if the girder or beam is *continuous* over the column, the loads will be equal to the reactions as given in Table XVII on pages 218 and 219 of Vol. I.

Load on column.

If the girders or beams overhang columns and are

built into the wall at the other end (such as gallery beams for instance) the respective loads on the column and wall and upward pressure on wall can be found from Formulæ 116 to 119 inclusive.

Gallery beams.

If the load on the overhang is uniform its reaction would be the same as a similar amount of load concentrated at one-half the span of overhang.

If the beams or girders are inclined they can be calculated the same as already explained (in the previous chapter), when calculating transverse strains on rafters; and the amount of anchoring necessary to prevent pulling out, can readily be found by obtaining the horizontal thrust by the graphical method, as already explained.

TABLE XLV.

STRENGTH OF HOLLOW CYLINDRICAL COLUMNS OF CAST-IRON.

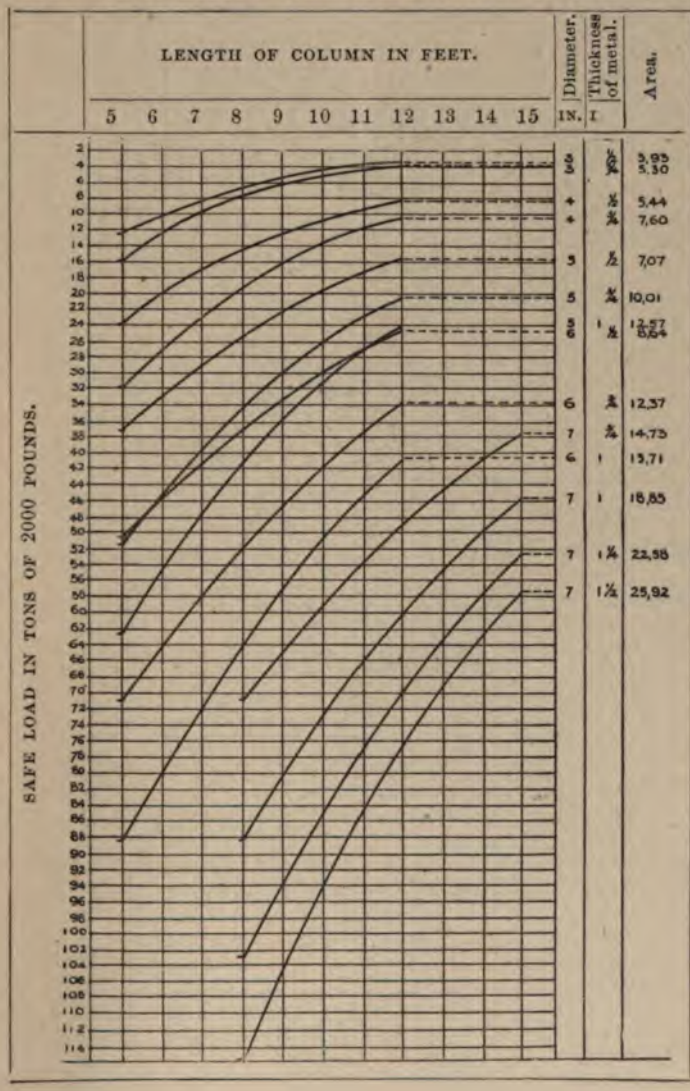


TABLE XLVI.

STRENGTH OF HOLLOW CYLINDRICAL COLUMNS OF CAST-IRON.

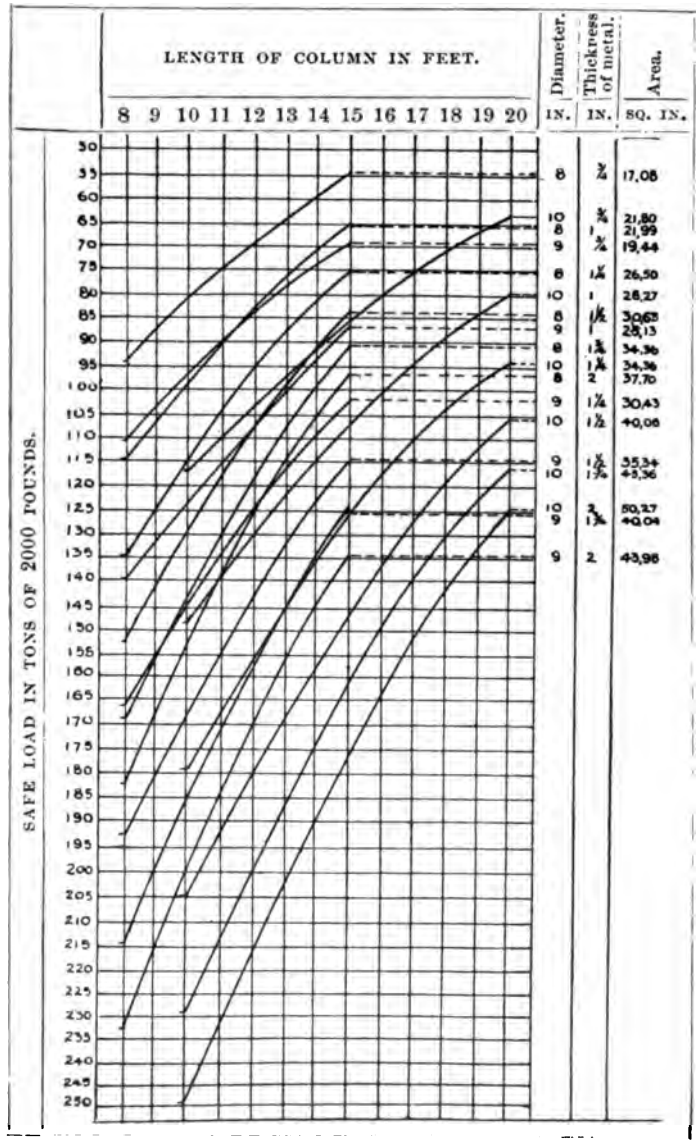


TABLE XLVII.

STRENGTH OF HOLLOW CYLINDRICAL COLUMNS OF CAST-IRON.

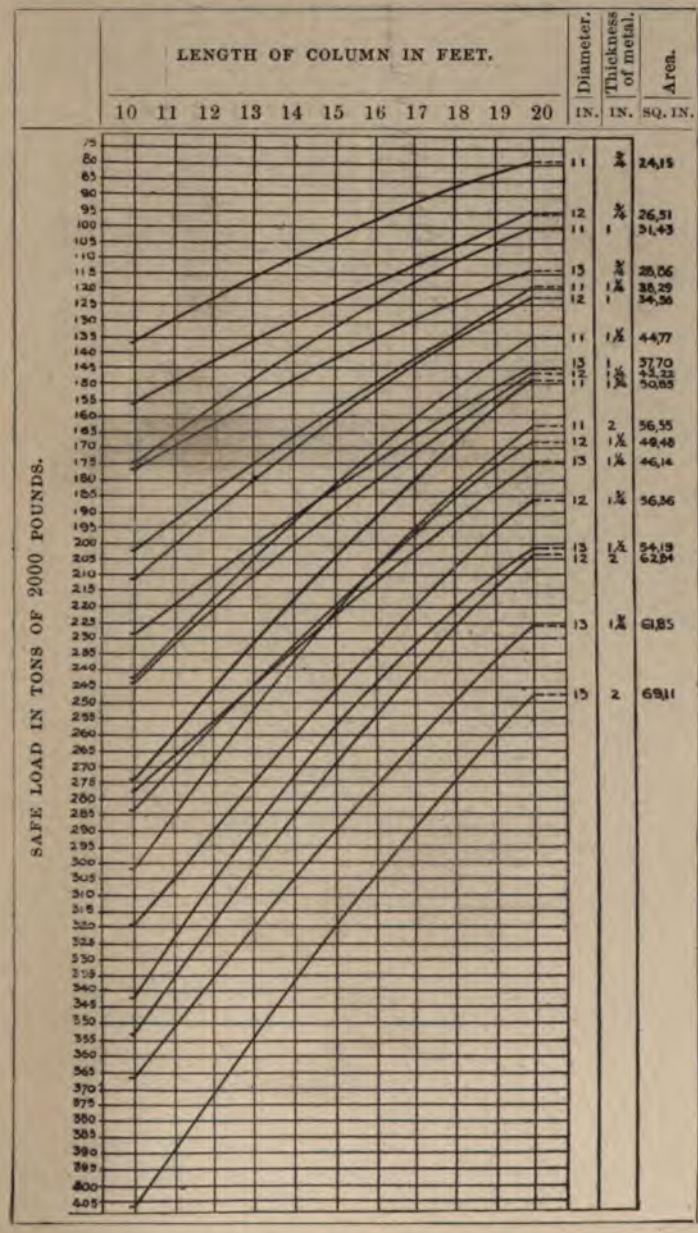


TABLE XLVIII.

STRENGTH OF HOLLOW CYLINDRICAL COLUMNS OF CAST-IRON.

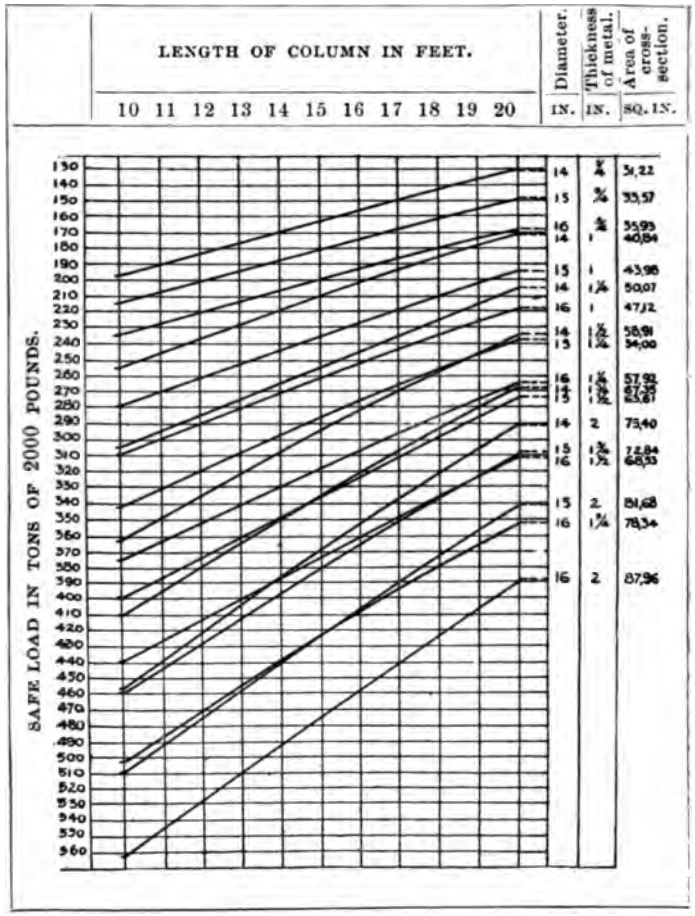


TABLE XLIX.

SAFE COMPRESSIVE LOADS PER SQUARE INCH ON WROUGHT-IRON COLUMNS.

Ratio of (in inches.)		Both ends smooth.			One end smooth one end pin end.			Both ends pin ends.		
Square of length to square of radius of gyration.	Length to radius of gyration.	For values of $\left(\frac{c}{f}\right)$ in lbs., equal.			For values of $\left(\frac{c}{f}\right)$ in lbs. equal.			For values of $\left(\frac{c}{f}\right)$ in lbs., equal.		
		8000	10000	12000	8000	10000	12000	8000	10000	12000
100	10	7989	9986	11983	7973	9966	11960	7960	9950	11940
225	15	7955	9944	11932	7940	9925	11911	7911	9888	11866
400	20	7921	9901	11881	7894	9868	11842	7843	9803	11764
625	25	7877	9846	11815	7836	9795	11755	7758	9697	11636
900	30	7821	9779	11735	7767	9708	11650	7656	9569	11483
1225	35	7762	9702	11643	7686	9607	11529	7538	9422	11307
1600	40	7692	9615	11538	7595	9493	11392	7407	9259	11111
2025	45	7614	9518	11422	7494	9367	11231	7264	9080	10897
2500	50	7529	9411	11293	7386	9232	11078	7105	8881	10657
3025	55	7437	9296	11155	7267	9084	10901	6949	8686	10423
3600	60	7339	9184	11009	7143	8928	10714	6780	8474	10169
4225	65	7236	9045	10855	7010	8762	10515	6605	8256	9907
4900	70	7127	8908	10690	6877	8596	10315	6426	8032	9638
5625	75	7015	8769	10523	6736	8420	10105	6244	7804	9365
6400	80	6896	8620	10345	6593	8241	9890	6058	7572	9086
7225	85	6777	8471	10165	6447	8059	9681	5877	7346	8815
8100	90	6653	8316	9979	6299	7874	9449	5694	7117	8541
9025	95	6527	8159	9791	6150	7687	9225	5512	6890	8269
10000	100	6400	8000	9600	6000	7500	9000	5333	6666	8000
11025	105	6271	7839	9407	5850	7312	8775	5157	6446	7736
12100	110	6140	7675	9210	5700	7125	8550	4980	6225	7470
13225	115	6010	7512	9015	5550	6937	8325	4820	6025	7230
14400	120	5880	7350	8820	5410	6762	8115	4650	5812	6975
15625	125	5750	7187	8625	5260	6575	7890	4490	5612	6735
16900	130	5620	7025	8430	5120	6400	7680	4340	5425	6510
18225	135	5500	6875	8250	4980	6225	7470	4180	5225	6270
19600	140	5370	6712	8055	4840	6050	7260	4040	5050	6060
21025	145	5240	6550	7860	4700	5875	7050	3900	4875	5850
22500	150	5120	6400	7680	4570	5712	6855	3770	4712	5655
24025	155	5000	6250	7500	4440	5550	6660	3630	4537	5445
25600	160	4880	6100	7320	4320	5400	6580	3510	4387	5265

TABLE L.

PROPERTIES OF PHOENIX COLUMNS.

Thickness in ins.	d inside.	d outside.	d, over flanges.	Area of cross-section sq. in.	Least radius of gyration in ins.
$\frac{1}{16}$	A — 3⅜	4	6 1/16	3,8	1,45
$\frac{1}{16}$		4¼	6 3/16	4,8	1,50
$\frac{1}{16}$		4¼	6 5/16	5,8	1,55
$\frac{1}{16}$		4¾	6 7/16	6,8	1,59
$\frac{1}{16}$	B¹ — 4⅛	5 5/16	8 1/16	6,4	1,92
$\frac{1}{16}$		5 7/16	8 3/16	7,8	1,96
$\frac{1}{16}$		5 9/16	8¼	9,2	2,02
$\frac{1}{16}$		5 11/16	8⅜	10,6	2,07
$\frac{1}{16}$		5 13/16	8 7/16	12,0	2,11
$\frac{1}{16}$		5 15/16	8½	13,4	2,16
$\frac{1}{16}$		6 1/16	8⅝	14.8	2,20
$\frac{1}{16}$	B² — 5⅛	6 7/16	9⅜	7,4	2,34
$\frac{1}{16}$		6 9/16	9¼	9,0	2,39
$\frac{1}{16}$		6 11/16	9 5/16	10,6	2,43
$\frac{1}{16}$		6 13/16	9⅜	12,2	2,48
$\frac{1}{16}$		6 15/16	9 7/16	13,8	2,52
$\frac{1}{16}$		7 1/16	9½	15,4	2,57
$\frac{1}{16}$		7 3/16	9 11/16	17,0	2,61
$\frac{1}{16}$	C — 7 8/16	7 1/16	11 9/16	10,0	2,80
$\frac{1}{16}$		7 3/16	11 11/16	12,0	2,85
$\frac{1}{16}$		7 5/16	11 13/16	14,0	2,90
$\frac{1}{16}$		8 1/16	11¾	16,0	2,94
$\frac{1}{16}$		8 3/16	11 15/16	18,0	2,98
$\frac{1}{16}$		8 5/16	12 1/16	19,2	3,03
$\frac{1}{16}$		8 7/16	12	21,2	3,08
$\frac{1}{16}$		8 9/16	12 1/16	23,2	3,12
$\frac{1}{16}$		8 11/16	12 3/16	25,2	3,16
$\frac{1}{16}$		8 13/16	12 5/16	27,2	3,21
$\frac{1}{16}$		8 15/16	12 7/16	29,2	3,26
1		9 3/16	12 9/16	33,2	3,34
1¼		9 7/16	12¾	37,2	3,43
1¼		9 11/16	12 15/16	41.2	3,52

Thickness in ins.	d inside.	d outside.	d, over flanges.	Area of cross-section sq. in.	Least radius of gyration in ins.
¼	E — 11	11¼	15 7/16	16,8	4,18
¼		11	15 9/16	19,2	4,23
¼		11	15¾	21,6	4,28
¼		11	15	24,0	4,32
¼		12	15 1/16	26,4	4,36
¼		12⅛	16	28,8	4,40
¼		12¼	16 1/16	31,8	4,45
¼		12	16 3/16	34,8	4,50
¼		12	16 5/16	37,8	4,55
¼		12	16 7/16	40,8	4,60
¼		12⅜	16½	43,8	4,64
1		13	16¾	49,8	4,73
1¼		13¼	17	55,8	4,82
1¼		13⅜	17 8/16	61,8	4.91
⅝	G — 14⅜	15	19¼	24,0	5,45
⅝		15⅛	19¼	28,0	5,50
⅝		15¼	19⅜	32,0	5,55
⅝		15	19 7/16	36,0	5,59
⅝		15	19½	40,0	5,63
⅝		15	19⅝	44,0	5,68
⅝		15⅜	19¾	48,0	5,72
⅝		15⅜	19 7/8	52,0	5,77
⅝		16	20	56,0	5,82
⅝		16⅛	20⅛	60,0	5,87
1		16⅜	20⅜	68,0	5,95
1¼		16⅜	20⅜	76,0	6,04
1¼		16	20¼	84,0	6,14
1¼		17⅛	21	92,0	6,23

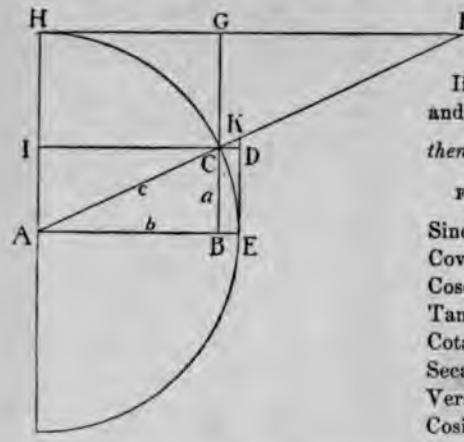
TABLE LI.

NATURAL SINES, ETC.

Degree.	Sine.	Covers.	Cosecnt.	Tangt.	Cotang.	Secant.	Versin.	Cosine.	
	For angles under 45°.								
0	0,00000	1,00000	Infinite.	0,00000	Infinite.	1,00000	0,0000	1,00000	90
1	0,01745	0,98254	57,2986	0,01745	57,2899	1,00015	0,0001	0,99984	89
2	0,03489	0,96510	28,6537	0,03492	28,6362	1,00060	0,0006	0,99939	88
3	0,05233	0,94766	19,1073	0,05240	19,0811	1,00137	0,0013	0,99862	87
4	0,06975	0,93024	14,3355	0,06992	14,3006	1,00244	0,0024	0,99756	86
5	0,08715	0,91284	11,4737	0,08748	11,4300	1,00381	0,0038	0,99619	85
6	0,10452	0,89547	9,5667	0,10510	9,5143	1,00550	0,0054	0,99452	84
7	0,12186	0,87813	8,2055	0,12278	8,1443	1,00750	0,0074	0,99254	83
8	0,13917	0,86082	7,1852	0,14054	7,1153	1,00982	0,0097	0,99026	82
9	0,15643	0,84356	6,3924	0,15838	6,3137	1,01246	0,0123	0,98768	81
10	0,17364	0,82635	5,7587	0,17632	5,6712	1,01542	0,0151	0,98480	80
11	0,19080	0,80919	5,2408	0,19438	5,1445	1,01871	0,0183	0,98162	79
12	0,20791	0,79208	4,8097	0,21255	4,7046	1,02234	0,0218	0,97814	78
13	0,22495	0,77504	4,4454	0,23086	4,3314	1,02630	0,0256	0,97437	77
14	0,24192	0,75807	4,1335	0,24932	4,0107	1,03061	0,0297	0,97029	76
15	0,25881	0,74118	3,8637	0,26794	3,7320	1,03527	0,0340	0,96592	75
16	0,27563	0,72436	3,6279	0,28674	3,4874	1,04029	0,0387	0,96126	74
17	0,29237	0,70762	3,4203	0,30573	3,2708	1,04569	0,0436	0,95630	73
18	0,30901	0,69098	3,2360	0,32491	3,0776	1,05146	0,0489	0,95105	72
19	0,32556	0,67443	3,0715	0,34432	2,9042	1,05762	0,0544	0,94551	71
20	0,34202	0,65797	2,9238	0,36397	2,7474	1,06417	0,0603	0,93969	70
21	0,35836	0,64163	2,7904	0,38386	2,6050	1,07114	0,0664	0,93358	69
22	0,37460	0,62539	2,6694	0,40402	2,4750	1,07853	0,0728	0,92718	68
23	0,39073	0,60926	2,5593	0,42447	2,3558	1,08636	0,0794	0,92050	67
24	0,40673	0,59326	2,4585	0,44522	2,2460	1,09463	0,0864	0,91354	66
25	0,42261	0,57738	2,3662	0,46630	2,1445	1,10337	0,0936	0,90630	65
26	0,43837	0,56162	2,2811	0,48773	2,0503	1,11260	0,1012	0,89879	64
27	0,45399	0,54600	2,2026	0,50952	1,9626	1,12232	0,1089	0,89100	63
28	0,46947	0,53052	2,1300	0,53170	1,8807	1,13257	0,1170	0,88294	62
29	0,48480	0,51519	2,0626	0,55430	1,8040	1,14335	0,1253	0,87461	61
30	0,50000	0,50000	2,0000	0,57735	1,7320	1,15470	0,1339	0,86602	60
31	0,51503	0,48496	1,9416	0,60086	1,6642	1,16663	0,1428	0,85716	59
32	0,52991	0,47008	1,8870	0,62486	1,6003	1,17917	0,1519	0,84804	58
33	0,54463	0,45536	1,8360	0,64940	1,5398	1,19236	0,1613	0,83867	57
34	0,55919	0,44080	1,7882	0,67450	1,4825	1,20621	0,1709	0,82903	56
35	0,57357	0,42642	1,7434	0,70020	1,4281	1,22077	0,1808	0,81915	55
36	0,58778	0,41221	1,7013	0,72654	1,3763	1,23606	0,1909	0,80901	54
37	0,60181	0,39818	1,6616	0,75355	1,3270	1,25213	0,2013	0,79863	53
38	0,61566	0,38433	1,6242	0,78128	1,2799	1,26901	0,2119	0,78801	52
39	0,62932	0,37067	1,5890	0,80978	1,2348	1,28675	0,2228	0,77714	51
40	0,64278	0,35721	1,5557	0,83909	1,1917	1,30540	0,2339	0,76604	50
41	0,65605	0,34394	1,5242	0,86928	1,1503	1,32501	0,2452	0,75470	49
42	0,66913	0,33086	1,4944	0,90040	1,1106	1,34563	0,2568	0,74314	48
43	0,68199	0,31800	1,4662	0,93251	1,0723	1,36732	0,2686	0,73135	47
44	0,69465	0,30534	1,4395	0,96568	1,0355	1,39016	0,2806	0,71933	46
45	0,70710	0,29289	1,4142	1,00000	1,0000	1,41421	0,2928	0,70710	45
	Cosine.	Versin.	Secant.	Cotang.	Tangt.	Cosecnt.	Covers.	Sine.	Degree.
	For angles over 45°.								

NOTE: If angle is smaller than 45° use top headings and read downwards, if greater than 45° use bottom headings and read upwards.

TABLE LI. (CONTINUED.)



If angle C B A = 90° and A C = 1

then:

FOR ANGLE C A B.

Sine	= C B
Coversine	= H I = G C
Cosecant	= A F
Tangent	= K E
Cotangent	= H F
Secant	= A K
Versine	= B E = C D
Cosine	= A B

If angle C A B = A and angle A C B = C; and A + C = 90°; and A C = c; C B = a; and A B = b, *then:*

Sine	A = Cosine	C =	$\frac{a}{c}$
Coversine	A = Versine	C =	$\frac{c-a}{c}$
Cosecant	A = Secant	C =	$\frac{c}{a}$
Tangent	A = Cotangent	C =	$\frac{a}{b}$
Cotangent	A = Tangent	C =	$\frac{b}{a}$
Secant	A = Cosecant	C =	$\frac{c}{b}$
Versine	A = Coversine	C =	$\frac{c-b}{c}$
Cosine	A = Sine	C =	$\frac{b}{c}$

$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

$a = \sqrt{c^2 - b^2} = \sqrt{(c+b).(c-b)}$

$b = \sqrt{c^2 - a^2} = \sqrt{(c+a).(c-a)}$

TABLE LII.

NATURAL SINES, TANGENTS AND SECANTS,

Advancing by 10 min.

Deg.	Min.	Sine.	Tangent.	Secant.	Deg.	Min.	Sine.	Tangent.	Secant.	Deg.	Min.	Sine.	Tangent.	Secant.
0	00	,0000	,0000	1,0000	8	00	,1392	,1405	1,0098	16	00	,2756	,2867	1,0403
	10	,0029	,0029	1,0000		10	,1421	,1435	1,0102		10	,2784	,2899	1,0412
	20	,0058	,0058	1,0000		20	,1449	,1465	1,0107		20	,2812	,2931	1,0421
	30	,0087	,0087	1,0000		30	,1478	,1495	1,0111		30	,2840	,2962	1,0429
	40	,0116	,0116	1,0001		40	,1507	,1524	1,0116		40	,2868	,2994	1,0439
	50	,0145	,0145	1,0001		50	,1536	,1554	1,0120		50	,2896	,3026	1,0448
1	00	,0175	,0175	1,0002	9	00	,1564	,1584	1,0125	17	00	,2924	,3057	1,0457
	10	,0204	,0204	1,0002		10	,1593	,1614	1,0129		10	,2952	,3089	1,0466
	20	,0233	,0233	1,0003		20	,1622	,1644	1,0134		20	,2979	,3121	1,0476
	30	,0262	,0262	1,0003		30	,1650	,1673	1,0139		30	,3007	,3153	1,0485
	40	,0291	,0291	1,0004		40	,1679	,1703	1,0144		40	,3035	,3185	1,0495
	50	,0320	,0320	1,0005		50	,1708	,1733	1,0149		50	,3062	,3217	1,0505
2	00	,0349	,0349	1,0006	10	00	,1736	,1763	1,0154	18	00	,3090	,3249	1,0515
	10	,0378	,0378	1,0007		10	,1765	,1793	1,0160		10	,3118	,3281	1,0525
	20	,0407	,0407	1,0008		20	,1794	,1823	1,0165		20	,3145	,3314	1,0535
	30	,0436	,0437	1,0010		30	,1822	,1853	1,0170		30	,3173	,3346	1,0545
	40	,0465	,0466	1,0011		40	,1851	,1883	1,0176		40	,3201	,3378	1,0555
	50	,0494	,0495	1,0012		50	,1880	,1914	1,0181		50	,3228	,3411	1,0566
3	00	,0523	,0524	1,0014	11	00	,1908	,1944	1,0187	19	00	,3256	,3443	1,0576
	10	,0552	,0553	1,0015		10	,1937	,1974	1,0193		10	,3283	,3476	1,0587
	20	,0581	,0582	1,0017		20	,1965	,2004	1,0199		20	,3311	,3508	1,0598
	30	,0610	,0612	1,0019		30	,1994	,2035	1,0205		30	,3338	,3541	1,0608
	40	,0640	,0641	1,0021		40	,2022	,2065	1,0211		40	,3365	,3574	1,0619
	50	,0669	,0670	1,0022		50	,2051	,2095	1,0217		50	,3393	,3607	1,0631
4	00	,0698	,0699	1,0024	12	00	,2079	,2126	1,0223	20	00	,3420	,3640	1,0642
	10	,0727	,0729	1,0027		10	,2108	,2156	1,0230		10	,3448	,3673	1,0653
	20	,0756	,0758	1,0029		20	,2136	,2186	1,0236		20	,3475	,3706	1,0665
	30	,0785	,0787	1,0031		30	,2164	,2217	1,0243		30	,3502	,3739	1,0676
	40	,0814	,0816	1,0033		40	,2193	,2247	1,0249		40	,3529	,3772	1,0688
	50	,0843	,0846	1,0036		50	,2221	,2278	1,0256		50	,3557	,3805	1,0700
5	00	,0872	,0875	1,0038	13	00	,2250	,2309	1,0263	21	00	,3584	,3839	1,0711
	10	,0901	,0904	1,0041		10	,2278	,2339	1,0270		10	,3611	,3872	1,0723
	20	,0929	,0934	1,0043		20	,2306	,2370	1,0277		20	,3638	,3906	1,0736
	30	,0958	,0963	1,0046		30	,2334	,2401	1,0284		30	,3665	,3939	1,0748
	40	,0987	,0992	1,0049		40	,2363	,2432	1,0291		40	,3692	,3973	1,0760
	50	,1016	,1022	1,0052		50	,2391	,2462	1,0299		50	,3719	,4006	1,0773
6	00	,1045	,1051	1,0055	14	00	,2419	,2493	1,0306	22	00	,3746	,4040	1,0785
	10	,1074	,1080	1,0058		10	,2447	,2524	1,0314		10	,3773	,4074	1,0798
	20	,1103	,1110	1,0061		20	,2476	,2555	1,0321		20	,3800	,4108	1,0811
	30	,1132	,1139	1,0065		30	,2504	,2586	1,0329		30	,3827	,4142	1,0824
	40	,1161	,1169	1,0068		40	,2532	,2617	1,0337		40	,3854	,4176	1,0837
	50	,1190	,1198	1,0072		50	,2560	,2648	1,0345		50	,3881	,4210	1,0850
7	00	,1219	,1228	1,0075	15	00	,2588	,2679	1,0353	23	00	,3907	,4245	1,0864
	10	,1248	,1257	1,0079		10	,2616	,2711	1,0361		10	,3934	,4279	1,0877
	20	,1276	,1287	1,0082		20	,2644	,2742	1,0369		20	,3961	,4314	1,0891
	30	,1305	,1317	1,0086		30	,2672	,2773	1,0377		30	,3987	,4348	1,0904
	40	,1334	,1346	1,0090		40	,2700	,2805	1,0386		40	,4014	,4383	1,0918
	50	,1363	,1376	1,0094		50	,2728	,2836	1,0394		50	,4041	,5417	1,0932

TABLE LII. — NATURAL SINES, TANGENTS AND SECANTS,

(CONTINUED).

Deg.	Min.	Sine.	Tangent.	Secant.	Deg.	Min.	Sine.	Tangent.	Secant.	Deg.	Min.	Sine.	Tangent.	Secant.
24	00	,4067	,4452	1,0946	32	00	,5299	,6249	1,1792	40	00	,6428	,8391	1,3054
	10	,4094	,4487	1,0961		10	,5324	,6289	1,1813		10	,6450	,8441	1,3086
	20	,4120	,4522	1,0975		20	,5348	,6330	1,1835		20	,6472	,8491	1,3118
	30	,4147	,4557	1,0989		30	,5373	,6371	1,1857		30	,6494	,8541	1,3151
	40	,4173	,4592	1,1004		40	,5398	,6412	1,1879		40	,6517	,8591	1,3184
	50	,4200	,4628	1,1019		50	,5422	,6453	1,1901		50	,6539	,8642	1,3217
25	00	,4226	,4663	1,1034	33	00	,5446	,6494	1,1924	41	00	,6561	,8693	1,3250
	10	,4253	,4699	1,1049		10	,5471	,6536	1,1946		10	,6583	,8744	1,3284
	20	,4279	,4734	1,1064		20	,5495	,6577	1,1969		20	,6604	,8796	1,3318
	30	,4305	,4770	1,1079		30	,5519	,6619	1,1992		30	,6626	,8847	1,3352
	40	,4331	,4806	1,1095		40	,5544	,6661	1,2015		40	,6648	,8899	1,3386
	50	,4358	,4841	1,1110		50	,5568	,6703	1,2039		50	,6670	,8952	1,3421
26	00	,4384	,4877	1,1126	34	00	,5592	,6745	1,2062	42	00	,6691	,9004	1,3456
	10	,4410	,4913	1,1142		10	,5616	,6787	1,2086		10	,6713	,9057	1,3492
	20	,4436	,4950	1,1158		20	,5640	,6830	1,2110		20	,6734	,9110	1,3527
	30	,4462	,4986	1,1174		30	,5664	,6873	1,2134		30	,6756	,9163	1,3563
	40	,4488	,5022	1,1190		40	,5688	,6916	1,2158		40	,6777	,9217	1,3600
	50	,4514	,5059	1,1207		50	,5712	,6959	1,2183		50	,6799	,9271	1,3636
27	00	,4540	,5095	1,1223	35	00	,5736	,7002	1,2208	43	00	,6820	,9325	1,3673
	10	,4566	,5132	1,1240		10	,5760	,7046	1,2233		10	,6841	,9380	1,3711
	20	,4592	,5169	1,1257		20	,5783	,7089	1,2258		20	,6862	,9435	1,3748
	30	,4617	,5206	1,1274		30	,5807	,7133	1,2283		30	,6884	,9490	1,3786
	40	,4643	,5243	1,1291		40	,5831	,7177	1,2309		40	,6905	,9545	1,3824
	50	,4669	,5280	1,1308		50	,5854	,7221	1,2335		50	,6926	,9601	1,3863
28	00	,4695	,5317	1,1326	36	00	,5878	,7265	1,2361	44	00	,6947	,9657	1,3902
	10	,4720	,5354	1,1343		10	,5901	,7310	1,2387		10	,6967	,9713	1,3941
	20	,4746	,5392	1,1361		20	,5925	,7355	1,2413		20	,6988	,9770	1,3980
	30	,4772	,5430	1,1379		30	,5948	,7400	1,2440		30	,7009	,9827	1,4020
	40	,4797	,5467	1,1397		40	,5972	,7445	1,2467		40	,7030	,9884	1,4061
	50	,4823	,5505	1,1415		50	,5995	,7490	1,2494		50	,7050	,9942	1,4101
29	00	,4848	,5543	1,1434	37	00	,6018	,7536	1,2521	45	00	,7071	1,0000	1,4142
	10	,4874	,5581	1,1452		10	,6041	,7581	1,2549		10	,7092	1,0058	1,4183
	20	,4899	,5619	1,1471		20	,6065	,7627	1,2577		20	,7112	1,0117	1,4225
	30	,4924	,5658	1,1490		30	,6088	,7673	1,2605		30	,7133	1,0176	1,4267
	40	,4950	,5696	1,1509		40	,6111	,7720	1,2633		40	,7153	1,0235	1,4310
	50	,4975	,5735	1,1528		50	,6134	,7766	1,2661		50	,7173	1,0295	1,4352
30	00	,5000	,5774	1,1547	38	00	,6157	,7813	1,2690	46	00	,7193	1,0355	1,4396
	10	,5025	,5812	1,1566		10	,6180	,7860	1,2719		10	,7214	1,0416	1,4439
	20	,5050	,5851	1,1586		20	,6202	,7907	1,2748		20	,7234	1,0477	1,4483
	30	,5075	,5890	1,1606		30	,6225	,7954	1,2778		30	,7254	1,0538	1,4527
	40	,5100	,5930	1,1626		40	,6248	,8002	1,2808		40	,7274	1,0599	1,4572
	50	,5125	,5969	1,1646		50	,6271	,8050	1,2837		50	,7294	1,0661	1,4617
31	00	,5150	,6009	1,1666	39	00	,6293	,8098	1,2868	47	00	,7314	1,0724	1,4663
	10	,5175	,6048	1,1687		10	,6316	,8146	1,2898		10	,7333	1,0786	1,4709
	20	,5200	,6088	1,1707		20	,6338	,8195	1,2929		20	,7353	1,0850	1,4755
	30	,5225	,6128	1,1728		30	,6361	,8243	1,2960		30	,7373	1,0913	1,4802
	40	,5250	,6168	1,1749		40	,6383	,8292	1,2991		40	,7392	1,0977	1,4849
	50	,5275	,6208	1,1770		50	,6406	,8342	1,3022		50	,7412	1,1041	1,4897

TABLE LII. — NATURAL SINES, TANGENTS AND SECANTS,

(CONTINUED).

Deg.	Min.	Sine.	Tangent.	Secant.	Deg.	Min.	Sine.	Tangent.	Secant.	Deg.	Min.	Sine.	Tangent.	Secant.
48	00	,7431	1,1106	1,4945	**56**	00	,8290	1,4826	1,7883	**64**	00	,8988	2,0503	2,2812
	10	,7451	1,1171	1,4993		10	,8307	1,4919	1,7960		10	,9001	2,0655	2,2949
	20	,7470	1,1237	1,5042		20	,8323	1,5013	1,8039		20	,9013	2,0809	2,3088
	30	,7490	1,1303	1,5092		30	,8339	1,5108	1,8118		30	,9026	2,0965	2,3228
	40	,7509	1,1369	1,5141		40	,8355	1,5204	1,8198		40	,9038	2,1123	2,3371
	50	,7528	1,1436	1,5192		50	,8371	1,5301	1,8279		50	,9051	2,1283	2,3515
49	00	,7547	1,1504	1,5243	**57**	00	,8387	1,5399	1,8361	**65**	00	,9063	2,1445	2,3662
	10	,7566	1,1571	1,5294		10	,8403	1,5497	1,8443		10	,9075	2,1609	2,3811
	20	,7585	1,1640	1,5345		20	,8418	1,5597	1,8527		20	,9088	2,1775	2,3961
	30	,7604	1,1708	1,5398		30	,8434	1,5697	1,8612		30	,9100	2,1943	2,4114
	40	,7623	1,1778	1,5450		40	,8450	1,5798	1,8699		40	,9112	2,2113	2,4269
	50	,7642	1,1847	1,5504		50	,8465	1,5900	1,8783		50	,9124	2,2286	2,4426
50	00	,7660	1,1918	1,5557	**58**	00	,8480	1,6003	1,8871	**66**	00	,9135	2,2460	2,4586
	10	,7679	1,1988	1,5611		10	,8496	1,6107	1,8959		10	,9147	2,2637	2,4748
	20	,7698	1,2059	1,5666		20	,8511	1,6213	1,9048		20	,9159	2,2817	2,4912
	30	,7716	1,2131	1,5721		30	,8526	1,6319	1,9139		30	,9171	2,2998	2,5078
	40	,7735	1,2203	1,5777		40	,8542	1,6426	1,9230		40	,9182	2,3183	2,5247
	50	,7753	1,2276	1,5833		50	,8557	1,6534	1,9323		50	,9194	2,3369	2,5419
51	00	,7771	1,2349	1,5890	**59**	00	,8572	1,6643	1,9416	**67**	00	,9205	2,3559	2,5593
	10	,7790	1,2423	1,5948		10	,8587	1,6753	1,9511		10	,9216	2,3750	2,5770
	20	,7808	1,2497	1,6005		20	,8601	1,6864	1,9606		20	,9228	2,3945	2,5949
	30	,7826	1,2572	1,6064		30	,8616	1,6977	1,9703		30	,9239	2,4141	2,6131
	40	,7844	1,2647	1,6123		40	,8631	1,7090	1,9801		40	,9250	2,4342	2,6316
	50	,7862	1,2723	1,6183		50	,8646	1,7205	1,9900		50	,9261	2,4545	2,6504
52	00	,7880	1,2799	1,6243	**60**	00	,8660	1,7321	2,0000	**68**	00	,9272	2,4751	2,6695
	10	,7898	1,2876	1,6303		10	,8675	1,7437	2,0101		10	,9283	2,4960	2,6888
	20	,7916	1,2954	1,6365		20	,8689	1,7556	2,0204		20	,9293	2,5172	2,7085
	30	,7934	1,3032	1,6427		30	,8704	1,7675	2,0308		30	,9304	2,5386	2,7285
	40	,7951	1,3111	1,6489		40	,8718	1,7796	2,0413		40	,9315	2,5603	2,7488
	50	,7969	1,3190	1,6553		50	,8732	1,7917	2,0519		50	,9325	2,5826	2,7695
53	00	,7986	1,3270	1,6616	**61**	00	,8746	1,8040	2,0627	**69**	00	,9336	2,6051	2,7904
	10	,8004	1,3351	1,6681		10	,8760	1,8165	2,0736		10	,9346	2,6279	2,8117
	20	,8021	1,3432	1,6746		20	,8774	1,8291	2,0846		20	,9356	2,6511	2,8334
	30	,8039	1,3514	1,6812		30	,8788	1,8418	2,0957		30	,9367	2,6746	2,8555
	40	,8056	1,3597	1,6878		40	,8802	1,8546	2,1070		40	,9377	2,6985	2,8779
	50	,8073	1,3680	1,6945		50	,8816	1,8676	2,1185		50	,9387	2,7228	2,9006
54	00	,8090	1,3764	1,7013	**62**	00	,8829	1,8807	2,1301	**70**	00	,9397	2,7475	2,9238
	10	,8107	1,3848	1,7081		10	,8843	1,8940	2,1418		10	,9407	2,7725	2,9474
	20	,8124	1,3934	1,7151		20	,8857	1,9074	2,1537		20	,9417	2,7980	2,9713
	30	,8141	1,4019	1,7221		30	,8870	1,9210	2,1657		30	,9426	2,8239	2,9957
	40	,8158	1,4106	1,7291		40	,8884	1,9347	2,1786		40	,9436	2,8502	3,0206
	50	,8175	1,4193	1,7362		50	,8897	1,9486	2,1902		50	,9446	2,8770	3,0458
55	00	,8192	1,4281	1,7434	**63**	00	,8910	1,9626	2,2027	**71**	00	,9455	2,9042	3,0716
	10	,8208	1,4370	1,7507		10	,8923	1,9768	2,2153		10	,9465	1,9319	3,0977
	20	,8225	1,4460	1,7581		20	,8936	1,9912	2,2282		20	,9474	2,9600	3,1244
	30	,8241	1,4550	1,7655		30	,8949	2,0057	2,2412		30	,9483	2,9887	3,1515
	40	,8258	1,4641	1,7730		40	,8962	2,0204	2,2543		40	,9492	3,0178	3,1792
	50	,8274	1,4733	1,7806		50	,8975	2,0353	2,2677		50	,9502	3,0475	3,2074

TABLE LII.—NATURAL SINES, TANGENTS AND SECANTS,

(CONTINUED).

Deg.	Min.	Sine.	Tangent.	Secant.	Deg.	Min.	Sine.	Tangent.	Secant.	Deg.	Min.	Sine.	Tangent.	Secant.
72	00	,9511	3,0777	3,2361	78	00	,9781	4,7046	4,8097	84	00	,9945	9,5144	9,5668
	10	,9520	3,1084	3,2653		10	,9787	4,7729	4,8765		10	,9948	9,7882	9,8391
	20	,9528	3,1397	3,2951		20	,9793	4,8430	4,9452		20	,9951	10,0780	10,1275
	30	,9537	3,1716	3,3255		30	,9799	4,9152	5,0159		30	,9954	10,3854	10,4334
	40	,9546	3,2041	3,3565		40	,9805	4,9894	5,0886		40	,9957	10,7119	10,7585
	50	,9555	3,2371	3,3881		50	,9811	5,0658	5,1636		50	,9959	11,0594	11,1045
73	00	,9563	3,2709	3,4203	79	00	,9816	5,1446	5,2408	85	00	,9962	11,430	11,474
	10	,9572	3,3052	3,4532		10	,9822	5,2257	5,3205		10	,9964	11,826	11,868
	20	,9580	3,3402	3,4867		20	,9827	5,3093	5,4026		20	,9967	12,251	12,291
	30	,9588	3,3759	3,5209		30	,9833	5,3955	5,4874		30	,9969	12,706	12,745
	40	,9596	3,4124	3,5559		40	,9838	5,4845	5,5749		40	,9971	13,197	13,235
	50	,9605	3,4496	3,5915		50	,9843	5,5764	5,6653		50	,9974	13,727	13,763
74	00	,9613	3,4874	3,6280	80	00	,9848	5,6713	5,5588	86	00	,9976	14,301	14,336
	10	,9621	3,5261	3,6652		10	,9853	5,7694	5,8554		10	,9978	14,924	14,958
	20	,9628	3,5656	3,7032		20	,9858	5,8708	5,9554		20	,9980	15,605	15,637
	30	,9636	3,6059	3,7420		30	,9863	5,9758	6,0589		30	,9981	16,350	16,380
	40	,9644	3,6470	3,7817		40	,9868	6,0844	6,1661		40	,9983	17,169	17,196
	50	,9652	3,6891	3,8222		50	,9872	6,1970	6,2772		50	,9985	18,075	18,103
75	00	,9659	3,7321	3,8637	81	00	,9877	6,3138	6,3925	87	00	,9986	19,081	19,107
	10	,9667	3,7760	3,9061		10	,9881	6,4348	6,5121		10	,9988	20,206	20,230
	20	,9674	3,8208	3,9495		20	,9886	6,5606	6,6363		20	,9989	21,470	21,494
	30	,9681	3,8667	3,9939		30	,9890	6,6912	6,7655		30	,9990	22,904	22,926
	40	,9689	3,9136	4,0394		40	,9894	6,8269	6,8998		40	,9992	24,542	24,562
	50	,9696	3,9617	4,0859		50	,9899	6,9682	7,0396		50	,9993	26,432	26,451
76	00	,9703	4,0108	4,1336	82	00	,9903	7,1154	7,1853	88	00	,9994	28,636	28,654
	10	,9710	4,0611	4,1824		10	,9907	7,2687	7,3372		10	,9995	31,242	31,258
	20	,9717	4,1126	4,2324		20	,9911	7,4287	7,4957		20	,9996	34,368	34,382
	30	,9724	4,1653	4,2837		30	,9914	7,5958	7,6613		30	,9997	38,188	38,202
	40	,9730	4,2193	4,3362		40	,9918	7,7704	7,8344		40	,9997	42,964	42,976
	50	,9737	4,2747	4,3901		50	,9922	7,9530	8,0156		50	,9998	49,104	49,114
77	00	,9744	4,3315	4,4454	83	00	,9925	8,1443	8,2055	89	00	,9998	57,290	57,299
	10	,9750	4,3897	4,5022		10	,9929	8,3450	8,4047		10	,9999	68,750	68,757
	20	,9757	4,4494	4,5604		20	,9932	8,5555	8,6138		20	,9999	85,940	85,946
	30	,9763	4,5107	4,6202		30	,9936	8,7769	8,8337		30	1,0000	114,589	114,593
	40	,9769	4,5736	4,6817		40	,9939	9,0098	9,0652		40	1,0000	171,885	171,888
	50	,9775	4,6382	4,7448		50	,9942	9,2553	9,3092		50	1,0000	343,774	343,775
										90	00	1,0000	Infinite	Infinite

TABLE LIII.

LOGARITHMS OF NUMBERS.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	40
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	37
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	33
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	31
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	29
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	27
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	25
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	24
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	23
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	21
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	18
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	17
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	16
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	14
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	13
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	13
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	12
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	12
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11
No.	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE LIII. — LOGARITHMS OF NUMBERS. (CONTINUED.)

No.	0	1	2	3	4	5	6	7	8	9	Diff.
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	11
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	10
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	10
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	10
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	10
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	9
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	9
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	8
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	8
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	8
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	8
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	8
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	7
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	7
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	7
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	7
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	6
No.	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE LIII. — LOGARITHMS OF NUMBERS. (CONTINUED.)

No.	0	1	2	3	4	5	6	7	8	9	Diff.
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	7
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	6
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	6
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	6
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	6
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	6
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	6
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	6
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	5
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	5
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	5
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	5
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	5
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	4
No.	0	1	2	3	4	5	6	7	8	9	Diff.

TABLE LIV.

SQUARES, CUBES, SQUARE AND CUBE ROOTS AND RECIPROCALS OF NUMBERS.

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
1	1	1	1,	1,	1,000000000
2	4	8	1,414214	1,259921	,500000000
3	9	27	1,732051	1,442250	,333333333
4	16	64	2,	1,587401	,250000000
5	25	125	2,236068	1,709976	,200000000
6	36	216	2,449490	1,817121	,166666667
7	49	343	2,645751	1,912931	,142857143
8	64	512	2,828427	2,	,125000000
9	81	729	3,	2,080084	,111111111
10	100	1000	3,162278	2,154435	,100000000
11	121	1331	3,316625	2,223980	,090909091
12	144	1728	3,464102	2,289429	,083333333
13	169	2197	3,605551	2,351335	,076923077
14	196	2744	3,741657	2,410142	,071428571
15	225	3375	3,872983	2,466212	,066666667
16	256	4096	4,	2,519842	,062500000
17	289	4913	4,123106	2,571282	,058823529
18	324	5832	4,242641	2,620741	,0555o5556
19	361	6859	4,358599	2,668402	,052631579
20	400	8000	4,472136	2,714418	,050000000
21	441	9261	4,582576	2,758924	,047619048
22	484	10648	4,690416	2,802039	,045454545
23	529	12167	4,795831	2,843867	,04347826o
24	576	13824	4,898979	2,884499	,041666667
25	625	15625	5,	2,924018	,040000000
26	676	17576	5,099019	2,962496	,038461538
27	729	19683	5,196152	3,	,037037037
28	784	21952	5,291503	3,036589	,035714286
29	841	24389	5,385165	3,072317	,034482759
30	900	27000	5,477226	3,107232	,033333333
31	961	29791	5,567764	3,141381	,032258065
32	1024	32768	5,656854	3,174802	,031250000
33	1089	35937	5,744563	3,207534	,030303030
34	1156	39304	5,830952	3,239612	,029411765
35	1225	42875	5,916080	3,271066	,028571429
36	1296	46656	6,	3,301927	,027777778
37	1369	50653	6,082762	3,332222	,027027027
38	1444	54872	6,164414	3,361975	,026315789
39	1521	59319	6,244998	3,391211	,025641026
40	1600	64000	6,324555	3,419952	,025000000
41	1681	68921	6,403124	3,448217	,024390244
42	1764	74088	6,480741	3,476027	,023809524
43	1849	79507	6,557438	3,503398	,023255814
44	1936	85184	6,633250	3,530348	,022727273
45	2025	91125	6,708204	3,556893	,022222222
46	2116	97336	6,782330	3,583048	,021739130
47	2209	103823	6,855655	3,608826	,021276600
48	2304	110592	6,928203	3,634241	,020833333
49	2401	117649	7,	3,659306	,020408163
50	2500	125000	7,071068	3,684031	,020000000
51	2601	132651	7,141428	3,708430	,019607843
52	2704	140608	7,211103	3,732511	,019230769
53	2809	148877	7,280110	3,756286	,018867925
54	2916	157464	7,348469	3,779763	,018518519
55	3025	166375	7,416198	3,802952	,018181818
56	3136	175616	7,4833 5	3,825862	,017857143
57	3249	185193	7,549834	3,848501	,017543860
58	3364	195112	7,615773	3,870877	,017241379
59	3481	205379	7,681146	3,892996	,016949153
60	3600	216000	7,745967	3,914868	,016666667
61	3721	226981	7,810250	3,936497	,016393443
62	3844	238328	7,874008	3,957891	,016129032

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
63	3969	250047	7,937254	3,979057	,015873016
64	4096	262144	8,	4,	,01562500
65	4225	274625	8,062258	4,020726	,015384615
66	4356	287496	8,124038	4,041240	,015151515
67	4489	300763	8,185353	4,061548	,014925373
68	4624	314432	8,246211	4,081655	,014705882
69	4761	328509	8,306624	4,101566	,014492754
70	4900	343000	8,366600	4,121285	,014285714
71	5041	357911	8,426150	4,140818	,014084507
72	5184	373248	8,485281	4,160168	,013888889
73	5329	389017	8,544004	4,179339	,013698630
74	5476	405224	8,602325	4,198336	,013513514
75	5625	421875	8,660254	4,217163	,013333333
76	5776	438976	8,717798	4,235824	,013157895
77	5929	456533	8,774964	4,254321	,012987013
78	6084	474552	8,831761	4,272659	,012820513
79	6241	493039	8,888194	4,290840	,012658228
80	6400	512000	8,944272	4,308869	,012500000
81	6561	531441	9,	4,326749	,012345679
82	6724	551368	9,055385	4,344481	,012195122
83	6889	571787	9,110434	4,362071	,012048193
84	7056	592704	9,165151	4,379519	,011904762
85	7225	614125	9,219544	4,396830	,011764706
86	7396	636056	9,273618	4,414005	,011627907
87	7569	658503	9,327379	4,431048	,011494253
88	7744	681472	9,380831	4,447960	,011363636
89	7921	704969	9,433981	4,464745	,011235955
90	8100	729000	9,486833	4,481405	,011111111
91	8281	753571	9,539392	4,497941	,010989011
92	8464	778688	9,591663	4,514357	,010869565
93	8649	804357	9,643651	4,530655	,010752688
94	8836	830584	9,695360	4,546836	,010638298
95	9025	857375	9,746794	4,562903	,010526316
96	9216	884736	9,797959	4,578857	,010416667
97	9409	912673	9,848858	4,594701	,010309278
98	9604	941192	9,899495	4,610436	,010204082
99	9801	970299	9,949874	4,626065	,010101010
100	10000	1000000	10,	4,641589	,010000000
101	10201	1030301	10,049876	4,657009	,009900990
102	10404	1061208	10,099505	4,672329	,009803922
103	10609	1092727	10,148892	4,687548	,009708738
104	10816	1124864	10,198039	4,702669	,009615385
105	11025	1157625	10,246951	4,717694	,009523810
106	11236	1191016	10,295630	4,732623	,009433962
107	11449	1225043	10,344080	4,747459	,009345794
108	11664	1259712	10,392305	4,762203	,009259259
109	11881	1295029	10,440306	4,776856	,009174312
110	12100	1331000	10,488088	4,791420	,009090909
111	12321	1367631	10,535653	4,805896	,009009009
112	12544	1404928	10,583005	4,8202-4	,008928571
113	12769	1442897	10,630145	4,834588	,008849558
114	12996	1481544	10,677078	4,848808	,008771930
115	13225	1520875	10,723805	4,862944	,008695652
116	13456	1560896	10,770329	4,876999	,008620690
117	13689	1601613	10,816653	4,890973	,008547009
118	13924	1643032	10,862780	4,904868	,008474576
119	14161	1685159	10,908712	4,918685	,008403361
120	14400	1728000	10,954451	4,932424	,008333333
121	14641	1771561	11,	4,946088	,008264463
122	14884	1815848	11,045361	4,959675	,008196721
123	15129	1860867	11,090536	4,973190	,008130081
124	15376	1906624	11,135528	4,986631	,008064516
125	15625	1953125	11,180339	5,	,008000000
126	15876	2000376	11,224972	5,013298	,007936508
127	16129	2048383	11,269427	5,026526	,007874016
128	16384	2097152	11,313708	5,039684	,007812500
129	16641	2146689	11,357816	5,052774	,007751938

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
130	16900	2197000	11,401754	5,065797	,007692308
131	17161	2248091	11,445523	5,078753	,007633588
132	17424	2299968	11,489125	5,091643	,007575758
133	17689	2352637	11,532562	5,104469	,007518797
134	17956	2406104	11,575836	5,117230	,007462687
135	18225	2460373	11,618950	5,129928	,007407407
136	18496	2515456	11,661903	5,142563	,007352941
137	18769	2571353	11,704699	5,155137	,007299270
138	19044	2628072	11,747344	5,167649	,007246377
139	19321	2685619	11,789826	5,180101	,007194245
140	19600	2744000	11,832159	5,192494	,007142857
141	19881	2803221	11,874342	5,204828	,007092199
142	20164	2863288	11,916375	5,217103	,007042254
143	20449	2924207	11,9582e0	5,229321	,006993007
144	20736	2985984	12,	5,241482	,006944444
145	21025	3048625	12,041594	5,253588	,006896552
146	21316	3112136	12,083046	5,265637	,006849315
147	21609	3176523	12,124355	5,277632	,006802721
148	21904	3241792	12,165525	5,289572	,006756757
149	22201	3307949	12,206555	5,301459	,006711409
150	22500	3375000	12,247448	5,313293	,006666667
151	22801	3442951	12,288205	5,325074	,006622517
152	23104	3511808	12,328828	5,336803	,006578947
153	23409	3581577	12,369316	5,348481	,006535948
154	23716	3652264	12,409673	5,360108	,006493506
155	24025	3723875	12,449899	5,371685	,006451613
156	24336	3796416	12,489996	5,383231	,006410256
157	24649	3869893	12,529964	5,394600	,006369427
158	24964	3944312	12,569805	5,406120	,006329114
159	25281	4019679	12,609520	5,417501	,006289308
160	25600	4096000	12,649110	5,428835	,006250000
161	25921	4173281	12,688577	5,440122	,006211180
162	26244	4251528	12,727922	5,451362	,006172840
163	26569	4330747	12,767145	5,462556	,006134969
164	26896	4410944	12,806248	5,473703	,006097561
165	27225	4492125	12,845232	5,484806	,006060606
166	27556	4574296	12,884098	5,495865	,006024096
167	27889	4657463	12,922848	5,506879	,005988024
168	28224	4741632	12,961481	5,517848	,005952381
169	28561	4826809	13,	5,528774	,005917160
170	28900	4913000	13,038405	5,539658	,005882353
171	29241	5000211	13,076697	5,550499	,005847953
172	29584	5088448	13,114877	5,561298	,005813953
173	29929	5177717	13,152946	5,572055	,005780347
174	30276	5268024	13,190906	5,582770	,005747126
175	30625	5359375	13,228757	5,593445	,005714286
176	30976	5451776	13,266499	5,604079	,005681818
177	31329	5545233	13,304135	5,614672	,005649718
178	31684	5639752	13,341664	5,625226	,005617978
179	32041	5735339	13,379088	5,635741	,005586592
180	32400	5832000	13,416408	5,646216	,005555556
181	32761	5929741	13,453624	5,656653	,005524862
182	33124	6028568	13,490738	5,667051	,005494505
183	33489	6128487	13,527749	5,677411	,005464481
184	33856	6229504	13,564660	5,687734	,005434783
185	34225	6331625	13,601470	5,698019	,005405405
186	34596	6434856	13,638182	5,708267	,005376344
187	34969	6539203	13,674794	5,718479	,005347594
188	35344	6644672	13,711309	5,728654	,005319149
189	35721	6751269	13,747727	5,738794	,005291005
190	36100	6859000	13,784049	5,748897	,005263158
191	36481	6967871	13,820275	5,758965	,005235602
192	36864	7077888	13,856406	5,768998	,005208333
193	37249	7189057	13,892440	5,778997	,005181347
194	37636	7301384	13,928388	5,788960	,005154639
195	38025	7414875	13,964240	5,798890	,005128205
196	38416	7529536	14,	5,808786	,005102041

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
197	38809	7645373	14,035669	5,818648	,005076142
198	39204	7762392	14,071247	5,828477	,005050505
199	39601	7880599	14,106736	5,838272	,005025126
200	40000	8000000	14,142136	5,848035	,005000000
201	40401	8120601	14,177447	5,857766	,004975124
202	40804	8242408	14,212670	5,867464	,004950495
203	41209	8365427	14,247807	5,877131	,004926108
204	41616	8489664	14,282857	5,886765	,004901961
205	42025	8615125	14,317821	5,896368	,004878049
206	42436	8741816	14,352700	5,905941	,004854369
207	42849	8869743	14,387495	5,915482	,004830918
208	43264	8998912	14,422205	5,924992	,004807692
209	43681	9129329	14,456832	5,934472	,004784689
210	44100	9261000	14,491377	5,943922	,004761905
211	44521	9393931	14,525839	5,953342	,004739336
212	44944	9528128	14,560220	5,962732	,004716981
213	45369	9663597	14,594519	5,972093	,004694836
214	45796	9800344	14,628739	5,981424	,004672897
215	46225	9938375	14,662878	5,990726	,004651163
216	46656	10077696	14,696938	6,	,004629630
217	47089	10218313	14,730920	6,009245	,004608295
218	47524	10360232	14,764823	6,018462	,004587156
219	47961	10503459	14,798649	6,027650	,004566210
220	48400	10648000	14,832397	6,036811	,004545455
221	48841	10793861	14,866069	6,045943	,004524887
222	49284	10941048	14,899664	6,055049	,004504505
223	49729	11089567	14,933184	6,064127	,004484305
224	50176	11239424	14,966629	6,073177	,004464286
225	50625	11390625	15,	6,082201	,004444444
226	51076	11543176	15,033296	6,091199	,004424779
227	51529	11697083	15,066519	6,100170	,004405286
228	51984	11852352	15,099668	6,109115	,004385965
229	52441	12008989	15,132746	6,118032	,004366812
230	52900	12167000	15,165750	6,126925	,004347826
231	53361	12326391	15,198684	6,135792	,004329004
232	53824	12487168	15,231546	6,144634	,004310345
233	54289	12649337	15,264337	6,153449	,004291845
234	54756	12812904	15,297058	6,162239	,004273504
235	55225	12977875	15,329709	6,171005	,004255319
236	55696	13144256	15,362291	6,179747	,004237288
237	56169	13312053	15,394804	6,188463	,004219409
238	56644	13481272	15,427248	6,197154	,004201681
239	57121	13651919	15,459624	6,205821	,004184100
240	57600	13824000	15,491933	6,214464	,004166667
241	58081	13997521	15,524174	6,223083	,004149378
242	58564	14172488	15,556349	6,231678	,004132231
243	59049	14348907	15,588457	6,240251	,004115226
244	59536	14526784	15,620499	6,248800	,004098361
245	60025	14706125	15,652475	6,257324	,004081633
246	60516	14886936	15,684387	6,265826	,004065041
247	61009	15069223	15,716233	6,274304	,004048583
248	61504	15252992	15,748015	6,282760	,004032258
249	62001	15438249	15,779733	6,291194	,004016064
250	62500	15625000	15,811388	6,299604	,004000000
251	63001	15813251	15,842979	6,307992	,003984064
252	63504	16003008	15,874507	6,316359	,003968254
253	64009	16194277	15,905973	6,324704	,003952569
254	64516	16387064	15,937377	6,333025	,003937008
255	65025	16581375	15,968719	6,341325	,003921569
256	65536	16777216	16,	6,349602	,003906250
257	66049	16974593	16,031219	6,357859	,003891051
258	66564	17173512	16,062378	6,366095	,003875969
259	67081	17373979	16,093476	6,374310	,003861004
260	67600	17576000	16,124515	6,382504	,003846154
261	68121	17779581	16,155494	6,390676	,003831418
262	68644	17984728	16,186414	6,398827	,003816794
263	69169	18191447	16,217274	6,406958	,003802281

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
284	[illegible]	1906744	16.248076	6.415068	.003518879
285	79225	1906625	16.278820	6.423157	.003473685
286	79524	1927464	16.309506	6.431228	.003496398
287	79250	1950453	16.340134	6.439275	.003545318
288	79524	1934952	16.370705	6.447306	.003531343
289	79521	1946969	16.401219	6.455314	.003517472
270	72900	1968300	16.431676	6.463304	.003703704
271	73441	1989511	16.462077	6.471274	.003689057
272	73984	2013248	16.492422	6.479224	.003676471
273	74529	2034417	16.522711	6.487153	.003663004
274	75076	2057824	16.552945	6.495064	.003649635
275	75625	2078075	16.583124	6.502956	.003636364
276	76176	2102576	16.613247	6.510829	.003623188
277	76729	2125333	16.643317	6.518684	.003610108
278	77284	2148352	16.673332	6.526519	.003597122
279	77841	2171639	16.703293	6.534335	.003584229
280	78400	2195200	16.733200	6.542132	.003571429
281	78961	2219041	16.763055	6.549912	.003558719
282	79524	2243778	16.792856	6.557672	.003546099
283	80089	2266187	16.822604	6.565414	.003533569
284	80656	2290304	16.852299	6.573138	.003521127
285	81225	2315125	16.881943	6.580844	.003508772
286	81796	2339656	16.911534	6.588532	.003496503
287	82369	2363903	16.941074	6.596202	.003484321
288	82944	2387872	16.970562	6.603854	.003472222
289	83521	2413569	17.	6.611489	.003460208
290	84100	2439000	17.029386	6.619106	.003448276
291	84681	2464171	17.058722	6.626705	.003436426
292	85264	2487088	17.088007	6.634287	.003424658
293	85849	2515757	17.117243	6.641852	.003412969
294	86436	2542184	17.146428	6.649400	.003401361
295	87025	2567375	17.175564	6.656930	.003389831
296	87616	2593336	17.204650	6.664444	.003378378
297	88209	2619073	17.233688	6.671940	.003367003
298	88804	2645592	17.262676	6.679420	.003355705
299	89401	2673899	17.291616	6.686883	.003344482
300	90000	27000000	17.320508	6.694329	.003333333
301	90601	2727001	17.349352	6.701759	.003322259
302	91204	2754008	17.378147	6.709173	.003311258
303	91809	2781827	17.406895	6.716570	.003300330
304	92416	2809464	17.435596	6.723951	.003289474
305	93025	2837925	17.464249	6.731315	.003278689
306	93636	2867216	17.492856	6.738664	.003267974
307	94249	2896443	17.521415	6.745997	.003257329
308	94864	2928112	17.549929	6.753313	.003246753
309	95481	2959629	17.578396	6.760614	.003236246
310	96100	2991000	17.606817	6.767899	.003225806
311	96721	30080231	17.635192	6.775169	.003215434
312	97344	30371328	17.663522	6.782423	.003205128
313	97969	30664297	17.691806	6.789661	.003194888
314	98596	30959144	17.720045	6.796884	.003184713
315	99225	31255875	17.748239	6.804092	.003174603
316	99856	31554496	17.776389	6.811285	.003164557
317	100489	31855013	17.804494	6.818462	.003154574
318	101124	32157432	17.832554	6.825624	.003144654
319	101761	32461759	17.860571	6.832771	.003134796
320	102400	32768000	17.888544	6.839904	.003125000
321	103041	33076161	17.916473	6.847021	.003115265
322	103684	33386248	17.944358	6.854124	.003105590
323	104329	33698267	17.972201	6.861212	.003095975
324	104976	34012224	18.	6.868285	.003086420
325	105625	34328125	18.027756	6.875344	.003076923
326	106276	34645976	18.055470	6.882389	.003067485
327	106929	34965783	18.083141	6.889419	.003058104
328	107584	35287552	18.110770	6.896434	.003048780
329	108241	35611289	18.138357	6.903436	.003039514
330	108900	35937000	18.165902	6.910423	.003030303

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
331	109561	36264691	18,193405	6,917396	,003021148
332	110224	36594368	18,220867	6,924356	,003012048
333	110889	36926037	18,248288	6,931309	,003003003
334	111556	37259704	18,275667	6,938232	,002994012
335	112225	37595375	18,303005	6,945150	,002985075
336	112896	37933056	18,330302	6,952053	,002976190
337	113569	38272753	18,357559	6,958943	,002967359
338	114244	38614472	18,384776	6,965819	,002958580
339	114921	38958219	18,411952	6,972682	,002949853
340	115600	39304000	18,439088	6,979532	,002941176
341	116281	39651821	18,466185	6,986369	,002932551
342	116964	40001688	18,493242	6,993491	,002923977
343	117649	40353607	18,520259	7,	,002915452
344	118336	40707584	18,547237	7,006796	,002906977
345	119025	41063625	18,574175	7,013579	,002898551
346	119716	41421736	18,601075	7,020349	,002890173
347	120409	41781923	18,627936	7,027106	,002881844
348	121104	42144192	18,654758	7,033850	,002873563
349	121801	42508549	18,681541	7,040581	,002865330
350	122500	42875000	18,708286	7,047208	,002857143
351	123201	43243551	18,734994	7,054003	,002849003
352	123904	43614208	18,761663	7,060696	,002840909
353	124609	43986977	18,788294	7,067376	,002832861
354	125316	44361864	18,814887	7,074043	,002824859
355	126025	44738875	18,841443	7,080698	,002816901
356	126736	45118016	18,867962	7,087341	,002808989
357	127449	45499293	18,894443	7,093970	,002801120
358	128164	45882712	18,920887	7,100588	,002793296
359	128881	46268279	18,947295	7,107193	,002785515
360	129600	46656000	18,973666	7,113786	,002777778
361	130321	47045881	19,	7,120367	,002770083
362	131044	47437928	19,026297	7,126935	,002762431
363	131769	47832147	19,052558	7,133492	,002754821
364	132496	48228544	19,078784	7,140037	,002747253
365	133225	48627125	19,104973	7,146569	,002739726
366	133956	49027896	19,131126	7,153090	,002732240
367	134689	49430863	19,157244	7,159599	,002724796
368	135424	49836032	19,183326	7,166095	,002717391
369	136161	50243409	19,209372	7,172580	,002710027
370	136900	50653000	19,235384	7,179054	,002702703
371	137641	51064811	19,261360	7,185516	,002695418
372	138384	51478848	19,287301	7,191966	,002688172
373	139129	51895117	19,313207	7,198405	,002680965
374	139876	52313624	19,339079	7,204832	,002673797
375	140625	52734375	19,364916	7,211247	,002666667
376	141376	53157376	19,390719	7,217652	,002659574
377	142129	53582633	19,416487	7,224045	,002652520
378	142884	54010152	19,442222	7,230427	,002645503
379	143641	54439939	19,467922	7,236797	,002638522
380	144400	54872000	19,493588	7,243156	,002631579
381	145161	55306341	19,519221	7,249504	,002624672
382	145924	55742968	19,544820	7,255841	,002617801
383	146689	56181887	19,570385	7,262167	,002610966
384	147456	56623104	19,595917	7,268482	,002604167
385	148225	57066625	19,621416	7,274786	,002597403
386	148996	57512456	19,646882	7,281079	,002590674
387	149769	57960603	19,672315	7,287362	,002583979
388	150544	58411072	19,697715	7,293633	,002577320
389	151321	58863869	19,723082	7,299893	,002570694
390	152100	59319000	19,748417	7,306143	,002564103
391	152881	59776471	19,773719	7,312383	,002557545
392	153664	60236288	19,798990	7,318611	,002551020
393	154449	60698457	19,824228	7,324829	,002544529
394	155236	61162984	19,849433	7,331037	,002538071
395	156025	61629875	19,874607	7,337234	,002531646
396	156816	62099136	19,899749	7,343420	,002525253
397	157609	62570773	19,924859	7,349597	,002518892

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
398	158404	63044792	19,949937	7,355762	,002512563
399	159201	63521199	19,974984	7,361918	,002506266
400	160000	64000000	20,	7,368063	,002500000
401	160801	64481201	20,024984	7,374198	,002493766
402	161604	64964808	20,049938	7,380323	,002487562
403	162409	65450827	20,074860	7,386437	,002481390
404	163216	65939264	20,099751	7,392542	,002475248
405	164025	66430125	20,124612	7,398636	,002469136
406	164836	66923416	20,149442	7,404721	,002463054
407	165649	67419143	20,174241	7,410795	,002457002
408	166464	67917312	20,199010	7,416859	,002450980
409	167281	68417929	20,223748	7,422914	,002444988
410	168100	68921000	20,248457	7,428959	,002439024
411	168921	69426531	20,273135	7,434994	,002433090
412	169744	69934528	20,297783	7,441019	,002427184
413	170569	70444997	20,322401	7,447034	,002421308
414	171396	70957944	20,346990	7,453040	,002415459
415	172225	71473375	20,371549	7,459036	,002409639
416	173056	71991296	20,396078	7,465022	,002403846
417	173889	72511713	20,420578	7,470999	,002398082
418	174724	73034632	20,445048	7,476966	,002392344
419	175561	73560059	20,469489	7,482924	,002386635
420	176400	74088000	20,493901	7,488872	,002380952
421	177241	74618461	20,518284	7,494811	,002375297
422	178084	75151448	20,542639	7,500741	,002369668
423	178929	75686967	20,566964	7,506661	,002364066
424	179776	76225024	20,591260	7,512571	,002358491
425	180625	76765625	20,615528	7,518473	,002352941
426	181476	77308776	20,639767	7,524365	,002347418
427	182329	77854483	20,663978	7,530248	,002341920
428	183184	78402752	20,688161	7,536122	,002336449
429	184041	78953589	20,712315	7,541987	,002331002
430	184900	79507000	20,736441	7,547842	,002325581
431	185761	80062991	20,760539	7,553689	,002320186
432	186624	80621568	20,784610	7,559526	,002314815
433	187489	81182737	20,808652	7,565355	,002309469
434	188356	81746504	20,832667	7,571174	,002304147
435	189225	82312875	20,856654	7,576985	,002298851
436	190096	82881856	20,880613	7,582786	,002293578
437	190969	83453453	20,904545	7,588579	,002288330
438	191844	84027672	20,928449	7,594363	,002283105
439	192721	84604519	20,952327	7,600138	,002277904
440	193600	85184000	20,976177	7,605905	,002272727
441	194481	85766121	21,	7,611663	,002267574
442	195364	86350888	21,023796	7,617412	,002262443
443	196249	86938307	21,047565	7,623152	,002257336
444	197136	87528384	21,071307	7,628884	,002252252
445	198025	88121125	21,095023	7,634607	,002247191
446	198916	88716536	21,118712	7,640321	,002242152
447	199809	89314623	21,142374	7,646027	,002237136
448	200704	89915392	21,166010	7,651725	,002232143
449	201601	90518849	21,189620	7,657414	,002227171
450	202500	91125000	21,213203	7,663094	,002222222
451	203401	91733851	21,236760	7,668766	,002217295
452	204304	92345408	21,260291	7,674430	,002212389
453	205209	92959677	21,283796	7,680085	,002207506
454	206106	93576664	21,307275	7,685732	,002202643
455	207025	94196375	21,330729	7,691371	,002197802
456	207936	94818816	21,354156	7,697002	,002192982
457	208849	95443993	21,377558	7,702624	,002188184
458	209764	96071912	21,400934	7,708238	,002183406
459	210681	96702579	21,424285	7,713844	,002178649
460	211600	97336000	21,447610	7,719442	,002173913
461	212521	97972181	21,470910	7,725032	,002169197
462	213444	98611128	21,494185	7,730614	,002164502
463	214369	99252847	21,517434	7,736187	,002159827
464	215296	99897344	21,540659	7,741753	,002155172

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
465	216225	100544625	21,563858	7,747310	,002150538
466	217156	101194696	21,587033	7,752860	,002145923
467	218089	101847563	21,610182	7,758402	,002141328
468	219024	102503232	21,633307	7,763936	,002136752
469	219961	103161709	21,656407	7,769462	,002132196
470	220900	103823000	21,679483	7,774980	,002127660
471	221841	104487111	21,702534	7,780490	,002123142
472	222784	105154048	21,725561	7,785992	,002118644
473	223729	105823817	21,748563	7,791487	,002114165
474	224676	106496424	21,771541	7,796974	,002109705
475	225625	107171875	21,794494	7,802453	,002105263
476	226576	107850176	21,817424	7,807925	,002100840
477	227529	108531333	21,840329	7,813389	,002096436
478	228484	109215352	21,863211	7,818845	,002092050
479	229441	109902239	21,886068	7,824294	,002087683
480	230400	110592000	21,908902	7,829735	,002083333
481	231361	111284641	21,931712	7,835168	,002079002
482	232324	111980168	21,954498	7,840594	,002074689
483	233289	112678587	21,977261	7,846013	,002070393
484	234256	113379904	22,	7,851424	,002066116
485	235225	114084125	22,022715	7,856828	,002061856
486	236196	114791256	22,045407	7,862224	,002057613
487	237169	115501303	22,068076	7,867613	,002053388
488	238144	116214272	22,090722	7,872994	,002049180
489	239121	116930169	22,113344	7,878368	,002044990
490	240100	117649000	22,135943	7,883734	,002040816
491	241081	118370771	22,158519	7,889094	,002036660
492	242064	119095488	22,181073	7,894446	,002032520
493	243049	119823157	22,203603	7,899791	,002028398
494	244036	120553784	22,226110	7,905129	,002024291
495	245025	121287375	22,248595	7,910460	,002020202
496	246016	122023936	22,271057	7,915784	,002016129
497	247009	122763473	22,293496	7,921100	,002012072
498	248004	123505992	22,315913	7,926408	,002008032
499	249001	124251499	22,338307	7,931710	,002004008
500	250000	125000000	22,360679	7,937005	,002000000
501	251001	125751501	22,383029	7,942293	,001996008
502	252004	126506008	22,405356	7,947573	,001992032
503	253009	127263527	22,427661	7,952847	,001988072
504	254016	128024064	22,449944	7,958114	,001984127
505	255025	128787625	22,472205	7,963374	,001980198
506	256036	129554216	22,494444	7,968627	,001976285
507	257049	130323843	22,516660	7,973873	,001972387
508	258064	131096512	22,538855	7,979112	,001968504
509	259081	131872229	22,561028	7,984344	,001964637
510	260100	132651000	22,583180	7,989570	,001960784
511	261121	133432831	22,605309	7,994788	,001956947
512	262144	134217728	22,627417	8,	,001953125
513	263169	135005697	22,649503	8,005205	,001949318
514	264196	135796744	22,671568	8,010403	,001945525
515	265225	136590875	22,693611	8,015595	,001941748
516	266256	137388096	22,715633	8,020779	,001937984
517	267289	138188413	22,737634	8,025957	,001934236
518	268324	138991832	22,759613	8,031129	,001930502
519	269361	139798359	22,781571	8,036293	,001926782
520	270400	140608000	22,803508	8,041451	,001923077
521	271441	141420761	22,825424	8,046603	,001919386
522	272484	142236648	22,847319	8,051748	,001915709
523	273529	143055667	22,869193	8,056886	,001912046
524	274576	143877824	22,891046	8,062018	,001908397
525	275625	144703125	22,912878	8,067143	,001904762
526	276676	145531576	22,934690	8,072262	,001901141
527	277729	146363183	22,956481	8,077374	,001897533
528	278784	147197952	22,978251	8,082480	,001893939
529	279841	148035889	23,	8,087579	,001890359
530	280900	148877000	23,021729	8,092672	,001886792
531	281961	149721291	23,043437	8,097759	,001883239

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
532	283024	150568768	23,065125	8,102839	,001879699
533	284089	151419437	23,086793	8,107913	,001876173
534	285156	152273304	23,108440	8,112980	,001872659
535	286225	153130375	23,130067	8,118041	,001869159
536	287296	153990656	23,151674	8,123096	,001865672
537	288369	154854153	23,173260	8,128145	,001862197
538	289444	155720872	23,194827	8,133187	,001858736
539	290521	156590819	23,216373	8,138223	,001855288
540	291600	157464000	23,237900	8,143253	,001851852
541	292681	158340421	23,259407	8,148276	,001848429
542	293764	159220088	23,280893	8,153294	,001845018
543	294849	160103007	23,302360	8,158305	,001841621
544	295936	160989184	23,323808	8,163310	,001838235
545	297025	161878625	23,345235	8,168309	,001834862
546	298116	162771336	23,366643	8,173302	,001831502
547	299209	163667323	23,388031	8,178289	,001828154
548	300304	164566592	23,409400	8,183269	,001824818
549	301401	165469149	23,430749	8,188244	,001821494
550	302500	166375000	23,452079	8,193213	,001818182
551	303601	167284151	23,473389	8,198175	,001814882
552	304704	168196608	23,494680	8,203132	,001811594
553	305809	169112377	23,515952	8,208082	,001808318
554	306916	170031464	23,537205	8,213027	,001805054
555	308025	170953875	23,558438	8,217966	,001801802
556	309136	171879616	23,579652	8,222898	,001798561
557	310249	172808693	23,600847	8,227825	,001795332
558	311364	173741112	23,622024	8,232746	,001792115
559	312481	174676879	23,643181	8,237661	,001788909
560	313600	175616000	23,664319	8,242570	,001785714
561	314721	176558481	23,685438	8,247474	,001782531
562	315844	177504328	23,706539	8,252371	,001779359
563	316969	178453547	22,727621	8,257263	,001776199
564	318096	179406144	23,748684	8,262149	,001773050
565	319225	180362125	23,769728	8,267029	,001769912
566	320356	181321496	23,790754	8,271903	,001766784
567	321489	182284263	23,811761	8,276772	,001763668
568	322624	183250432	23,832750	8,281635	,001760563
569	323761	184220009	23,853720	8,286493	,001757469
570	324900	185193000	23,874672	8,291344	,001754386
571	326041	186169411	23,895606	8,296190	,001751313
572	327184	187149248	23,916521	8,301030	,001748252
573	328329	188132517	23,937418	8,305865	,001745201
574	329476	189119224	23,958297	8,310694	,001742160
575	330625	190109375	23,979157	8,315517	,001739130
576	331776	191102976	24,	8,320335	,001736111
577	332929	192100033	24,020824	8,325147	,001733102
578	334084	193100552	24,041630	8,329954	,001730104
579	335241	194104539	24,062418	8,334755	,001727116
580	336400	195112000	24,083189	8,339551	,001724138
581	337561	196122941	24,103941	8,344341	,001721170
582	338724	197137368	24,124676	8,349125	,001718213
583	339889	198155287	24,145392	8,353904	,001715266
584	341056	199176704	24,166091	8,358678	,001712329
585	342225	200201625	24,186773	8,363446	,001709402
586	343396	201230056	24,207436	8,368209	,001706485
587	344569	202262003	24,228082	8,372966	,001703578
588	345744	203297472	24,248711	8,377718	,001700680
589	346921	204336469	24,269322	8,382465	,001697793
590	348100	205379000	24,289915	8,387206	,001694915
591	349281	206425071	24,310491	8,391942	,001692047
592	350464	207474688	24,331050	8,396673	,001689189
593	351649	208527857	24,351591	8,401398	,001686341
594	352836	209584584	24,372115	8,406118	,001683502
595	354025	210644875	24,392621	8,410832	,001680672
596	355216	211708736	24,413111	8,415541	,001677852
597	356409	212776173	24,433583	8,420245	,001675042
598	357604	213847192	24,454038	8,424944	,001672241

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
599	358801	214921799	24,474476	8,429638	,001669449
600	360000	216000000	24,494897	8,434327	,001666667
601	361201	217081801	24,515301	8,439009	,001663894
602	362404	218167208	24,535688	8,443687	,001661130
603	363609	219256227	24,556058	8,448360	,001658375
604	364816	220348864	24,576411	8,453027	,001655629
605	366025	221445125	24,596747	8,457689	,001652893
606	367236	222545016	24,617067	8,462347	,001650165
607	368449	223648543	24,637370	8,466999	,001647446
608	369664	224755712	24,657656	8,471647	,001644737
609	370881	225866529	24,677925	8,476289	,001642036
610	372100	226981000	24,698178	8,480926	,001639344
611	373321	228099131	24,718414	8,485557	,001636661
612	374544	229220928	24,738633	8,490184	,001633987
613	375769	230346397	24,758836	8,494806	,001631321
614	376996	231475544	24,779023	8,499423	,001628664
615	378225	232608375	24,799193	8,504034	,001626016
616	379456	233744896	24,819347	8,508642	,001623377
617	380689	234885113	24,839485	8,513243	,001620746
618	381924	236029032	24,859606	8,517840	,001618123
619	383161	237176659	24,879711	8,522432	,001615509
620	384400	238328000	24,899799	8,527019	,001612903
621	385641	239483061	24,919872	8,531601	,001610306
622	386884	240641848	24,939928	8,536178	,001607717
623	388129	241804367	24,959968	8,540750	,001605136
624	389376	242970624	24,979992	8,545317	,001602564
625	390625	244140625	25,	8,549880	,001600000
626	391876	245134376	25,019992	8,554437	,001597444
627	393129	246491883	25,039968	8,558990	,001594896
628	394384	247673152	25,059928	8,563538	,001592357
629	395641	248858189	25,079872	8,568081	,001589825
630	396900	250047000	25,099801	8,572619	,001587302
631	398161	251239591	25,119713	8,577152	,001584786
632	399424	252435968	25,139610	8,581681	,001582278
633	400689	253636137	25,159491	8,586205	,001579779
634	401956	254840104	25,179357	8,590724	,001577287
635	403225	256047875	25,199206	8,595238	,001574803
636	404496	257259456	25,219040	8,599748	,001572327
637	405769	258474853	25,238859	8,604252	,001569859
638	407044	259694072	25,258662	8,608753	,001567398
639	408321	260917119	25,278449	8,613248	,001564945
640	409600	262144000	25,298221	8,617739	,001562500
641	410881	263374721	25,317978	8,622225	,001560062
642	412164	264609288	25,337719	8,626706	,001557632
643	413449	265847707	25,357445	8,631183	,001555210
644	414736	267089984	25,377155	8,635655	,001552795
645	416025	268336125	25,396850	8,640123	,001550388
646	417316	269585136	25,416530	8,644585	,001547988
647	418609	270840023	25,436195	8,649044	,001545595
648	419904	272097792	25,455844	8,653497	,001543210
649	421201	273359549	25,475478	8,657946	,001540832
650	422500	274625000	25,495098	8,662391	,001538462
651	423801	275894451	25,514702	8,666831	,001536098
652	425104	277167808	25,534291	8,671266	,001533742
653	426409	278445077	25,553865	8,675697	,001531394
654	427716	279726264	25,573424	8,680124	,001529052
655	429025	281011375	25,592968	8,684546	,001526718
656	430336	282300416	25,612497	8,688963	,001524390
657	431649	283593393	25,632011	8,693376	,001522070
658	432964	284890312	25,651511	8,697784	,001519757
659	434281	286191179	25,670995	8,702188	,001517451
660	435600	287496000	25,690465	8,706588	,001515152
661	436921	288804781	25,709920	8,710983	,001512859
662	438244	290117528	25,729361	8,715373	,001510574
663	439569	291434247	25,748786	8,719760	,001508296
664	440896	292754944	25,768197	8,724141	,001506024
665	442225	294079625	25,787594	8,728519	,001503759

No.	Square	Cube	Sq. Root.	Cube Root.	Reciprocal.
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
733	537289	393832837	27,073973	9,016431	,001364256
734	538756	395446904	27,092434	9,020529	,001362398
735	540225	397065375	27,110883	9,024624	,001360544
736	541696	398688256	27,129320	9,028715	,001358696
737	543169	400315553	27,147744	9,032802	,001356852
738	544644	401947272	27,166155	9,036886	,001355014
739	546121	403583419	27,184554	9,040965	,001353180
740	547600	405224000	27,202941	9,045042	,001351351
741	549081	406869021	27,221315	9,049114	,001349528
742	550564	408518488	27,239677	9,053183	,001347709
743	552049	410172407	27,258026	9,057248	,001345895
744	553536	411830784	27,276363	9,061310	,001344086
745	555025	413493625	27,294688	9,065368	,001342282
746	556516	415160936	27,313001	9,069422	,001340483
747	558009	416832723	27,331301	9,073473	,001338688
748	559504	418508992	27,349589	9,077520	,001336898
749	561001	420189749	27,367864	9,081563	,001335113
750	562500	421875000	27,386128	9,085603	,001333333
751	564001	423564751	27,404379	9,089639	,001331558
752	565504	425259008	27,422618	9,093672	,001329787
753	567009	426957777	27,440845	9,097701	,001328021
754	568516	428661064	27,459060	9,101726	,001326260
755	570025	430368875	27,477263	9,105748	,001324503
756	571536	432081216	27,495454	9,109767	,001322751
757	573049	433798093	27,513633	9,113782	,001321004
758	574564	435519512	27,531800	9,117793	,001319261
759	576081	437245479	27,549955	9,121801	,001317523
760	577600	438976000	27,568097	9,125805	,001315789
761	579121	440711081	27,586228	9,129806	,001314060
762	580644	442450728	27,604347	9,133803	,001312336
763	582169	444194947	27,622455	9,137797	,001310616
764	583696	445943744	27,640550	9,141787	,001308901
765	585225	447697125	27,658633	9,145774	,001307190
766	586756	449455096	27,676705	9,149758	,001305483
767	588289	451217663	27,694765	9,153737	,001303781
768	589824	452984832	27,712813	9,157714	,001302083
769	591361	454756609	27,730849	9,161687	,001300390
770	592900	456533000	27,748874	9,165656	,001298701
771	594441	458314011	27,766887	9,169622	,001297017
772	595984	460099648	27,784888	9,173585	,001295337
773	597529	461889917	27,802877	9,177544	,001293661
774	599076	463684824	27,820855	9,181500	,001291990
775	600625	465484375	27,838822	9,185453	,001290323
776	602176	467288576	27,856777	9,189402	,001288660
777	603729	469097433	27,874720	9,193347	,001287001
778	605284	470910952	27,892651	9,197290	,001285347
779	606841	472729139	27,910571	9,201229	,001283697
780	608400	474552000	27,928480	9,205164	,001282051
781	609961	476379541	27,946377	9,209096	,001280410
782	611524	478211768	27,964263	9,213025	,001278772
783	613089	480048687	27,982137	9,216950	,001277139
784	614656	481890304	28,	9,220872	,001275510
785	616225	483736025	28,017851	9,224791	,001273885
786	617796	485587656	28,035691	9,228706	,001272265
787	619369	487443403	28,053520	9,232618	,001270648
788	620944	489303872	28,071337	9,237527	,001269036
789	622521	491169069	28,089143	9,240433	,001267427
790	624100	493039000	28,106938	9,244335	,001265823
791	625681	494913671	28,124722	9,248234	,001264223
792	627264	496793088	28,142494	9,252130	,001262626
793	628849	498677257	28,160255	9,256022	,001261034
794	630436	500566184	28,178005	9,259911	,001259446
795	632025	502459875	28,195744	9,263797	,001257862
796	633616	504358336	28,213472	9,267679	,001256281
797	635209	506261573	28,231188	9,271559	,001254705
798	636804	508169592	28,248893	9,275435	,001253133
799	638401	510082399	28,266588	9,279308	,001251564

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
800	640000	512000000	28.28427	9.28377	.001250000
801	641601	513922401	28.30583	9.29044	.001248439
802	643204	515849608	28.32738	9.29709	.001246883
803	644809	517781627	28.34892	9.30375	.001245330
804	646416	519718464	28.37046	9.31041	.001243781
805	648025	521660125	28.39199	9.31707	.001242236
806	649636	523606616	28.41351	9.32372	.001240695
807	651249	525557943	28.43503	9.33037	.001239157
808	652864	527514112	28.45654	9.33703	.001237624
809	654481	529475129	28.47805	9.34368	.001236094
810	656100	531441000	28.49956	9.35034	.001234568
811	657721	533411731	28.52106	9.35699	.001233046
812	659344	535387328	28.54255	9.36364	.001231527
813	660969	537367797	28.56404	9.37029	.001230012
814	662596	539353144	28.58553	9.37694	.001228501
815	664225	541343375	28.60701	9.38359	.001226994
816	665856	543338496	28.62849	9.39024	.001225490
817	667489	545338513	28.64996	9.39688	.001223990
818	669124	547343432	28.67143	9.40353	.001222494
819	670761	549353259	28.69290	9.41017	.001221001
820	672400	551368000	28.71436	9.41681	.001219512
821	674041	553387661	28.73582	9.42346	.001218027
822	675684	555412248	28.75728	9.43010	.001216545
823	677329	557441767	28.77873	9.43674	.001215067
824	678976	559476224	28.80019	9.44338	.001213592
825	680625	561515625	28.82163	9.45002	.001212121
826	682276	563559976	28.84308	9.45666	.001210654
827	683929	565609283	28.86452	9.46330	.001209190
828	685584	567663552	28.88596	9.46994	.001207729
829	687241	569722789	28.90740	9.47658	.001206273
830	688900	571787000	28.92883	9.48321	.001204819
831	690561	573856191	28.95026	9.48985	.001203369
832	692224	575930368	28.97168	9.49648	.001201923
833	693889	578009537	28.99310	9.50312	.001200480
834	695556	580093704	29.01451	9.50975	.001199041
835	697225	582182875	29.03592	9.51639	.001197605
836	698896	584277056	29.05733	9.52302	.001196172
837	700569	586376253	29.07873	9.52965	.001194743
838	702244	588480472	29.10013	9.53628	.001193317
839	703921	590589719	29.12152	9.54291	.001191895
840	705600	592704000	29.14291	9.54954	.001190476
841	707281	594823321	29.16429	9.55617	.001189061
842	708964	596947688	29.18567	9.56280	.001187648
843	710649	599077107	29.20705	9.56942	.001186240
844	712336	601211584	29.22842	9.57604	.001184834
845	714025	603351125	29.24979	9.58266	.001183432
846	715716	605495736	29.27115	9.58928	.001182033
847	717409	607645423	29.29252	9.59589	.001180638
848	719104	609800192	29.31387	9.60251	.001179245
849	720801	611960049	29.33523	9.60912	.001177856
850	722500	614125000	29.35658	9.61573	.001176471
851	724201	616295051	29.37793	9.62234	.001175088
852	725904	618470208	29.39928	9.62894	.001173709
853	727609	620650477	29.42062	9.63554	.001172333
854	729316	622835864	29.44195	9.64215	.001170960
855	731025	625026375	29.46329	9.64875	.001169591
856	732736	627222016	29.48462	9.65535	.001168224
857	734449	629422793	29.50594	9.66195	.001166861
858	736164	631628712	29.52727	9.66854	.001165501
859	737881	633839779	29.54859	9.67514	.001164144
860	739600	636056000	29.56991	9.68173	.001162791
861	741321	638277381	29.59122	9.68832	.001161440
862	743044	640503928	29.61253	9.69491	.001160093
863	744769	642735647	29.63384	9.70150	.001158749
864	746496	644972544	29.65515	9.70808	.001157407
865	748225	647214625	29.67644	9.71467	.001156069
866	749956	649461896	29.69774	9.72125	.001154734

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
867	751689	651714363	29,444864	9,535417	,001153403
868	753424	653972032	29,461840	9,539082	,001152074
869	755161	656234909	29,478806	9,542744	,001150748
870	756900	658503000	29,495762	9,546403	,001149425
871	758641	660776311	29,512709	9,550059	,001148106
872	760384	663054848	29,529646	9,553712	,001146789
873	762129	665338617	29,546573	9,557363	,001145475
874	763876	667627624	29,563491	9,561011	,001144165
875	765625	669921875	29,580399	9,564656	,001142857
876	767376	672221376	29,597297	9,568298	,001141553
877	769129	674526133	29,614186	9,571938	,001140251
878	770884	676836152	29,631065	9,575574	,001138952
879	772641	679151439	29,647934	9,579208	,001137656
880	774400	681472000	29,664794	9,582840	,001136364
881	776161	683797841	29,681644	9,586468	,001135074
882	777924	686128968	29,698485	9,590094	,001133787
883	779689	688465387	29,715316	9,593717	,001132503
884	781456	690807104	29,732137	9,597337	,001131222
885	783225	693154125	29,748950	9,600955	,001129944
886	784996	695506456	29,765752	9,604570	,001128668
887	786769	697864103	29,782545	9,608182	,001127396
888	788544	700227072	29,799329	9,611791	,001126126
889	790321	702595369	29,816103	9,615398	,001124859
890	792100	704969000	29,832868	9,619002	,001123596
891	793881	707347971	29,849623	9,622603	,001122334
892	795664	709732288	29,866369	9,626202	,001121076
893	797449	712121957	29,883106	9,629797	,001119821
894	799236	714516984	29,899833	9,633391	,001118568
895	801025	716917375	29,916551	9,636981	,001117318
896	802816	719323136	29,933259	9,640569	,001116071
897	804609	721734273	29,949958	9,644154	,001114782
898	806404	724150792	29,966648	9,647736	,001113586
899	808201	726572699	29,983328	9,651316	,001112347
900	810000	729000000	30,	9,654893	,001111111
901	811804	731432701	30,016662	9,658468	,001109878
902	813604	733870808	30,033314	9,662040	,001108647
903	815409	736314327	30,049958	9,665609	,001107420
904	817216	738763264	30,066592	9,669176	,001106195
905	819025	741217625	30,083217	9,672740	,001104972
906	820836	743677416	30,099833	9,676301	,001103753
907	822649	746142643	30,116440	9,679860	,001102536
908	824464	748613312	30,133038	9,683416	,001101322
909	826281	751089429	30,149626	9,686970	,001100110
910	828100	753571000	30,166206	9,690521	,001098901
911	829921	756058031	30,182776	9,694069	,001097695
912	831744	758550528	30,199337	9,697615	,001096491
913	833569	761048497	30,215889	9,701158	,001095290
914	835396	763551944	30,232432	9,704698	,001094092
915	837225	766060875	30,248966	9,708236	,001092896
916	839056	768575296	30,265491	9,711772	,001091703
917	840889	771095213	30,282007	9,715305	,001090513
918	842724	773620632	30,298514	9,718835	,001089325
919	844561	776151559	30,315012	9,722363	,001088139
920	846400	778688000	30,331501	9,725888	,001086957
921	848241	781229961	30,347981	9,729410	,001085776
922	850084	783777448	30,364452	9,732930	,001084599
923	851929	786330467	30,380915	9,736448	,001083423
924	853776	788889024	30,397368	9,739963	,001082251
925	855625	791453125	30,413812	9,743475	,001081081
926	857476	794022776	30,430248	9,746985	,001079914
927	859329	796597983	30,446674	9,750493	,001078749
928	861184	799178752	30,463092	9,753998	,001077586
929	863041	801765089	30,479501	9,757500	,001076426
930	864900	804357000	30,495901	9,761000	,001075269
931	866761	806954491	30,512292	9,764497	,001074114
932	868624	809557568	30,528675	9,767992	,001072961
933	870489	812166237	30,545048	9,771484	,001071811

No.	Square.	Cube.	Sq. Root.	Cube Root.	Reciprocal.
934	872356	814780504	30,561413	9.774974	,001070664
935	874225	817400375	30,577769	9.778461	,001068519
936	876096	820025856	30,594117	9.782946	,001068376
937	877969	822656053	30,610455	9.785428	,001067236
938	879844	825293672	30,626785	9.788908	,001066096
939	881721	827936019	30,643106	9.792386	,001064963
940	883600	830584000	30,659419	9.795861	,001063830
941	885481	833237621	30,675723	9.799333	,001062699
942	887364	835896888	30,692018	9.802803	,001061571
943	889249	838561807	30,708305	9.806271	,001060445
944	891136	841232384	30,724583	9.809736	,001059322
945	893025	843908625	30,740852	9.813198	,001058201
946	894916	846590536	30,757113	9.816659	,001057082
947	896809	849278123	30,773365	9.820117	,001055966
948	898704	851971392	30,789608	9.823572	,001054852
949	900601	854670349	30,805843	9.827025	,001053741
950	902500	857375000	30,822070	9.830475	,001052632
951	904401	860085351	30,838288	9.833924	,001051525
952	906304	862801408	30,854497	9.837369	,001050420
953	908209	865523177	30,870698	9.840813	,001049318
954	910116	868250664	30,886890	9.844254	,001048218
955	912025	870983875	30,903074	9.847692	,001047120
956	913936	873722816	30,919248	9.851128	,001046025
957	915849	876467493	30,935417	9.854562	,001044932
958	917764	879217912	30,951575	9.857993	,001043841
959	919681	881974079	30,967725	9.861422	,001042753
960	921600	884736000	30,983867	9.864848	,001041667
961	923521	887503681	31,	9.868272	,001040583
962	925444	890277128	31,016125	9.871694	,001039501
963	927369	893056347	31,032241	9.875113	,001038422
964	929296	895841344	31,048349	9.878530	,001037344
965	931225	898632125	31,064449	9.881945	,001036269
966	933156	901428696	31,080540	9.885357	,001035197
967	935089	904231063	31,096624	9.888767	,001034126
968	937024	907039232	31,112698	9.892175	,001033058
969	938961	909853209	31,128765	9.895580	,001031992
970	940900	912673000	31,144823	9.898983	,001030928
971	942841	915498611	31,160873	9.902383	,001029866
972	944784	918330048	31,176914	9.905782	,001028807
973	946729	921167317	31,192948	9.909178	,001027749
974	948676	924010424	31,208973	9.912571	,001026694
975	950625	926859375	31,224990	9.915962	,001025641
976	952576	929714176	31,240999	9.919351	,001024590
977	954529	932574833	31,256999	9.922738	,001023541
978	956484	935441352	31,272991	9.926122	,001022495
979	958441	938313739	31,288976	9.929504	,001021450
980	960400	941192000	31,304952	9.932884	,001020408
981	962361	944076141	31,320919	9.936261	,001019368
982	964324	946966168	31,336879	9.939636	,001018330
983	966289	949862087	31,352831	9.943009	,001017294
984	968256	952763904	31,368774	9.946380	,001016260
985	970225	955671625	31,384710	9.949748	,001015228
986	972196	958585256	31,400637	9.953114	,001014199
987	974169	961504803	31,416556	9.956477	,001013171
988	976144	964430272	31,432467	9.959839	,001012146
989	978121	967361669	31,448370	9.963198	,001011122
990	980100	970299000	31,464265	9.966555	,001010101
991	982081	973242271	31,480152	9.969909	,001009082
992	984064	976191488	31,496031	9.973262	,001008065
993	986049	979146657	31,511903	9.976612	,001007049
994	988036	982107784	31,527765	9.979960	,001006036
995	990025	985074875	31,543621	9.983305	,001005025
996	992016	988047936	31,559468	9.986649	,001004016
997	994009	991026973	31,575307	9.989990	,001003009
998	996004	994011992	31,591138	9.993329	,001002004
999	998001	997002999	31,606961	9.996666	,001001001
1000	1000000	1000000000	31,622777	10,	,001000000

TABLE LV.

MULTIPLICATION TABLE IN INCHES FOR RECTANGULAR SECTIONS.

Thickness in inches.	1″	1¼″	1½″	1¾″	2″	2¼″	2½″	2¾″	3″	3¼″	3½″	3¾″	4″	4¼″	4½″	4¾″	12″
1/16	,063	,078	,094	,109	,125	,141	,156	,172	,188	,203	,219	,234	,250	,266	,281	,297	,750
1/8	,125	,156	,188	,219	,250	,281	,313	,344	,375	,406	,438	,469	,500	,531	,563	,594	1,50
3/16	,188	,234	,281	,328	,375	,422	,469	,516	,563	,609	,656	,703	,750	,797	,844	,891	2,25
1/4	,250	,313	,375	,438	,500	,563	,625	,688	,750	,813	,875	,938	1,00	1,06	1,13	1,19	3,00
5/16	,313	,391	,469	,547	,625	,703	,781	,859	,938	1,02	1,09	1,17	1,25	1,33	1,41	1,48	3,75
3/8	,375	,469	,563	,656	,750	,844	,938	1,03	1,13	1,22	1,31	1,41	1,50	1,59	1,69	1,78	4,50
7/16	,438	,547	,656	,766	,875	,984	1,09	1,20	1,31	1,42	1,53	1,64	1,75	1,86	1,97	2,08	5,25
1/2	,500	,625	,750	,875	1,00	1,13	1,25	1,38	1,50	1,63	1,75	1,88	2,00	2,13	2,25	2,38	6,00
9/16	,563	,703	,844	,984	1,13	1,27	1,41	1,55	1,69	1,83	1,97	2,11	2,25	2,39	2,53	2,67	6,75
5/8	,625	,781	,938	1,09	1,25	1,41	1,56	1,72	1,88	2,03	2,19	2,34	2,50	2,66	2,81	2,97	7,50
11/16	,688	,859	1,03	1,20	1,38	1,55	1,72	1,89	2,06	2,23	2,41	2,58	2,75	2,92	3,09	3,27	8,25
3/4	,750	,938	1,13	1,31	1,50	1,69	1,88	2,06	2,25	2,44	2,63	2,81	3,00	3,19	3,38	3,56	9,00
13/16	,813	1,02	1,22	1,42	1,63	1,83	2,03	2,23	2,44	2,64	2,84	3,05	3,25	3,45	3,66	3,86	9,75
7/8	,875	1,09	1,31	1,53	1,75	1,97	2,19	2,41	2,63	2,84	3,06	3,28	3,50	3,72	3,94	4,16	10,50
15/16	,938	1,17	1,41	1,64	1,88	2,11	2,34	2,58	2,81	3,05	3,28	3,52	3,75	3,98	4,22	4,45	11,25
1	1,00	1,25	1,50	1,75	2,00	2,25	2,50	2,75	3,00	3,25	3,50	3,75	4,00	4,25	4,50	4,75	12,00
1 1/16	1,06	1,33	1,59	1,86	2,13	2,39	2,66	2,92	3,19	3,45	3,72	3,98	4,25	4,52	4,78	5,05	12,75
1 1/8	1,13	1,41	1,69	1,97	2,25	2,53	2,81	3,09	3,38	3,66	3,94	4,22	4,50	4,78	5,06	5,34	13,50
1 3/16	1,19	1,48	1,78	2,08	2,38	2,67	2,97	3,27	3,56	3,86	4,16	4,45	4,75	5,05	5,34	5,64	14,25
1 1/4	1,25	1,56	1,88	2,19	2,50	2,81	3,13	3,44	3,75	4,06	4,38	4,69	5,00	5,31	5,63	5,94	15,00
1 5/16	1,31	1,64	1,97	2,30	2,63	2,95	3,28	3,61	3,94	4,27	4,59	4,92	5,25	5,58	5,91	6,23	15,75
1 3/8	1,38	1,72	2,06	2,41	2,75	3,09	3,44	3,78	4,13	4,47	4,81	5,16	5,50	5,84	6,19	6,53	16,50
1 7/16	1,44	1,80	2,16	2,52	2,88	3,23	3,59	3,95	4,31	4,67	5,03	5,39	5,75	6,11	6,47	6,83	17,25
1 1/2	1,50	1,88	2,25	2,63	3,00	3,38	3,75	4,13	4,50	4,88	5,25	5,63	6,00	6,38	6,75	7,13	18,00
1 9/16	1,56	1,95	2,34	2,73	3,13	3,52	3,91	4,30	4,69	5,08	5,47	5,86	6,25	6,64	7,03	7,42	18,75
1 5/8	1,63	2,03	2,44	2,84	3,25	3,66	4,06	4,47	4,88	5,28	5,69	6,09	6,50	6,91	7,31	7,72	19,50
1 11/16	1,69	2,11	2,53	2,95	3,38	3,80	4,22	4,64	5,06	5,48	5,91	6,33	6,75	7,17	7,59	8,02	20,25
1 3/4	1,75	2,19	2,63	3,06	3,50	3,94	4,38	4,81	5,25	5,69	6,13	6,56	7,00	7,44	7,88	8,31	21,00
1 13/16	1,81	2,27	2,72	3,17	3,63	4,08	4,53	4,98	5,44	5,89	6,34	6,80	7,25	7,70	8,16	8,61	21,75
1 7/8	1,88	2,34	2,81	3,28	3,75	4,22	4,69	5,16	5,63	6,09	6,56	7,03	7,50	7,97	8,44	8,91	22,50
1 15/16	1,94	2,42	2,91	3,39	3,88	4,36	4,84	5,33	5,81	6,30	6,78	7,27	7,75	8,23	8,72	9,20	23,25
2	2,00	2,50	3,00	3,50	4,00	4,50	5,00	5,50	6,00	6,50	7,00	7,50	8,00	8,50	9,00	9,50	24,00

TABLE LV. (CONTINUED.)

Thickness in inches.	5"	5¼"	5½"	5¾"	6"	6¼"	6½"	6¾"	7"	7¼"	7½"	7¾"	8"	8¼"	8½"	8¾"	9"
1/16	,313	,328	,344	,359	,375	,391	,406	,422	,438	,453	,469	,484	,500	,516	,531	,547	,563
1/8	,625	,656	,688	,719	,750	,781	,813	,844	,875	,906	,938	,969	1,00	1,03	1,06	1,09	1,13
3/16	,938	,984	1,03	1,08	1,13	1,17	1,22	1,27	1,31	1,36	1,41	1,45	1,50	1,55	1,59	1,64	1,69
¼	1,25	1,31	1,38	1,44	1,50	1,56	1,63	1,69	1,75	1,81	1,88	1,94	2,00	2,06	2,13	2,19	2,25
5/16	1,56	1,64	1,72	1,80	1,88	1,95	2,03	2,11	2,19	2,27	2,34	2,42	2,50	2,58	2,66	2,73	2,81
3/8	1,88	1,97	2,06	2,16	2,25	2,34	2,44	2,53	2,63	2,72	2,81	2,91	3,00	3,09	3,19	3,28	3,38
7/16	2,19	2,30	2,41	2,52	2,63	2,73	2,84	2,95	3,06	3,17	3,28	3,39	3,50	3,61	3,72	3,84	3,94
½	2,50	2,63	2,75	2,88	3,00	3,13	3,25	3,38	3,50	3,64	3,75	3,88	4,00	4,13	4,25	4,38	4,50
9/16	2,81	2,95	3,09	3,23	3,38	3,52	3,66	3,80	3,94	4,08	4,22	4,36	4,50	4,64	4,78	4,92	5,06
5/8	3,13	3,28	3,44	3,59	3,75	3,91	4,06	4,22	4,38	4,53	4,69	4,84	5,00	5,16	5,31	5,47	5,63
11/16	3,44	3,61	3,78	3,95	4,13	4,30	4,47	4,64	4,81	4,98	5,16	5,33	5,50	5,67	5,84	6,02	6,19
¾	3,75	3,94	4,13	4,31	4,50	4,69	4,88	5,06	5,25	5,44	5,63	5,81	6,00	6,19	6,38	6,56	6,75
13/16	4,06	4,27	4,47	4,67	4,88	5,08	5,28	5,48	5,69	5,89	6,09	6,30	6,50	6,70	6,91	7,11	7,31
7/8	4,38	4,59	4,81	5,03	5,25	5,47	5,69	5,91	6,13	6,34	6,56	6,78	7,00	7,22	7,44	7,66	7,88
15/16	4,69	4,92	5,16	5,39	5,63	5,86	6,09	6,33	6,56	6,80	7,03	7,27	7,50	7,73	7,97	8,20	8,44
1	5,00	5,25	5,50	5,75	6,00	6,25	6,50	6,75	7,00	7,25	7,50	7,75	8,00	8,25	8,50	8,75	9,00
1 1/16	5,31	5,58	5,84	6,11	6,38	6,64	6,91	7,17	7,44	7,70	7,97	8,23	8,50	8,77	9,04	9,30	9,56
1 1/8	5,63	5,91	6,19	6,47	6,75	7,03	7,31	7,59	7,88	8,16	8,44	8,72	9,00	9,28	9,56	9,84	10,12
1 3/16	5,94	6,23	6,53	6,83	7,13	7,42	7,72	8,02	8,31	8,61	8,91	9,20	9,50	9,80	10,09	10,39	10,69
1 ¼	6,25	6,56	6,88	7,19	7,50	7,81	8,13	8,44	8,75	9,06	9,38	9,69	10,00	10,31	10,63	10,94	11,25
1 5/16	6,56	6,89	7,22	7,55	7,88	8,20	8,53	8,86	9,19	9,52	9,84	10,17	10,50	10,83	11,16	11,48	11,81
1 3/8	6,88	7,22	7,56	7,91	8,25	8,59	8,94	9,28	9,63	9,97	10,31	10,66	11,00	11,34	11,69	12,03	12,38
1 7/16	7,19	7,55	7,91	8,27	8,63	8,98	9,34	9,70	10,06	10,42	10,78	11,14	11,50	11,86	12,23	12,58	12,94
1 ½	7,50	7,88	8,25	8,63	9,00	9,38	9,75	10,13	10,50	10,88	11,25	11,63	12,00	12,38	12,75	13,13	13,50
1 9/16	7,81	8,20	8,59	8,98	9,38	9,77	10,16	10,55	10,94	11,33	11,72	12,11	12,50	12,89	13,28	13,67	14,06
1 5/8	8,13	8,53	8,94	9,34	9,75	10,16	10,56	10,97	11,38	11,78	12,19	12,59	13,00	13,41	13,81	14,22	14,63
1 11/16	8,44	8,86	9,28	9,70	10,13	10,55	10,97	11,39	11,81	12,23	12,66	13,08	13,50	13,92	14,34	14,77	15,19
1 ¾	8,75	9,19	9,63	10,06	10,50	10,94	11,38	11,81	12,25	12,69	13,13	13,56	14,00	14,44	14,88	15,31	15,75
1 13/16	9,06	9,52	9,97	10,42	10,88	11,33	11,78	12,23	12,69	13,14	13,59	14,05	14,50	14,95	15,41	15,86	16,31
1 7/8	9,38	9,84	10,31	10,78	11,27	11,72	12,19	12,66	13,13	13,59	14,06	14,53	15,00	15,47	15,94	16,41	16,88
1 15/16	9,69	10,17	10,66	11,14	11,63	12,11	12,59	13,08	13,56	14,05	14,53	15,02	15,50	15,98	16,47	16,95	17,44
2	10,00	10,50	11,00	11,50	12,00	12,50	13,00	13,50	14,00	14,50	15,00	15,50	16,00	16,50	17,00	17,50	18,00

TABLE LV. (CONTINUED.)

Thickness in inches.	9″	9¼″	9½″	9¾″	10″	10¼″	10½″	10¾″	11″	11¼″	11½″	11¾″	12″	12¼″	12½″	12¾″
1/16	,563	,578	,594	,609	,625	,641	,656	,672	,688	,703	,719	,734	,750	,766	,781	,797
1/8	1,13	1,16	1,19	1,22	1,25	1,28	1,31	1,34	1,38	1,41	1,44	1,47	1,50	1,53	1,56	1,59
3/16	1,69	1,73	1,78	1,83	1,88	1,92	1,97	2,02	2,06	2,11	2,16	2,20	2,25	2,30	2,34	2,39
1/4	2,25	2,31	2,38	2,44	2,50	2,56	2,63	2,69	2,75	2,81	2,88	2,94	3,00	3,06	3,13	3,19
5/16	2.81	2.89	2.97	3,05	3,13	3,20	3,28	3,36	3,44	3,52	3,59	3,67	3,75	3,83	3,91	3,98
3/8	3.38	3.47	3,56	3,66	3,75	3,84	3,94	4,03	4,13	4,22	4,31	4,41	4,50	4,59	4,69	4,78
7/16	3,94	4.05	4,16	4,27	4,38	4,48	4,59	4,70	4,81	4,92	5,03	5,14	5,25	5,36	5,47	5,58
1/2	4,50	4,63	4,75	4,88	5,00	5,13	5,25	5,38	5,50	5,63	5,75	5,88	6,00	6,13	6,25	6,38
9/16	5,06	5,20	5,34	5,48	5,63	5,77	5,91	6,05	6,19	6,33	6,47	6,61	6,75	6,89	7,03	7,17
5/8	5,63	5,78	5,94	6,09	6,25	6,41	6,56	6,72	6,88	7,03	7,19	7,34	7,50	7,66	7,81	7,97
11/16	6,19	6,36	6,53	6,70	6,88	7,05	7,22	7,39	7,56	7,73	7,91	8,08	8,25	8,42	8,59	8,77
3/4	6,75	6,94	7,13	7,31	7,50	7,69	7,88	8,06	8,25	8,44	8,63	8,81	9,00	9,19	9,38	9,56
13/16	7,31	7,52	7,72	7,92	8,13	8,33	8,53	8,73	8,94	9,14	9,34	9,55	9,75	9,95	10,16	10,36
7/8	7,88	8,09	8,31	8,53	8,75	8,97	9,19	9,41	9,63	9,84	10,06	10,28	10,50	10,72	10,94	11,16
15/16	8,44	8,67	8,91	9,14	9,38	9,61	9,84	10,08	10,31	10,55	10,78	11,02	11,25	11,48	11,72	11,95
1	9,00	9,25	9,50	9,75	10,00	10,25	10,50	10,75	11,00	11,25	11,50	11,75	12,00	12,25	12,50	12,75
1 1/16	9,56	9,83	10,09	10,36	10,63	10,89	11,16	11,42	11,69	11,95	12,22	12,48	12,75	13,02	13,28	13,55
1 1/8	10,13	10,41	10,69	10,97	11,25	11,53	11,81	12,09	12,38	12,66	12,94	13,22	13,50	13,78	14,06	14,34
1 3/16	10,69	10,98	11,28	11,58	11,88	12,17	12,47	12,77	13,06	13,36	13,66	13,95	14,25	14,55	14,84	15,14
1 1/4	11,25	11,56	11,88	12,19	12,50	12,81	13,13	13,44	13,75	14,06	14,38	14,69	15,00	15,31	15,63	15,94
1 5/16	11,81	12,14	12,47	12,80	13,13	13,45	13,78	14,11	14,44	14,77	15,09	15,42	15,75	16,08	16,41	16,73
1 3/8	12,38	12,72	13,06	13,41	13,75	14,09	14,44	14,78	15,13	15,47	15,81	16,16	16,50	16,84	17,19	17,53
1 7/16	12,94	13,30	13,66	14,02	14,38	14,73	15,09	15,45	15,81	16,17	16,53	16,89	17,25	17,61	17,97	18,33
1 1/2	13,50	13,88	14,25	14,63	15,00	15,38	15,75	16,13	16,50	16,88	17,25	17,63	18,00	18,38	18,75	19,13
1 9/16	14,06	14,45	14,84	15,23	15,63	16,02	16,41	16,80	17,19	17,58	17,97	18,36	18,75	19,14	19,53	19,92
1 5/8	14,63	15,03	15,44	15,84	16,25	16,66	17,06	17,47	17,88	18,28	18,69	19,09	19,50	19,91	20,31	20,72
1 11/16	15,19	15,61	16,03	16,45	16,88	17,30	17,72	18,14	18,56	18,98	19,41	19,83	20,25	20,67	21,09	21,52
1 3/4	15,75	16,19	16,63	17,06	17,50	17,94	18,38	18,81	19,25	19,69	20,13	20,56	21,00	21,44	21,88	22,31
1 13/16	16,31	16,77	17,22	17,67	18,13	18,58	19,03	19,48	19,94	20,39	20,84	21,30	21,75	22,20	22,66	23,11
1 7/8	16,88	17,34	17,81	18,28	18,75	19,22	19,69	20,16	20,63	21,09	21,56	22,03	22,50	22,97	23,44	23,91
1 15/16	17,44	17,92	18,41	18,89	19,38	19,86	20,34	20,83	21,31	21,80	22,28	22,77	23,25	23,73	24,22	24,70
2	18,00	18,50	19,00	19,50	20,00	20,50	21,00	21,50	22,00	22,50	23,00	23,50	24,00	24,50	25,00	25,50

TABLE LVI.

CIRCUMFERENCES AND AREAS OF CIRCLES FROM 0 TO 100, ALSO SQUARES UP TO 12.

Diameter or side.	Circumference.	Area.	Square.	Diameter or side.	Circumference.	Area.	Square.
0				2	6,2832	3,1416	4,0000
1/16	,1963	,0031	,0039	1/16	6,4795	3,3410	4,2539
1/8	,3927	,0123	,0156	1/8	6,6759	3,5466	4,5156
3/16	,5890	,0276	,0352	3/16	6,8722	3,7583	4,7852
1/4	,7854	,0491	,0625	1/4	7,0686	3,9761	5,0625
5/16	,9817	,0767	,0977	5/16	7,2649	4,2000	5,3477
3/8	1,1781	,1104	,1406	3/8	7,4613	4,4301	5,6406
7/16	1,3744	,1503	,1914	7/16	7,6576	4,6664	5,9414
1/2	1,5708	,1963	,2500	1/2	7,8540	4,9087	6,2500
9/16	1,7671	,2485	,3164	9/16	8,0503	5,1572	6,5664
5/8	1,9635	,3068	,3906	5/8	8,2467	5,4119	6,8906
11/16	2,1598	,3712	,4727	11/16	8,4430	5,6727	7,2227
3/4	2,3562	,4418	,5625	3/4	8,6394	5,9396	7,5625
13/16	2,5525	,5185	,6602	13/16	8,8357	6,2126	7,9102
7/8	2,7489	,6013	,7656	7/8	9,0321	6,4918	8,2656
15/16	2,9452	,6903	,8789	15/16	9,2284	6,7771	8,6289
1	3,1416	,7854	1,0000	3	9,4248	7,0686	9,0000
1/16	3,3379	,8866	1,1289	1/16	9,6211	7,3662	9,3789
1/8	3,5343	,9940	1,2656	1/8	9,8175	7,6699	9,7656
3/16	3,7306	1,1075	1,4102	3/16	10,014	7,9798	10,160
1/4	3,9270	1,2272	1,5625	1/4	10,210	8,2958	10,563
5/16	4,1233	1,3530	1,7227	5/16	10,407	8,6179	10,973
3/8	4,3197	1,4849	1,8906	3/8	10,603	8,9462	11,391
7/16	4,5160	1,6230	2,0664	7/16	10,799	9,2806	11,816
1/2	4,7124	1,7671	2,2500	1/2	10,996	9,6211	12,256
9/16	4,9087	1,9175	2,4414	9/16	11,192	9,9678	12,691
5/8	5,1051	2,0739	2,6406	5/8	11,388	10,321	13,141
11/16	5,3014	2,2365	2,8477	11/16	11,585	10,680	13,598
3/4	5,4978	2,4053	3,0625	3/4	11,781	11,045	14,063
13/16	5,6941	2,5802	3,2852	13/16	11,977	11,416	14,535
7/8	5,8905	2,7612	3,5156	7/8	12,174	11,793	15,016
15/16	6,0868	2,9483	3,7539	15/16	12,370	12,177	15,504

TABLE LVI. — CIRCUMFERENCES AND AREAS OF CIRCLES FROM 0 TO 100, ALSO SQUARES UP TO 12, (CONTINUED).

Diameter or side.	Circumference.	Area.	Square.	Diameter or side.	Circumference.	Area.	Square.
4	12,566	12,566	16,000	6	18,850	28,274	36,000
1/16	12,763	12,962	16,504	1/16	19,046	28,866	36,754
1/8	12,959	13,364	17,016	1/8	19,242	29,465	37,516
3/16	13,155	13,772	17,535	3/16	19,439	30,069	38,285
1/4	13,352	14,186	18,063	1/4	19,635	30,680	39,063
5/16	13,548	14,607	18,598	5/16	19,831	31,296	39,848
3/8	13,744	15,033	19,141	3/8	20,028	31,919	40,641
7/16	13,941	15,466	19,691	7/16	20,224	32,548	41,441
1/2	14,137	15,904	20,250	1/2	20,420	33,183	42,250
9/16	14,334	16,349	20,816	9/16	20,617	33,824	43,066
5/8	14,530	16,800	21,391	5/8	20,813	34,472	43,891
11/16	14,726	17,257	21,973	11/16	21,009	35,125	44,723
3/4	14,923	17,721	22,563	3/4	21,206	35,785	45,563
13/16	15,119	18,190	23,160	13/16	21,402	36,450	46,410
7/8	15,315	18,665	23,766	7/8	21,598	37,122	47,266
15/16	15,512	19,147	24,379	15/16	21,795	37,800	48,129
5	15,708	19,635	25,000	7	21,991	38,485	49,000
1/16	15,904	20,129	25,629	1/16	22,187	39,175	49,879
1/8	16,101	20,629	26,266	1/8	22,384	39,871	50,766
3/16	16,297	21,135	26,910	3/16	22,580	40,574	51,660
1/4	16,493	21,648	27,563	1/4	22,777	41,282	52,563
5/16	16,690	22,166	28,223	5/16	22,973	41,997	53,473
3/8	16,886	22,691	28,891	3/8	23,169	42,718	54,391
7/16	17,082	23,221	29,566	7/16	23,366	43,445	55,316
1/2	17,279	23,758	30,250	1/2	23,562	44,179	56,250
9/16	17,475	24,301	30,941	9/16	23,758	44,918	57,191
5/8	17,671	24,850	31,641	5/8	23,955	45,664	58,141
11/16	17,868	25,406	32,348	11/16	24,151	46,415	59,098
3/4	18,064	25,967	33,063	3/4	24,347	47,173	60,063
13/16	18,261	26,535	33,785	13/16	24,544	47,937	61,035
7/8	18,457	27,109	34,516	7/8	24,740	48,707	62,016
15/16	18,653	27,688	35,254	15/16	24,936	49,483	63,004

TABLE LVI. — CIRCUMFERENCES AND AREAS OF CIRCLES FROM 0 TO 100, ALSO SQUARES UP TO 12, (CONTINUED).

Diameter or side.	Circumference.	Area.	Square.	Diameter or side.	Circumference.	Area.	Square.
8	25,133	50,265	64,000	10	31,416	78,540	100,00
1/16	25,329	51,054	65,004	1/16	31,612	79,525	101,25
1/8	25,525	51,849	66,016	1/8	31,809	80,516	102,52
3/16	25,722	52,649	67,035	3/16	32,005	81,513	103,79
1/4	25,918	53,456	68,063	1/4	32,201	82,516	105,06
5/16	26,114	54,269	69,098	5/16	32,398	83,525	106,35
3/8	26,311	55,088	70,141	3/8	32,594	84,541	107,64
7/16	26,507	55,914	71,191	7/16	32,790	85,562	108,94
1/2	26,704	56,745	72,250	1/2	32,987	86,590	110,25
9/16	26,900	57,583	73,316	9/16	33,183	87,624	111,57
5/8	27,096	58,426	74,391	5/8	33,379	88,664	112,89
11/16	27,293	59,276	75,473	11/16	33,576	89,710	114,22
3/4	27,489	60,132	76,563	3/4	33,772	90,763	115,56
13/16	27,685	60,994	77,660	13/16	33,968	91,821	116,91
7/8	27,882	61,862	78,766	7/8	34,165	92,886	118,27
15/16	28,078	62,737	79,879	15/16	34,361	93,956	119,63
9	28,274	63,617	81,000	11	34,558	95,033	121,00
1/16	28,471	64,504	82,129	1/16	34,754	96,116	122,38
1/8	28,667	65,397	83,266	1/8	34,950	97,205	123,77
3/16	28,863	66,296	84,410	3/16	35,147	98,301	125,16
1/4	29,060	67,201	85,563	1/4	35,343	99,402	126,56
5/16	29,256	68,112	86,723	5/16	35,539	100,51	127,97
3/8	29,452	69,029	87,891	3/8	35,736	101,62	129,39
7/16	29,649	69,953	89,066	7/16	35,932	102,74	130,82
1/2	29,845	70,882	90,250	1/2	36,128	103,87	132,25
9/16	30,041	71,818	91,441	9/16	36,325	105,00	133,69
5/8	30,238	72,760	92,641	5/8	36,521	106,14	135,14
11/16	30,434	73,708	93,848	11/16	36,717	107,28	136,60
3/4	30,631	74,662	95,063	3/4	36,914	108,43	138,06
13/16	30,827	75,622	96,285	13/16	37,110	109,59	139,54
7/8	31,023	76,589	97,516	7/8	37,306	110,75	141,02
15/16	31,220	77,561	98,754	15/16	37,503	111,92	142,50

TABLE LVI. — CIRCUMFERENCES AND AREAS OF CIRCLES FROM 0 TO 100, (CONTINUED).

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
12	37,6991	113,10	16	50,2655	201,06	20	62,8319	314,16
1/8	38,0918	115,47	1/8	50,6582	204,22	1/8	63,2246	318,10
1/4	38,4845	117,86	1/4	51,0509	207,39	1/4	63,6173	322,06
3/8	38,8772	120,28	3/8	51,4436	210,60	3/8	64,0100	326,05
1/2	39,2699	122,72	1/2	51,8363	213,82	1/2	64,4026	330,06
5/8	39,6626	125,19	5/8	52,2290	217,08	5/8	64,7953	334,10
3/4	40,0553	127,68	3/4	52,6217	220,35	3/4	65,1880	338,16
7/8	40,4480	130,19	7/8	53,0144	223,65	7/8	65,5807	342,25
13	40,8407	132,73	17	53,4071	226,98	21	65,9734	346,36
1/8	41,2334	135,30	1/8	53,7998	230,33	1/8	66,3661	350,50
1/4	41,6261	137,89	1/4	54,1925	233,71	1/4	66,7588	354,66
3/8	42,0188	140,50	3/8	54,5852	237,10	3/8	67,1515	358,84
1/2	42,4115	143,14	1/2	54,9779	240,53	1/2	67,5442	363,05
5/8	42,8042	145,80	5/8	55,3706	243,98	5/8	67,9369	367,28
3/4	43,1969	148,49	3/4	55,7633	247,45	3/4	68,3296	371,54
7/8	43,5896	151,20	7/8	56,1560	250,95	7/8	68,7223	375,83
14	43,9823	153,94	18	56,5487	254,47	22	69,1150	380,13
1/8	44,3750	156,70	1/8	56,9414	258,02	1/8	69,5077	384,46
1/4	44,7677	159,48	1/4	57,3341	261,59	1/4	69,9004	388,82
3/8	45,1604	162,30	3/8	57,7268	265,18	3/8	70,2931	393,20
1/2	45,5531	165,13	1/2	58,1195	268,80	1/2	70,6858	397,61
5/8	45,9458	167,99	5/8	58,5122	272,45	5/8	71,0785	402,04
3/4	46,3385	170,87	3/4	58,9049	276,12	3/4	71,4712	406,49
7/8	46,7312	173,78	7/8	59,2976	279,81	7/8	71,8639	410,97
15	47,1239	176,71	19	59,6903	283,53	23	72,2566	415,48
1/8	47,5166	179,67	1/8	60,0830	287,27	1/8	72,6493	420,00
1/4	47,9093	182,65	1/4	60,4757	291,04	1/4	73,0420	424,56
3/8	48,3020	185,66	3/8	60,8684	294,83	3/8	73,4347	429,13
1/2	48,6947	188,69	1/2	61,2611	298,65	1/2	73,8274	433,74
5/8	49,0874	191,75	5/8	61,6538	302,49	5/8	74,2201	438,36
3/4	49,4801	194,83	3/4	62,0465	306,35	3/4	74,6128	443,01
7/8	49,8728	197,93	7/8	62,4392	310,24	7/8	75,0055	447,69

TABLE LVI. — CIRCUMFERENCES AND AREAS OF CIRCLES FROM 0 TO 100, (CONTINUED).

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
24	75,3982	452,39	28	87,9646	615,75	32	100,531	804,25
	75,7909	457,11		88,3573	621,26		100,924	810,54
	76,1836	461,86		88,7500	626,80		101,316	816,86
	76,5763	466,64		89,1427	632,36		101,709	823,21
	76,9690	471,44		89,5354	637,94		102,102	829,58
	77,3617	476,26		89,9281	643,55		102,494	835,97
	77,7544	481,11		90,3208	649,18		102,887	842,39
	78,1471	485,98		90,7135	654,84		103,280	848,83
25	78,5398	490,87	29	91,1062	660,52	33	103,676	855,30
	78,9325	495,79		91,4989	666,23		104,065	861,79
	79,3252	500,74		91,8916	671,96		104,458	868,31
	79,7179	505,71		92,2843	677,71		104,851	874,85
	80,1106	510,71		92,6770	683,49		105,243	881,41
	80,5033	515,72		93,0697	689,30		105,636	888,00
	80,8960	520,77		93,4624	695,13		106,029	894,62
	81,2887	525,84		93,8551	700,98		106,421	901,26
26	81,6814	530,93	30	94,2478	706,86	34	106,814	907,92
	82,0741	536,05		94,6405	712,76		107,207	914,61
	82,4668	541,19		95,0332	718,69		107,600	921,32
	82,8595	546,35		95,4259	724,64		107,992	928,06
	83,2522	551,55		95,8186	730,62		108,385	934,82
	83,6449	556,76		96,2113	736,62		108,778	941,61
	84,0376	562,00		96,6040	742,64		109,170	948,42
	84,4303	567,27		96,9967	748,69		109,563	955,25
27	84,8230	572,56	31	97,3894	754,77	35	109,956	962,11
	85,2157	577,87		97,7821	760,87		110,348	969,00
	85,6084	583,21		98,1748	766,99		110,741	975,91
	86,0011	588,57		98,5675	773,14		111,134	982,84
	86,3938	593,96		98,9602	779,31		111,527	989,80
	86,7865	599,37		99,3529	785,51		111,919	996,78
	87,1792	604,81		99,7456	791,73		112,312	1003,8
	87,5719	610,27		100,138	797,98		112,705	1010,8

TABLE LVI. — CIRCUMFERENCES AND AREAS OF CIRCLES FROM 0 TO 100, (CONTINUED).

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
36	113,097	1017,9	40	125,664	1256,6	44	138,230	1520,5
1/8	113,490	1025,0	1/8	126,056	1264,5	1/8	138,623	1529,2
1/4	113,883	1032,1	1/4	126,449	1272,4	1/4	139,015	1537,9
3/8	114,275	1039,2	3/8	126,842	1280,3	3/8	139,408	1546,6
1/2	114,668	1046,3	1/2	127,235	1288,2	1/2	139,801	1555,3
5/8	115,061	1053,5	5/8	127,627	1296,2	5/8	140,194	1564,0
3/4	115,454	1060,7	3/4	128,020	1304,2	3/4	140,586	1572,8
7/8	115,846	1068,0	7/8	128,413	1312,2	7/8	140,979	1581,6
37	116,239	1075,2	41	128,805	1320,3	45	141,372	1590,4
1/8	116,632	1082,5	1/8	129,198	1328,3	1/8	141,764	1599,3
1/4	117,024	1089,8	1/4	129,591	1336,4	1/4	142,157	1608,2
3/8	117,417	1097,1	3/8	129,993	1344,5	3/8	142,550	1617,0
1/2	117,810	1104,5	1/2	130,376	1352,7	1/2	142,942	1626,0
5/8	118,202	1111,8	5/8	130,769	1360,8	5/8	143,335	1634,9
3/4	118,596	1119,2	3/4	131,161	1369,0	3/4	143,728	1643,9
7/8	118,988	1126,7	7/8	131,554	1377,2	7/8	144,121	1652,9
38	119,381	1134,1	42	131,947	1385,4	46	144,513	1661,9
1/8	119,773	1141,6	1/8	132,340	1393,7	1/8	144,906	1670,9
1/4	120,166	1149,1	1/4	132,732	1402,0	1/4	145,299	1680,0
3/8	120,559	1156,6	3/8	133,125	1410,3	3/8	145,691	1689,1
1/2	120,951	1164,2	1/2	133,518	1418,6	1/2	146,084	1698,2
5/8	121,344	1171,7	5/8	133,910	1427,0	5/8	146,477	1707,4
3/4	121,737	1179,3	3/4	134,303	1435,4	3/4	146,869	1716,5
7/8	122,129	1186,9	7/8	134,696	1443,8	7/8	147,262	1725,7
39	122,522	1194,6	43	135,088	1452,2	47	147,655	1734,9
1/8	122,915	1202,3	1/8	135,481	1460,7	1/8	148,048	1744,2
1/4	123,308	1210,0	1/4	135,874	1469,1	1/4	148,440	1753,5
3/8	123,700	1217,7	3/8	136,267	1477,6	3/8	148,833	1762,7
1/2	124,093	1225,4	1/2	136,659	1486,2	1/2	149,226	1772,1
5/8	124,486	1233,2	5/8	137,052	1494,7	5/8	149,618	1781,4
3/4	124,878	1241,0	3/4	137,445	1503,3	3/4	150,011	1790,8
7/8	125,271	1248,8	7/8	137,837	1511,9	7/8	150,404	1800,1

TABLE LVI.—CIRCUMFERENCES AND AREAS OF CIRCLES FROM [illegible]. CONTINUED.

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
49	153.938	1885.7	53	166.504	2206.2	57	179.071	2551.8
1/8	154.331	1895.4	1/8	166.897	2216.9	1/8	179.463	2563.0
1/4	154.723	1905.0	1/4	167.289	2227.6	1/4	179.856	2574.2
3/8	155.116	1914.7	3/8	167.683	2237.5	3/8	180.249	2585.4
1/2	155.509	1924.4	1/2	168.075	2248.0	1/2	180.642	2596.7
5/8	155.902	1934.2	5/8	168.468	2258.5	5/8	181.034	2608.0
3/4	156.294	1943.9	3/4	168.861	2269.1	3/4	181.427	2619.4
7/8	156.687	1953.7	7/8	169.253	2279.6	7/8	181.820	2630.7
50	157.080	1963.5	54	169.646	2290.2	58	182.212	2642.1
1/8	157.472	1973.3	1/8	170.038	2300.8	1/8	182.605	2653.5
1/4	157.865	1983.2	1/4	170.431	2311.5	1/4	182.998	2664.9
3/8	158.258	1993.1	3/8	170.824	2322.1	3/8	183.390	2676.4
1/2	158.650	2003.0	1/2	171.217	2332.8	1/2	183.783	2687.8
5/8	159.043	2012.9	5/8	171.609	2343.5	5/8	184.176	2699.3
3/4	159.436	2022.8	3/4	172.002	2354.3	3/4	184.569	2710.9
7/8	159.829	2032.8	7/8	172.395	2365.0	7/8	184.961	2722.4
51	160.221	2042.8	55	172.788	2375.8	59	185.354	2734.0
1/8	160.614	2052.8	1/8	173.180	2386.6	1/8	185.747	2745.6
1/4	161.007	2062.9	1/4	173.573	2397.5	1/4	186.139	2757.2
3/8	161.399	2073.0	3/8	173.966	2408.3	3/8	186.532	2768.8
1/2	161.792	2083.1	1/2	174.358	2419.2	1/2	186.925	2780.5
5/8	162.185	2093.2	5/8	174.751	2430.1	5/8	187.317	2792.2
3/4	162.577	2103.3	3/4	175.144	2441.1	3/4	187.710	2803.9
7/8	162.970	2113.5	7/8	175.536	2452.0	7/8	188.103	2815.7

TABLE LVI. — CIRCUMFERENCES AND AREAS OF CIRCLES FROM 0 TO 100, (CONTINUED).

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
60	188,496	2827,4	64	201,062	3217,0	68	213,628	3631,7
1/8	188,888	2839,2	1/8	201,455	3229,6	1/8	214,021	3645,0
1/4	189,281	2851,0	1/4	201,847	3242,2	1/4	214,414	3658,4
3/8	189,674	2862,9	3/8	202,240	3254,8	3/8	214,806	3671,8
1/2	190,066	2874,8	1/2	202,633	3267,5	1/2	215,199	3685,3
5/8	190,459	2886,6	5/8	203,025	3280,1	5/8	215,592	3698,7
3/4	190,852	2898.6	3/4	203,418	3292,8	3/4	215,984	3712,2
7/8	191,244	2910,5	7/8	203,811	3305,6	7/8	216,377	3725,7
61	191,637	2922,5	65	204,204	3318,3	69	216,770	3739,3
1/8	192,030	2934,5	1/8	204,596	3331,1	1/8	217,163	3752,8
1/4	192,423	2946,5	1/4	204,989	3343,9	1/4	217,555	3766,4
3/8	192,815	2958,5	3/8	205,382	3356,7	3/8	217,948	3780,0
1/2	193,208	2970,6	1/2	205,774	3369,6	1/2	218,341	3793,7
5/8	193,601	2982,7	5/8	206,167	3382,4	5/8	218,733	3807,3
3/4	193,993	2994,8	3/4	206,560	3395,3	3/4	219.126	3821,0
7/8	194,386	3006,9	7/8	206,952	3408,2	7/8	219,519	3834,7
62	194,779	3019,1	66	207,345	3421,2	70	219,911	3848,5
1/8	195,171	3031,3	1/8	207,738	3434,2	1/8	220,304	3862.2
1/4	195,564	3043,5	1/4	208,131	3447,2	1/4	220,697	3876,0
3/8	195,957	3055,7	3/8	208,523	3460,2	3/8	221,090	3889,8
1/2	196,350	3068,0	1/2	208,916	3473,2	1/2	221,482	3903,6
5/8	196,742	3080,3	5/8	209,309	3486,3	5/8	221,875	3917,5
3/4	197,135	3092,6	3/4	209,701	3499,4	3/4	222,268	3631.4
7/8	197,528	3104,9	7/8	210,094	3512,5	7/8	222,660	3945,3
63	197,920	3117,2	67	210,487	3525,7	71	223,053	3959,2
1/8	198,313	3129,6	1/8	210,879	3538.8	1/8	223,446	3973,1
1/4	198,706	3142.0	1/4	211,272	3552,0	1/4	223,838	3987,1
3/8	199,098	3154,5	3/8	211,665	3565,2	3/8	224,231	4001,1
1/2	199,491	3166.9	1/2	212,058	3578,5	1/2	224,624	4015,2
5/8	199,884	3179,4	5/8	212,450	3591.7	5/8	225,017	4029,2
3/4	200,277	3191,9	3/4	212,843	3605,0	3/4	225,409	4043,3
7/8	200,669	3204,4	7/8	213,236	3618,3	7/8	225,802	4057,4

TABLE LVI. — CIRCUMFERENCES AND AREAS OF CIRCLES FROM 0 TO 100. (CONTINUED).

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
72	226,195	4071,5	76	238,761	4536,5	80	251,327	5026,5
¼	226,587	4085,7	¼	239,154	4551,4	¼	251,720	5042,3
½	226,980	4099,8	½	239,546	4566,4	½	252,113	5058,0
¾	227,373	4114,0	¾	239,939	4581,3	¾	252,506	5073,8
1	227,765	4128,2	1	240,332	4596,3	1	252,898	5089,6
¼	228,158	4142,5	¼	240,725	4611,4	¼	253,291	5105,4
½	228,551	4156,8	½	241,117	4626,4	½	253,684	5121,2
¾	228,944	4171,1	¾	241,510	4641,5	¾	254,076	5137,1
73	229,336	4185,4	77	241,903	4656,6	81	254,469	5153,0
¼	229,729	4199,7	¼	242,295	4671,8	¼	254,862	5168,9
½	230,122	4214,1	½	242,688	4686,9	½	255,254	5184,9
¾	230,514	4228,5	¾	243,081	4702,1	¾	255,647	5200,8
1	230,907	4242,9	1	243,473	4717,3	1	256,040	5216,8
¼	231,300	4257,4	¼	243,866	4732,5	¼	256,433	5232,8
½	231,692	4271,8	½	244,259	4747,8	½	256,825	5248,9
¾	232,085	4286,3	¾	244,652	4763,1	¾	257,218	5264,9
74	232,478	4300,8	78	245,044	4778,4	82	257,611	5281,0
¼	232,871	4315,4	¼	245,437	4793,7	¼	258,003	5297,1
½	233,263	4329,9	½	245,830	4809,0	½	258,396	5313,3
¾	233,656	4344,5	¾	246,222	4824,4	¾	258,789	5329,4
1	234,049	4359,2	1	246,615	4839,8	1	259,181	5345,6
¼	234,441	4373,8	¼	247,008	4855,2	¼	259,574	5361,8
½	234,834	4388,5	½	247,400	4870,7	½	259,967	5378,1
¾	235,227	4403,1	¾	247,793	4886,2	¾	260,359	5394,3
75	235,619	4417,9	79	248,186	4901,7	83	260,752	5410,6
¼	236,012	4432,6	¼	248,579	4917,2	¼	261,145	5426,9
½	236,405	4447,4	½	248,971	4932,7	½	261,538	5443,3
¾	236,798	4462,2	¾	249,364	4948,3	¾	261,930	5459,6
1	237,190	4477,0	1	249,757	4963,9	1	262,323	5476,0
¼	237,583	4491,8	¼	250,149	4979,5	¼	262,716	5492,4
½	237,976	4506,7	½	250,542	4995,2	½	263,108	5508,8
¾	238,368	4521,5	¾	250,935	5010,9	¾	263,501	5525,3

TABLE LVI. — CIRCUMFERENCE AND AREAS OF CIRCLES FROM 0 TO 100, (CONTINUED).

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
84	263,894	5541,8	88	276,460	6082,1	92	289,027	6647,6
1/8	264,286	5558,3	1/8	276,853	6099,4	1/8	289,419	6665,7
1/4	264,679	5574,8	1/4	277,246	6116,7	1/4	289,812	6683,8
3/8	265,072	5591,4	3/8	277,638	6134,1	3/8	290,205	6701,9
1/2	265,465	5607,9	1/2	278,031	6151,4	1/2	290,597	6720,1
5/8	265,857	5624,5	5/8	278,424	6168,8	5/8	290,990	6738,2
3/4	266,250	5641,2	3/4	278,816	6186,2	3/4	291,383	6756,4
7/8	266,643	5657,8	7/8	279,209	6203,7	7/8	291,775	6774,7
85	267,035	5674,5	89	279,602	6221,1	93	292,168	6792,9
1/8	267,428	5691,2	1/8	279,994	6238,6	1/8	292,561	6811,2
1/4	267,821	5707,9	1/4	280,387	6256,1	1/4	292,954	6829,5
3/8	268,213	5724,7	3/8	280,780	6273,7	3/8	293,346	6847,8
1/2	268,606	5741,5	1/2	281,173	6291,2	1/2	293,739	6866,1
5/8	268,999	5758,3	5/8	281,565	6308,8	5/8	294,132	6884,5
3/4	269,392	5775,1	3/4	281,958	6326,4	3/4	294,524	6902,9
7/8	269,784	5791,9	7/8	282,351	6344,1	7/8	294,917	6921,3
86	270,177	5808,8	90	282,743	6361,7	94	295,310	6939,8
1/8	270,570	5825,7	1/8	283,136	6379,4	1/8	295,702	6958,2
1/4	270,962	5842,6	1/4	283,529	6397,1	1/4	296,095	6976,7
3/8	271,355	5859,6	3/8	283,921	6414,9	3/8	296,488	6995,3
1/2	271,748	5876,5	1/2	284,314	6432,6	1/2	296,881	7013,8
5/8	272,140	5893,5	5/8	284,707	6450,4	5/8	297,273	7032,4
3/4	272,533	5910,6	3/4	285,100	6468,2	3/4	297,666	7051,0
7/8	272,926	5927,6	7/8	285,492	6486,0	7/8	298,059	7069,6
87	273,319	5944,7	91	285,885	6503,9	95	298,451	7088,2
1/8	273,711	5961,8	1/8	286,278	6521,8	1/8	298,844	7106,9
1/4	274,104	5978,9	1/4	286,670	6539,7	1/4	299,237	7125,6
3/8	274,497	5996,0	3/8	287,063	6557,6	3/8	299,629	7144,3
1/2	274,889	6013,2	1/2	287,456	6575,5	1/2	300,022	7163,0
5/8	275,282	6030,4	5/8	287,848	6593,5	5/8	300,415	7181,8
3/4	275,675	6047,6	3/4	288,241	6611,5	3/4	300,807	7200,6
7/8	276,067	6064,9	7/8	288,634	6629,6	7/8	301,200	7219,4

TABLE LVI. — CIRCUMFERENCE AND AREAS OF CIRCLES FROM 0 TO 100, (CONTINUED).

Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.	Diameter.	Circumference.	Area.
96	301,593	7238,2	97	305,913	7447,1	98	310,232	7658,9
	301,986	7257,1					310,625	7678,3
	302,378	7276,0		306,305	7466,2			
	302,771	7294,9		306,698	7485,3			
				307,091	7504,5	99	311,018	7697,7
	303,164	7313,8		307,483	7523,7		311,410	7717,1
	303,556	7332,8					311,803	7736,6
	303,949	7351,8	98	307,876	7543,0		312,196	7756,1
	304,342	7370,8		308,269	7562,2		312,588	7775,6
				308,661	7581,5		312,981	7795,2
97	304,734	7389,8		309,054	7600,8		313,374	7814,8
	305,127	7408,9		309,447	7620,1		313,767	7834,4
	305,520	7428,0		309,840	7639,5	100	314,159	7854,0

TABLE LVII.

DECIMAL PARTS OF AN INCH FOR EACH $\frac{1}{64}$ OF AN INCH.

FRACTION.	DECIMAL.	FRACTION.	DECIMAL.	FRACTION.	DECIMAL.
$\frac{1}{64}$	0,015625	$\frac{21}{64}$	0,328125	$\frac{45}{64}$	0,703125
$\frac{2}{64}$	0,03125	$\frac{22}{64}$	0,34375	$\frac{46}{64}$	0,71875
$\frac{3}{64}$	0,046875	$\frac{23}{64}$	0,359375	$\frac{47}{64}$	0,734375
$\frac{1}{16}$	0,0625	$\frac{24}{64}$	0,375	$\frac{48}{64}$	0,75
		$\frac{25}{64}$	0,390625		
$\frac{5}{64}$	0,078125	$\frac{26}{64}$	0,40625	$\frac{49}{64}$	0.765625
$\frac{6}{64}$	0,09375	$\frac{27}{64}$	0,421875	$\frac{50}{64}$	0,78125
$\frac{7}{64}$	0,109375	$\frac{7}{16}$	0,4375	$\frac{51}{64}$	0,796875
$\frac{1}{8}$	0,125	$\frac{29}{64}$	0,453125	$\frac{13}{16}$	0,8125
		$\frac{15}{32}$	0,46875		
$\frac{9}{64}$	0,140625	$\frac{31}{64}$	0,484375	$\frac{53}{64}$	0,828125
$\frac{5}{32}$	0,15625	$\frac{1}{2}$	0,5	$\frac{54}{64}$	0,84375
$\frac{11}{64}$	0,171875	$\frac{33}{64}$	0,515625	$\frac{55}{64}$	0,859375
$\frac{3}{16}$	0,1875	$\frac{17}{32}$	0,53125	$\frac{7}{8}$	0,875
		$\frac{35}{64}$	0,546875		
$\frac{13}{64}$	0,203125	$\frac{9}{16}$	0,5625		
$\frac{7}{32}$	0,21875	$\frac{37}{64}$	0,578125	$\frac{57}{64}$	0,890625
$\frac{15}{64}$	0,234375	$\frac{19}{32}$	0,59375	$\frac{29}{32}$	0,90625
$\frac{1}{4}$	0,25	$\frac{39}{64}$	0,609375	$\frac{59}{64}$	0,921875
		$\frac{5}{8}$	0,625	$\frac{15}{16}$	0,9375
$\frac{17}{64}$	0,265625	$\frac{41}{64}$	0,640625		
$\frac{9}{32}$	0,28125	$\frac{21}{32}$	0,65625	$\frac{61}{64}$	0,953125
$\frac{19}{64}$	0,296875	$\frac{43}{64}$	0,671875	$\frac{31}{32}$	0,96875
$\frac{5}{16}$	0,3125	$\frac{11}{16}$	0,6875	$\frac{63}{64}$	0,984375

TABLE LVIII.

DECIMAL PARTS OF A FOOT FOR EACH $\frac{1}{64}$ OF AN INCH, FROM 0″ TO 12″.

Inch.	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″
0	0	,0833	,1667	,2500	,3333	,4167	,5000	,5833	,6667	,7500	,8333	,9167
$\frac{1}{64}$	,0013	,0846	,1680	,2513	,3346	,4180	,5013	,5846	,6680	,7513	,8346	,9180
$\frac{1}{32}$	,0026	,0859	,1693	,2526	,3359	,4193	,5026	,5859	,6693	,7526	,8359	,9193
$\frac{3}{64}$	,0039	,0872	,1706	,2539	,3372	,4206	,5039	,5872	,6706	,7539	,8372	,9206
$\frac{1}{16}$	,0052	,0885	,1719	,2552	,3385	,4219	,5052	,5885	,6719	,7552	,8385	,9219
$\frac{5}{64}$	,0065	,0898	,1732	,2565	,3398	,4232	,5065	,5898	,6732	,7565	,8398	,9232
$\frac{3}{32}$	,0078	,0911	,1745	,2578	,3411	,4245	,5078	,5911	,6745	,7578	,8411	,9245
$\frac{7}{64}$	,0091	,0924	,1758	,2591	,3424	,4258	,5091	,5924	,6758	,7591	,8424	,9258
$\frac{1}{8}$	,0104	,0937	,1771	,2604	,3437	,4271	,5104	,5937	,6771	,7604	,8437	,9271
$\frac{9}{64}$	,0117	,0951	,1784	,2617	,3451	,4284	,5117	,5951	,6784	,7617	,8451	,9284
$\frac{5}{32}$	,0130	,0964	,1797	,2630	,3464	,4297	,5130	,5964	,6797	,7630	,8464	,9297
$\frac{11}{64}$	,0143	,0977	,1810	,2643	,3477	,4310	,5143	,5977	,6810	,7643	,8477	,9310
$\frac{3}{16}$	,0156	,0990	,1823	,2656	,3490	,4323	,5156	,5990	,6823	,7656	,8490	,9323
$\frac{13}{64}$	,0169	,1003	,1836	,2669	,3503	,4336	,5169	,6003	,6836	,7669	,8503	,9336
$\frac{7}{32}$	,0182	,1016	,1849	,2682	,3516	,4349	,5182	,6016	,6849	,7682	,8516	,9349
$\frac{15}{64}$	,0195	,1029	,1862	,2695	,3529	,4362	,5195	,6029	,6862	,7695	,8529	,9362
$\frac{1}{4}$	,0208	,1042	,1875	,2708	,3542	,4375	,5208	,6042	,6875	,7708	,8542	,9375
$\frac{17}{64}$	,0221	,1055	,1888	,2721	,3555	,4388	,5221	,6055	,6888	,7721	,8555	,9388
$\frac{9}{32}$	,0234	,1068	,1901	,2734	,3568	,4401	,5234	,6068	,6901	,7734	,8568	,9401
$\frac{19}{64}$	,0247	,1081	,1914	,2747	,3581	,4414	,5247	,6081	,6914	,7747	,8581	,9414
$\frac{5}{16}$	,0260	,1094	,1927	,2760	,3594	,4427	,5260	,6094	,6927	,7760	,8594	,9427
$\frac{21}{64}$	,0273	,1107	,1940	,2773	,3607	,4440	,5273	,6107	,6940	,7773	,8607	,9440
$\frac{11}{32}$	,0286	,1120	,1953	,2786	,3620	,4453	,5286	,6120	,6953	,7786	,8620	,9453
$\frac{23}{64}$	,0299	,1133	,1966	,2799	,3633	,4466	,5299	,6133	,6966	,7799	,8633	,9466
$\frac{3}{8}$	,0312	,1146	,1979	,2812	,3646	,4479	,5312	,6146	,6979	,7812	,8646	,9479
$\frac{25}{64}$	,0326	,1159	,1992	,2826	,3659	,4492	,5326	,6159	,6992	,7826	,8659	,9492
$\frac{13}{32}$	,0339	,1172	,2005	,2839	,3672	,4505	,5339	,6172	,7005	,7839	,8672	,9505
$\frac{27}{64}$	,0352	,1185	,2018	,2852	,3685	,4518	,5352	,6185	,7018	,7852	,8685	,9518
$\frac{7}{16}$	,0365	,1198	,2031	,2865	,3698	,4531	,5365	,6198	,7031	,7865	,8698	,9531
$\frac{29}{64}$	,0378	,1211	,2044	,2878	,3711	,4544	,5378	,6211	,7044	,7878	,8711	,9544
$\frac{15}{32}$	,0391	,1224	,2057	,2891	,3724	,4557	,5391	,6224	,7057	,7891	,8724	,9557
$\frac{31}{64}$	,0404	,1237	,2070	,2904	,3737	,4570	,5404	,6237	,7070	,7904	,8737	,9570
$\frac{1}{2}$	,0417	,1250	,2083	,2917	,3750	,4583	,5417	,6250	,7083	,7917	,8750	,9583

TABLE LVIII.— DECIMAL PARTS OF A FOOT FOR EACH $\frac{1}{64}$ OF AN INCH FROM 0″ TO 12″, (CONTINUED.)

Inch.	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″
1/64	,0430	,1263	,2096	,2930	,3763	,4596	,5430	,6263	,7096	,7930	,8763	,9596
1/32	,0443	,1276	,2109	,2943	,3776	,4609	,5443	,6276	,7109	,7943	,8776	,9609
3/64	,0456	,1289	,2122	,2956	,3789	,4622	,5456	,6289	,7122	,7956	,8789	,9622
1/16	,0469	,1302	,2135	,2969	,3802	,4635	,5469	,6302	,7135	,7969	,8802	,9635
5/64	,0482	,1315	,2148	,2982	,3815	,4648	,5482	,6315	,7148	,7982	,8815	,9648
3/32	,0495	,1328	,2161	,2995	,3828	,4661	,5495	,6328	,7161	,7995	,8828	,9661
7/64	,0508	,1341	,2174	,3008	,3841	,4674	,5508	,6341	,7174	,8008	,8841	,9674
1/8	,0521	,1354	,2188	,3021	,3854	,4688	,5521	,6354	,7188	,8021	,8854	,9688
9/64	,0534	,1367	,2201	,3034	,3867	,4701	,5534	,6367	,7201	,8034	,8867	,9701
5/32	,0547	,1380	,2214	,3047	,3880	,4714	,5547	,6380	,7214	,8047	,8880	,9714
11/64	,0560	,1393	,2227	,3060	,3893	,4727	,5560	,6393	,7227	,8060	,8893	,9727
3/16	,0573	,1406	,2240	,3073	,3906	,4740	,5573	,6406	,7240	,8073	,8906	,9740
13/64	,0586	,1419	,2253	,3086	,3919	,4753	,5586	,6419	,7253	,8086	,8919	,9753
7/32	,0599	,1432	,2266	,3099	,3932	,4766	,5599	,6432	,7266	,8099	,8932	,9766
15/64	,0612	,1445	,2279	,3112	,3945	,4779	,5612	,6445	,7279	,8112	,8945	,9779
1/4	,0625	,1458	,2292	,3125	,3958	,4792	,5625	,6458	,7292	,8125	,8958	,9792
17/64	,0638	,1471	,2305	,3138	,3971	,4805	,5638	,6471	,7305	,8138	,8971	,9805
9/32	,0651	,1484	,2318	,3151	,3984	,4818	,5651	,6484	,7318	,8151	,8984	,9818
19/64	,0664	,1497	,2331	,3164	,3997	,4831	,5664	,6497	,7331	,8164	,8997	,9831
5/16	,0677	,1510	,2344	,3177	,4010	,4844	,5677	,6510	,7344	,8177	,9010	,9844
21/64	,0690	,1523	,2357	,3190	,4023	,4857	,5690	,6523	,7357	,8190	,9023	,9857
11/32	,0703	,1536	,2370	,3203	,4036	,4870	,5703	,6536	,7370	,8203	,9036	,9870
23/64	,0716	,1549	,2383	,3216	,4049	,4883	,5716	,6549	,7383	,8216	,9049	,9883
3/8	,0729	,1562	,2396	,3229	,4062	,4896	,5729	,6562	,7396	,8229	,9062	,9896
25/64	,0742	,1576	,2409	,3242	,4076	,4909	,5742	,6576	,7409	,8242	,9076	,9909
13/32	,0755	,1589	,2422	,3255	,4089	,4922	,5755	,6589	,7422	,8255	,9089	,9922
27/64	,0768	,1602	,2435	,3268	,4102	,4935	,5768	,6602	,7435	,8268	,9102	,9935
7/16	,0781	,1615	,2448	,3281	,4115	,4948	,5781	,6615	,7448	,8281	,9115	,9948
29/64	,0794	,1628	,2461	,3294	,4128	,4961	,5794	,6628	,7461	,8294	,9128	,9961
15/32	,0807	,1641	,2474	,3307	,4141	,4974	,5807	,6641	,7474	,8307	,9141	,9974
31/64	,0820	,1654	,2487	,3320	,4154	,4987	,5820	,6654	,7487	,8320	,9154	,9987
1												1.0000

TABLE LIX.

REDUCTION OF ENGLISH FEET AND INCHES TO METRES.

	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″
0′	0	0,025	0,051	0,076	0,102	0,127	0,152	0,178	0,203	0,229	0,254	0,279
1′	0,305	0,330	0,356	0,381	0,406	0,432	0,457	0,483	0,508	0,533	0,559	0,584
2′	0,610	0,635	0,660	0,686	0,711	0,737	0,762	0,787	0,813	0,838	0,864	0,889
3′	0,914	0,940	0,965	0,991	1,016	1,041	1,067	1,092	1,118	1,143	1,168	1,194
4′	1,219	1,245	1,270	1,295	1,321	1,346	1,372	1,397	1,422	1,448	1,473	1,499
5′	1,524	1,549	1,575	1,600	1,626	1,651	1,676	1,702	1,727	1,753	1,778	1,803
6′	1,829	1,854	1,880	1,905	1,930	1,956	1,981	2,007	2,032	2,057	2,083	2,108
7′	2,134	2,159	2,184	2,210	2,235	2,261	2,286	2,311	2,337	2,362	2,388	2,413
8′	2,438	2,464	2,489	2,515	2,540	2,565	2,591	2,616	2,642	2,667	2,692	2,718
9′	2,743	2,769	2,794	2,819	2,845	2,870	2,896	2,921	2,946	2,972	2,997	3,023
10′	3,048	3,073	3,099	3,124	3,150	3,175	3,200	3,226	3,251	3,277	3,302	3,327
11′	3,353	3,378	3,404	3,429	3,454	3,480	3,505	3,531	3,556	3,581	3,607	3,632
12′	3,658	3,683	3,708	3,734	3,759	3,785	3,810	3,835	3,861	3,886	3,912	3,937
13′	3,962	3,988	4,013	4,039	4,064	4,089	4,115	4,140	4,166	4,191	4,216	4,242
14′	4,267	4,293	4,318	4,343	4,369	4,394	4,420	4,445	4,470	4,496	4,521	4,547
15′	4,572	4,597	4,623	4,648	4,674	4,699	4,724	4,750	4,775	4,801	4,826	4,851
16′	4,877	4,902	4,928	4,953	4,978	5,004	5,029	5,055	5,080	5,105	5,131	5,156
17′	5,182	5,207	5,232	5,258	5,283	5,309	5,334	5,359	5,385	5,410	5,436	5,461
18′	5,486	5,512	5,537	5,563	5,588	5,613	5,639	5,664	5,690	5,715	5,740	5,766
19′	5,791	5,817	5,842	5,867	5,893	5,918	5,944	5,969	5,994	6,020	6,045	6,071
20′	6,096	6,121	6,147	6,172	6,198	6,223	6,248	6,274	6,299	6,325	6,350	6,375
21′	6,401	6,426	6,452	6,477	6,502	6,528	6,553	6,579	6,604	6,629	6,655	6,680
22′	6,706	6,731	6,756	6,782	6,807	6,833	6,858	6,883	6,909	6,934	6,960	6,985
23′	7,010	7,036	7,061	7,087	7,112	7,137	7,163	7,188	7,214	7,239	7,264	7,290
24′	7,315	7,341	7,366	7,391	7,417	7,442	7,468	7,493	7,518	7,544	7,569	7,595
25′	7,620	7,645	7,671	7,696	7,722	7,747	7,772	7,798	7,823	7,849	7,874	7,899
26′	7,925	7,950	7,976	8,001	8,026	8,052	8,077	8,103	8,128	8,153	8,179	8,204
27′	8,230	8,255	8,280	8,306	8,331	8,357	8,382	8,407	8,433	8,458	8,484	8,509
28′	8,534	8,560	8,585	8,611	8,636	8,661	8,687	8,712	8,738	8,763	8,788	8,814
29′	8,839	8,865	8,890	8,915	8,941	8,966	8,992	9,017	9,042	9,068	9,093	9,119
30′	9,144	9,169	9,195	9,220	9,246	9,271	9,296	9,322	9,347	9,373	9,398	9,423
31′	9,449	9,474	9,500	9,525	9,550	9,576	9,601	9,627	9,652	9,677	9,703	9,728
32′	9,754	9,779	9,804	9,830	9,855	9,881	9,906	9,931	9,957	9,982	10,008	10,033
33′	10,058	10,084	10,109	10,135	10,160	10,185	10,211	10,236	10,262	10,287	10,312	10,338
34′	10,363	10,389	10,414	10,439	10,465	10,490	10,516	10,541	10,566	10,592	10,617	10,643
35′	10,668	10,693	10,719	10,744	10,770	10,795	10,820	10,846	10,871	10,897	10,922	10,947
36′	10,973	10,998	11,024	11,049	11,074	11,100	11,125	11,151	11,176	11,201	11,227	11,252
37′	11,278	11,303	11,328	11,354	11,379	11,405	11,430	11,455	11,481	11,506	11,532	11,557
38′	11,532	11,608	11,633	11,659	11,684	11,709	11,735	11,760	11,786	11,811	11,836	11,862
39′	11,887	11,913	11,938	11,963	11,989	12,014	12,040	12,065	12,090	12,116	12,141	12,167
40′	12,192	12,217	12,243	12,268	12,294	12,319	12,344	12,370	12,395	12,421	12,446	12,471
41′	12,497	12,522	12,548	12,573	12,598	12,624	12,649	12,675	12,700	12,725	12,751	12,776
42′	12,802	12,827	12,852	12,878	12,902	12,929	12,954	12,979	13,005	13,030	13,056	13,081
43′	13,106	13,132	13,157	13,183	13,208	13,233	13,259	13,284	13,310	13,335	13,360	13,386
44′	13,411	13,437	13,462	13,487	13,513	13,538	13,564	13,589	13,614	13,640	13,665	13,691
	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″

Metres.

1000 millimetres = 100 centimetres = 10 decimetres = 1 metre.
1000 metres = 100 decametres = 10 hectometres = 1 kilometre.

TABLE LX.

REDUCTION OF ENGLISH INCHES TO MILLIMETRES.

1/32″	0,79	19/32″	15,08	1 5/16″	33,34	2 7/8″	73,02
1/16″	1,59	5/8″	15,87	1 3/8″	34,92	3″	76,20
3/32″	2,38	21/32″	16,67	1 7/16″	36,51	3 1/8″	79,37
1/8″	3,17	11/16″	17,46	1 1/2″	38,10	3 1/4″	82,55
5/32″	3,97	23/32″	18,26	1 9/16″	39,69	3 3/8″	85,72
3/16″	4,76	3/4″	19,05	1 5/8″	41,27	3 1/2″	88,90
7/32″	5,56	25/32″	19,84	1 11/16″	42,86	3 5/8″	92,07
1/4″	6,35	13/16″	20,64	1 3/4″	44,45	3 3/4″	95,25
9/32″	7,14	27/32″	21,43	1 13/16″	46,04	3 7/8″	98,42
5/16″	7,94	7/8″	22,22	1 7/8″	47,62	4″	101,60
11/32″	8,73	29/32″	23,02	1 15/16″	49,21	4 1/4″	107,95
3/8″	9,52	15/16″	23,81	2″	50,80	4 1/2″	114,30
13/32″	10,32	31/32″	24,61	2 1/8″	53,97	4 3/4″	120,65
7/16″	11,11	1″	25,40	2 1/4″	57,15	5″	127,00
15/32″	11,91	1 1/16″	26,99	2 3/8″	60,32	5 1/4″	133,35
1/2″	12,70	1 1/8″	28,57	2 1/2″	63,50	5 1/2″	139,70
17/32″	13,49	1 3/16″	30,16	2 5/8″	66,67	5 3/4″	146,05
9/16″	14,29	1 1/4″	31,75	2 3/4″	69,85	6″	152,40

TABLE LXI.

REDUCTION OF ENGLISH WEIGHTS TO KILOGRAMS.

Lbs.	Kilos.	Lbs.	Kilos.	Cwts.	Kilos.	Cwts.	Kilos.
1	0,454	17	7,711	1	50,80	17	863,64
2	0,907	18	8,165	2	101,60	18	914,44
3	1,361	19	8,618	3	152,41	19	965,24
4	1,814	20	9,072	4	203,21	Long Tons.	
5	2,268	21	9,525	5	254,01	1	1016,05
6	2,722	22	9,979	6	304,81	2	2032,1
7	3,175	23	10,433	7	355,62	3	3048,1
8	3,629	24	10,886	8	406,42	4	4064,2
9	4,082	25	11,340	9	457,22	5	5080,2
10	4,536	26	11,793	10	508,02	6	6096,3
11	4,990	27	12,247	11	558,83	7	7112,3
12	5,443			12	609,63	8	8128,4
13	5,897	Qu.		13	660,43	9	9144,4
14	6,350	1	12,701	14	711,23	10	10160,5
15	6,804	2	25,401	15	762,04	11	11176,5
16	7,257	3	38,102	16	812,84	12	12192,6

TABLE LXII.

ENGLISH MEASURES AND WEIGHTS.

(U. S. standards are the same.)

LONG MEASURE.

12 inches = 1 foot.
3 feet = 1 yard.
16½ feet = 5½ yards = 1 rod, or pole, or perch.
660 feet = 220 yards = 40 rods = 1 furlong.
5280 feet = 1760 yards = 320 rods = 8 furlongs = 1 mile (land).
6 feet = 1 fathom.
720 feet = 120 fathoms = 1 cable's length.
6086,37 feet = 2028,79 yards = 1,1527 land miles = 1 knot or sea-mile.

SQUARE MEASURE.

144 square inches = 1 square foot.
100 square feet = 1 square.
9 square feet = 1 square yard.
272¼ square feet = 30¼ square yards = 1 square rod.
10890 square feet = 1210 square yards = 40 square rods = 1 rood.
43560 square feet = 4840 square yards = 160 square rods = 4 roods = 1 acre.
27878400 square feet = 3097600 square yards = 102400 square rods = 2560 roods = 640 acres = 1 square mile.
1 acre = 208,71 feet square = 69,57 yards square = 220 feet × 198 feet.

SOLID MEASURE.

1728 cubic inches = 1 cubic foot = 28,3161 litres.
27 cubic feet = 1 cubic yard = 764,534 litres.
1 cubic inch = 0,0005787 cubic feet = 0,0163863 litres.
1 litre = 1 cubic decimetre = 61,0254 cubic inches.
1 perch of stonework = 24,75 cubic feet.

LIQUID MEASURE.

(U. S. standard.)

1 gill = 7,21875 cubic inches.
4 gills = 1 pint = 28,875 cubic inches.
8 gills = 2 pints = 1 quart = 57,75 cubic inches.
32 gills = 8 pints = 4 quarts = 1 gallon = 231 cubic inches = 0,13368 cubic feet.
1 cubic foot = 7,48052 gallons (approximately 7½ gallons).
1 gallon of water weighs = 8,33888 pounds.
63 gallons = 1 hogshead.
1 gallon = 4,405 litres.

AVOIRDUPOIS (COMMERCIAL) WEIGHT.

16 drams = 1 ounce (oz.).
16 ounces = 1 pound (lb.).
14 pounds = 1 stone.
28 pounds = 1 quarter.
112 pounds = 1 hundredweight (cwts.).
2240 pounds = 20 cwts. = 1 ton (gross or long).
2000 pounds = 1 short ton (used in this work).

27,34375 troy grains = 1 dram avoirdupois.
1 troy pound = 0,822857 avoirdupois pound = 12 troy ounces = 240 troy pennyweights = 5760 troy grains.
1 avoirdupois pound = 1,215278 troy pound.

GENERAL INDEX.

ABUTMENT, see Examples; thrust of roof, II, 164, 165; to arches, I, 163-170; to truss, II, 164, 165.
Acre, see Table LXII, II, 262.
Analysis, see Graphical analysis.
Anchors, of metal, I, 128.
Angles, list of steel and iron, Tables XXII and XXIII, End of Volume I; plate girder, rivets in, II, 129-131; plate girder, strength of, Table XLII, End of Vol. II; plate girder value, II, 120; to plate girders, II, 114, 115.
Annealing of steel, II, 37.
Approximate Formulæ, arches, depth of crown, I, 179; arches, rule for, I, 179; deflection, iron beams and girders, I, 223, 234; plate girders, I, 165.
Arch, see Bond, Examples; chapter on, I, 154-179; abutting force at base, I, 75; approximate rule, I, 179; brick floor arch, I, 158, 159; curve of pressure, I, 157; depth at crown, I, 179; depth of crown, approximate formula, I, 179; fireproof floor arch I, 159-161; groined or vaulted, I, 179; horizontal force at crown, where located, I, 75; horizontal pressure, I, 157; how load acts, I, 75; inverted arch, I, 89, 90, 168-170; in wall, I, 161-167; line of pressure, effect if not central, I, 77-79; line of pressure, theory of, I, 76; of cambered plank, I, 176; of Spanish tile, I, 176-179; stress at intrados and extrados, I, 77-79, 158; strain on edges, formulæ, I, 78, 79; theory of, I, 75-79; three forces acting on, I, 75; to find neutral axes, I, 154, 155; truss, II, 154; vaulted, I, 179; with abutments, I, 163-170; with abutting arch, I, 166-168; what to avoid, I, 125, 126.
Area of any section found graphically, I, 268, 269; of circles, *see Table LVI*, II, 246-256; of cross-section for different sections, Table I, I, 12-21; of squares, *see Table LVI*, II, 246-256.
Asphalt, in machinery foundations, to avoid vibrations, I, 131, 132; qualities of, I, 132; used in damp-proofing, I, 87-119.
Avoirdupois, see Table LXII, II, 262.
Axis, see Neutral axis.

BAR Iron, strongest wrought-iron, II, 35.
Barrels, thrust on walls, I, 146-148.
Base-course, see Examples; over piles, I, 94, 95; width of, I, 85, 88.
Bases, of columns, II, 203, 204; wrought-iron preferable, II, 207.
Beams, see Bending-moment, Cantilever, Continuous Girders, Deflection, Examples, Floors, Gallery Beam, Girders, Inclined Beam, Load, Transverse Strength, Trussed Beams, Tubular Beam; chapter on, I, 180-240; average strain in flanges, I, 225-227, 243; bending-moment, Table VII, I, 58, 59; bending-moment, any point, several loads, formula, I, 53; central bending-moment, centre load, formula, I, 52; central bending-moment, uniform load, formula, I, 52; comparative deflection of, different cross-sections, I, 61; comparative deflection of, different lengths, I, 61; comparative deflection of, different lengths and cross-sections, I, 62; comparative strength, different cross-sections, I, 60; comparative strength, different lengths, I, 60; comparative strength, different lengths and cross-sections, I, 61; comparison with different loads and cantilever (strength and stiffness,) I, 57; continuous, II, 211; cross-bridging, I, 191; deflection Table VII, I, 58, 59; deflection, any load, formula, I, 67; deflection, centre load, formula, I, 66; deflection, comparative formulæ, I, 61, 62; deflection, different loads, I, 66, 67; deflection, formula, I, 228; deflection, iron,

approximate formulæ, I. 223, 224; deflection, not to crack plaster, formula, I. 56; deflection, not to crack plaster uneven loads) formulæ, I. 57; deflection, point of greatest, formula, I. 67; deflection, uniform load, formula, I. 66; effect of load on, I. 182-186; end in wall, I. 125; framing of, I. 192, 193; if rectangular, strength of, I. 144-147; if trussed, I. 220-223; inclined, II. 212; trussed, analysis of load, II. 192, 193, 195, 196; inclined, transverse strain on, II. 192, 193, 195, 196; iron, approximate deflection, I. 223, 224; iron, doubled, I. 226; iron, length not to crack plaster, formulæ, I. 224; iron, strength of, as girders, Table XV, I. 199; lateral flexure, I. 204-206; list of steel and iron, Table XX, End of Vol. I; longitudinal shearing, formula, I. 55; moment of inertia, I. 180-182; most economical, I. 224, 227; most economical shape, I. 56; one inch thick (uniform load) strength of, Table IX, I. 63; overhanging, II. 211, 212; rectangular, transverse strength, formulæ, I. 134, 135; shearing, Table VIII, I. 58, 59; shearing, vertical, any point, formula, I. 54; steel, use of, I. 203, 204; steel, new deep beam, II. 135; strength, comparative formulæ, I. 60, 62; strength of wooden floor, Table XII, End of Vol. I; strength of wooden one inch thick, Table VIII, I. 62; strength of iron floor, Table XIV, End of Volume I; tie-rods to iron beams, I. 206, 207; to ascertain amount of load on, I. 80; to find required cross-section, I. 54; wrought, built in, deflection of, I. 77; with ends built in, strength of, I. 77; with overhang, II. 86-88.

Bearing, see *Compression*; effective length of, for girders, I. 227, 228; for columns, II. 293, 294; of plate girder, II. 151.

Bearing area, see *Examples* of truss, II. 186, 188, 189.

Bending, of rivet, II. 65-70; on rivets, pins and rods, Table XXXVIII, End of Vol. II.

Bending-moment, see *Beam*, *Cantilever*, *Centre Load*, *Continuous Girder*, *Load*, *Eccentric Load*, *Equivalent Load*, *Inclined Beam*, *Overhang*, *Several Loads*, *Uniform Load*, *Uniformly Distributed Load*; amount of greatest, I. 52-54; beam, any point, several loads, formula, I. 53; beam, central one, centre load, formula, I. 52; beam, central uniform load, formula, I. 52; beam with overhang, II. 86-88; beams and cantilevers, Table VII, I. 58, 59; cantilever, any loading, formula, I. 54; cantilever, end load, formula, I. 53; cantilever, uniform load, formula, I. 53; continuous girders, Table XVII, I. 218, 219; defined, I. 49; graphical analysis of, I. 245, 246; graphical method, formula, I. 265; pin, II. 94-105; pin, graphically, II. 102-104; pin unequally loaded, II. 86-88; plate girder, II. 124, 125, 132, 133; point of greatest, I. 51, 52; rivets and pins, II. 80-82; rivets, increase modulus, II. 61.

Bent strut or tie, see *Curved*.

Bessemer, see *Table XXIX*, End of Vol. II; steel process, II. 30-32; steel, tests, Table XXXIV, End of Vol. II.

Blast, hot and cold, II. 3, 4.

Blast furnace, description of, II. 2-6.

Blocks, iron or steel, Table XXIX, End of Vol. II.

Blue heat, effect on steel, II. 36; effect on wrought-iron, II. 36.

Boilers, riveting, II. 64, 65.

Bolt, see *Examples*, *Foot-bolt*, *Pin*, *Strap*; calculated, II. 187; heads, standard dimensions for, Table XXXIII, End of Vol II.

Bond, of arches, I. 125, 126; of brick facings, I. 126; use of bond stones, I, 127, 128.

Bow-Herffe, process of, II. 32, 33.

Box Girder, see *Plate Girder*; description of, II. 107.

Braces for wind, II. 173, 174.

Brass, effect of different kinds of strains, II. 48, 49.

Bricks, facings, how bonded, I. 126; weight of, Table V, I. 45; strength of, I. 130; strength of, Table V, I, 45.

Brickwork, how to use bond stones, I, 127, 128; strength of, I. 135.

Bridging, cross-bridging of beams, I, 131.

Bridle Irons, see *Stirrup Irons*.

Bridging of web, II. 175.

Bridle-rod, see *Examples*, *Joint*; in riveting, II. 63.

Brittleness, *Wire*, average weight, formulæ, I. 7.

C

CANNON, tests, Table XXXIV, End of Vol. II.

Cantilever, see *Beams*, *Bending-moment*, *Deflection*, *Inclined Beam*, *Load*, *Overhang*, *Transverse Strength*; bending-moment, Table VII, I. 58, 59; bending-moment, any loading, formula, I, 54; bending-moment, end load, formula, I, 53; bending-moment, uniform load, formula, I, 53; comparative deflection of, different cross-sections, I, 61; comparative deflection of, different lengths, I, 61; comparative deflec-

tion of, different lengths and cross-sections, I, 62; comparative strength, different cross-sections, I, 60; comparative strength, different lengths, I, 60; comparative strength, different lengths and cross-sections, I, 61; comparison with different loads and beams, (strength and stiffness), I, 57; deflection, Table VII, I, 58, 59; deflection, any load, formula, I, 67; deflection, comparative formulæ, I, 61, 62; deflection, different loads, I, 66, 67; deflection, end load, formula, I, 66; deflection, not to crack plaster, formula, I, 57; deflection, uniform load, formula, I, 66; longitudinal shearing, formula, I, 35; rectangular, transverse strength, formulæ, I, 195; shearing, Table VII, I, 58, 59; shearing, vertical, any point, formula, I, 35; strength, comparative formulæ, I, 60, 62, to find required cross-section, I, 54.

Caps of columns, II, 203.

Carbon, in metal, tests, Table XXXIV, End of Vol. II.

Case hardened iron, classified, Table XXIX, End of Vol. II.

Case hardening of wrought-iron, II, 30, 31.

Cast-iron, see Core, Flask, Iron, Malleable Iron, Moulds; analysis of qualities, II, 8; avoid bases of, II, 207; avoid sharp angles, II, 12; big casting bad, II, 11; cheap castings, II, 9; classification of, II, 25-27; classified, Table XXIX, End of Vol. II; columns, circular, strength of, Tables XLV to XLVIII, II, 213-216; column, hollow, example, II, 205, 206; contraction from strain, II, 43-45; danger of sudden cooling, II, 11; densities and weights of, II, 8; dressed and undressed, tests, Table XXXIV, End of Vol. II; ductility of, II, 41; economical castings, II, 10; economical designing, II, 50-54; effect of different kinds of strains, II, 48, 49; elasticity of, II, 41-46; extension from strain, II, 43-45; how tested, II, 39; inspection of, II, 10; malleable, II, 18; malleable, tests, Table XXXIV, End of Vol. II; manufacture of, II, 9-12; nature and use of, chapter on, II, 1-55; nature due to carbon, II, 1; pipes, how tarred, II, 32; produced from pig, II, 9; remelting, defect of, II, 33, 34; résumé of qualities, II, 27; shapes for columns, II, 202, 203; shrinkage of, II, 12; skin of, II, 10, 11; skin protects from rust, II, 31; stronger if thin, II, 11; superseded by wrought, II, 188; testing of, II, 39-42; tests, Table XXXIV, End of Vol. II; thickness, defect of, II, 34, 35; thickness even, II, 10; thickness for columns, II, 50, 51; to prevent unequal shrinkage, II, 11.

Cast steel, see Steel; classified, Table XXIX, End of Vol. II; description of, II, 12; should be annealed, II, 12; tests, Table XXXIV, End of Vol. II.

Cellars, damp-proofing of, I, 87, 119.

Cellar walls, see Examples, Underpinning; and retaining walls, chapter on, I, 96-116; line of pressure, arithmetical theory of, I, 98-104; line of pressure, graphical method, I, 104-107; pressure against, formulæ, I, 101.

Cement, see Mortar; how stains avoided, I, 122; of iron, II, 33; strength of, Tables IV and V, I, 42, 45; weight of, I, 42, 45.

Centimetres, Table LIX, II, 260.

Centre load (beam), see Table VII, I, 59; *see also*, II, 182, 195; *see Examples*, I, 196, 207, 213, 252; compared to other loads, I, 57, 63; arithmetically, reactions, I, 48, 51; arithmetically, point of greatest bending-moment, I, 51, 52, 59; arithmetically, amount of greatest bending-moment, I, 52, 59; arithmetically, required moment of resistance at point of greatest bending-moment, I, 50, 54; arithmetically, bending-moment and required moment of resistance at any point, I, 50, 54, 59; arithmetically, greatest deflection, I, 56, 57, 59, 66, 224; arithmetically, shearing, I, 33-35, 59; arithmetically, strength of, I, 194; graphically, reactions, I, 245; graphically, point of greatest bending-moment, I, 243; graphically, amount of greatest bending-moment, I, 245; graphically, required moment of resistance at point of greatest bending-moment, I, 243, 244; graphically, bending-moment and required moment of resistance at any point, I, 243, 244; graphically, greatest deflection, I, 246-248; graphically, shearing, I, 246.

Chain riveting, II, 62.

Channels, list of steel and iron, Table XXI, End of Vol. I; tests, Table XXXIV, End of Vol. II.

Chimney, see Examples; flue area, formula, I, 138; how to build top, I, 140; size of flue, I, 138; smoke flues, I, 119, 120; strength of, I, 134-141; strength of, formulæ, I, 134; wind pressure on, I, 148-150.

Chords, average strain, formulæ, I, 225-227.

Circles, area of, *see Table LVI*, II, 246-256; circumference of, *see Table LVI*, II, 246-256.

Circular cast-iron columns, strength of, Tables XLV to XLVIII, II, 213-216; surface, wind, effect on, II, 177, 178.

CIR

Circumference of circles, *see Table LVI*, II, 246–256.
Clapp-Griffiths steel process, II, 23.
Classification of iron and steels, Table XXIX, End of Vol. II.
Clay, safe compression on, Table V, I, 45; weight of, Table V, I, 45.
Coal, weight of, Table VI, I, 46.
Cold rolling, effect of, II, 36.
Column, *see Compression*, *Core*, *Examples*; chapter on, II, 200–212; bearing of, II, 203, 204; brackets, design of, II, 203; caps and bases, II, 203, 204; cast-iron, hollow, example, II, 205, 206; cast-iron, shapes of, II, 202, 203; cast-iron, should be drilled, II, 202; comparative formulæ, I, 65; curved, II, 160-164; curved example, II, 162, 163; example (Table XLIX), II, 209, 210; long, formula, I, 24; Phœnix, II, 210, 211; Phœnix, properties of, Table L, II, 218; planing end, II, 202; projections, design of, II, 203; rectangular wrought-iron, not to wrinkle, I, 25-27; short, formula, I, 22; strength of cast-iron circular, Tables XLV to XLVIII, II, 213-216; strength of different cross-sections, I, 65; strength of different lengths, I, 64; thickness for cast-iron, II, 50, 51; to find radius of gyration, II, 200; to prevent water freezing in, II, 202; turning end, II, 202; value of different bearings, II, 202; with continuous girder, II, 211; with overhanging beam, II, 211, 212; wrought-iron (Table XLIX), II, 208-210; wrought-iron, safe loads on, Table XLIX, II, 217; wrought-iron, shapes, II, 206–209; why smooth end stronger, II, 201, 202; zee bars, II, 207-209.
Commercial weight, *see Table LXII*, II, 262.
Comparative formulæ, I, 60–62, 65.
Compression, *see Bearing*, *Column*, *Curved*, *Examples*, *Strut*: curved strut, example, II, 162, 163; example, Table XLIX, II, 209, 210; formula, short columns, I, 22; in curved member, II, 160 164; in riveting, II, 65-70; large block, nature of, II, 183, 184; long columns, formula, I, 24; metals, recent tests, Table XXXIV, End of Vol. II; of rivets, II, 65-70; on rafter, II, 196; on rivets and pins, II, 81, 82; pins, iron and steel, Tables XXXV to XXXVII, End of Vol. II; rivets, iron and steel, Table XXXV, End of Vol. II; safe, on cast-iron, circular columns, Tables XLV to XLVIII, II, 213 216; safe, on clay, earth, gravel, loam, sand, Table V, I, 45; short columns, formula, I, 22; wrought-iron columns, safe loads on, Table XLIX, II, 217.
Concentrated Load, *not central*, *Beam and Cantilever*, *see Table VII*, I, 58, 59; *see also*, II, 87, 88, 182, 195, 212; *see Examples*, I, 196, 249; arithmetically, reactions, beam, I, 47, 51; arithmetically, reactions, cantilever (equals load); arithmetically, point of greatest bending-moment, beam, I, 51, 52, 59; arithmetically, point of greatest bending-moment, cantilever, I, 51, 52, 58; arithmetically, amount of greatest bending-moment, beam, I, 52, 59; arithmetically, amount of greatest bending-moment, cantilever, I, 54, 58; arithmetically, required moment of resistance at point of greatest bending-moment, beam, I, 50, 54; arithmetically, required moment of resistance at point of greatest bending-moment, cantilever, I, 50, 54; arithmetically, bending-moment and required moment of resistance at any point, beam, I, 50, 54, 59; arithmetically, bending-moment and required moment of resistance at any point, cantilever, I, 50, 54, 58; arithmetically, greatest deflection, beam, I, 56, 59, 67; arithmetically, greatest deflection, cantilever, I, 57, 58, 67; arithmetically, shearing, beam, I, 33–35, 59; arithmetically, shearing, cantilever, I, 35, 58; arithmetically, strength of beam, I, 195; arithmetically, strength of cantilever, I, 195; graphically, beam (cantilever similar), reactions, I, 245; graphically, beam (cantilever similar), point of greatest bending-moment, I, 243; graphically, beam (cantilever similar), amount of greatest bending-moment, I, 245; graphically, beam (cantilever similar), required moment of resistance at point of greatest bending-moment, I, 243, 244, graphically, beam (cantilever similar), bending-moment and required moment of resistance at any point, I, 243, 244; graphically, beam (cantilever similar), greatest deflection, I, 246–248; graphically, beam (cantilever similar), shearing, I, 246.
Concrete, *see Cement*: strength of, Table V, I, 45; weight of, Table V, I, 45.
Contents, table of, Beginning of Vols. I and II.
Continuous girders, II, 211; bending-moment, Table XVII, I, 218, 219; deflection, Table XVII, I, 218, 219; explained, I, 217-222; reactions, Table XVII, I, 218, 219.
Contraction, cast-iron under strain, II, 43-45; due to strain, I, 227-237; from strain, formula, I, 227; guarded against, if rollers used, II, 199; metals, recent tests, Table XXXIV, End of Vol. II; of materials, I, 67–69; wrought-iron under strain, II, 43-45.
Copper, effect of different kinds of strains, II, 48, 49.

COP

COR

Corbel, see Examples; strength of, I, 150–153.
Core, supported by chaplets, II, 10; used in casting, II, 9, 10.
Cosecant, natural, Table LI, II, 219, 220.
Cosine, natural, Table LI, II, 219, 220.
Cotangent, natural, Table LI, II, 219, 220.
Cover plate, see Joint; in riveting, II, 63, 64.
Coversine, natural, Table LI, II, 219, 220.
Cracks, see Plastering, Timber, Wood; in wood, I, 187–189.
Cross-bridging, explained, I, 191.
Cross-section, see Examples, Sections; area of, for different sections, Table I, I, 12–21; to find required, of beam or cantilever, I, 54.
Cross shearing, graphical analysis of, I, 246.
Crucible, see Table XXIX, End of Vol. II; description, II, 2, 3; steel, tests, Table XXXIV, End of Vol. II.
Cube roots, Table LIV, II, 228–242.
Cubes, Table LIV, II, 228–242.
Curved, see Examples; strut, II, 160–164; tie, II, 160–164.
Cwts., see Table LXII, II, 262; reduced to kilograms, Table LXI, II, 261.

DAMP-PROOFING of walls and cellars, I, 87, 119.
Decay of wood, I, 190, 191.
Decimal, of inch, expressed in common fraction, Table LVII, II, 257; parts of foot per 1-64 inch, Table LVIII, II, 258, 259.
Decimetres, see Table LXII, II, 262; Table LIX, II, 260.
Deck beams, list of steel and iron, Table XXV, End of Vol. I.
Deflection, see Beams, Cantilever, Centre load, Concentrated load, End load, Examples, Load, Plate girder, Several loads, Uniform and concentrated loads, Uniform load; approximate rules for iron beams and girders, I, 223, 224; beam, any load, formula, I, 67; beam, centre load, formula, I, 66; beam, comparative formulæ, I, 61, 62; beam, formula, I, 228; beam, load at any joint, I, 67; beam, not to crack plaster, formula, I, 56; beam, point of greatest, formula, I, 67; beam, (uneven loads) not to crack plaster, formula, I, 57; beam, uniform load, formula, I, 66; cantilever, any load, formula, I, 67; cantilever, comparative formulæ, I, 61, 62; cantilever, end load, formula, I, 66; cantilever, load at any point, I, 67; cantilever, load at free end, I, 66; cantilever, not to crack plaster, formula, I, 56; cantilever, uniform load, formula, I, 66; comparison of beams and cantilevers with different loads, I, 57; continuous girders, Table XVII, I, 218, 219; due to strain, I, 228–237; graphical analysis of, I, 246–249; graphical method, formulæ, I, 248; iron beams and girders, approximate formulæ, I, 223, 224; iron beams and girders, not to crack plastering, I, 224; not to crack plastering, I, 56, 57; of trussed beams or girders, I, 225, 231–237; on beams and cantilevers, Table VII, I, 58, 59; plate girder, Table XLI, End of Vol. II; plate girder, II, 117; plate girder with diminishing flange, I, 229–231; plate girder, arithmetically, II, 132; plate girder, by table, II, 132; plate girder, formulæ, I, 228, 229; plate girder, graphically, II, 131, 132; plate girder, not to crack plaster, formula, I, 230; truss, formulæ, I, 228, 229; truss, not to crack plaster, formula, I, 230; when to be overlooked (also Table VIII), I, 47, 62; when to calculate, Table IX, I, 63; when to calculate for, in wooden beams, Table VIII, I, 62.
Dekametres, Table LIX, II, 260.
Designing pin joint, II, 83–105.
Diagram, see Graphical analysis, Strain diagram,
Distance of extreme fibres from neutral axis, for different sections, Table I, I, 12–21; *of neutral axis from extreme fibres*, for different sections, Table I, I, 12–21.
Dome, see Examples; calculation of, I, 170–176; metal bands, formula, I, 174.
Drainage of soils, I, 87.
Dram, see Table LXII, II, 262.
Drilling, cast-iron columns, II, 202; for rivets, II, 58, 59.
Ductility, in wrought-iron, cast-iron and steel, II, 41.
Dynamic, see Impact.

END

EARTH, safe compression on, Table V, I, 45; weight of, Table V, I, 45.
Economical, see Beams, Cast-iron, Examples; cast-iron column section, II, 206; designing cast-iron, wrought-iron and steel, II, 50–54.
Elasticity, in wrought-iron, cast-iron and steel, II, 41–46; modulus for plate girders, II, 117; of metals, tests, Table XXXIV, End of Vol. II.
Elastic limit, metals, recent tests, Table XXXIV, End of Vol. II.
Electro-plating, process of, II, 32.
Elongation, see Expansion; metals, recent tests, Table XXXIV, End of Vol. II.
End Load (cantilever), see Table VII, I, 58; *see also*, II, 87, 88, 212; *see*

ENG

Examples, I, 196; compared to other loads, I, 57, 63; arithmetically, reaction (equals load); arithmetically, point of greatest bending-moment, I, 51, 52, 58; arithmetically, amount of greatest bending-moment, I, 53, 58; arithmetically, required moment of resistance at point of greatest bending-moment, I, 50, 54; arithmetically, bending-moment and required moment of resistance at any point, I, 50, 54, 58; arithmetically, greatest deflection, I, 57, 58, 66; arithmetically, shearing, I, 35, 58; arithmetically, strength of, I, 195; graphically, point of greatest bending-moment, I, 243; graphically, amount of greatest bending-moment, I, 245; graphically, required moment of resistance at point of greatest bending-moment, I, 243, 244; graphically, bending moment and required moment of resistance at any point, I, 243, 244; graphically, greatest deflection, I, 246-248; graphically, shearing, I, 246.

English inch reduced to millimetres, Table LX, II, 261; (U. S.) measures and weights, Table LXII, II, 262; weights, reduced to kilograms, Table LXI, II, 261.

Examples, abutment, thrust on, of arch, I, 164; arch abutting on pier, I, 166; arch, four inch rowlock in floor, I, 158; arch, hollow fireproof floor arch, I, 159; arch, in wall carrying piers, I, 161; arch, Spanish floor arch, I, 177; arch, tie-rod in place of abutment, I, 165; arch to opening in wall, I, 156; arch with abutment, in fence or bridge wall, I, 163; base-course, size of, I, 85; beam, comparative deflection, I, 61; beam, comparative strength, I, 60; beam, most economical, I, 63; beam, shearing, any point, I, 34; beam, shearing at support, I, 33; beam, to resist rupture, I, 55; bearing area, II, 198, 199; bearing area of truss, II, 186; bending-moment, amount of greatest, arithmetically, I, 55; bending-moment, amount of greatest, graphically, I, 250; bending-moment, point of greatest, arithmetically, I, 55; bending-moment, point of, graphically, I, 249; bolts in strap, II, 187; cellar wall, deeper than adjoining building, I, 110; cellar wall, to resist earth pressure, I, 108; centre load (beam), I, 196, 207, 213, 252; centre load on wooden girder calculated graphically, I, 252; chimney, size of flue, I, 138; chimney, thickness of wall, I, 138; column, (Table XLIX), II, 209, 210; compression, (Table XLIX), II, 209, 210; compression on rafter, II, 196; compression, short block, I, 22; concentrated load, not central (beam and cantilever), I, 196, 249; concentrated load on wooden girder, calculated graphically, I, 249; corbel of stone, strength of, I, 157; cross-section, required of beam, I, 56; curved strut, II, 162, 163; curved tie, II, 163, 164; deflection, comparative, of beams, I, 61; deflection of trussed wooden beam, I, 236; detailing plate girder, II, 120-135; dome, calculation of, I, 170; dome, metal bands, I, 174; dome, size of dowels, I, 174; economical beam, I, 63; end load, cantilever, I, 196; expansion, iron, I, 67; expansion, stone coping, I, 69; flitch plate girder, I, 207; flue, size of, I, 138; foundations, size of base-courses, I, 85; girder, bearing, I, 216; hollow cast-iron column, II, 205, 206; how to use Table XII, I, 197; how to use Table XIII, I, 198; how to use Table XIV, I, 202; how to use Table XV, I, 203; how to use Table XVI, I, 206; how to use Tables XIX-XXV, I, 237; how to use Table XXXII, II, 48; how to use Tables XXXV-XL, II, 80-83, 90, 93, 94; how to use Tables XLI-XLIII, II, 118-120; how to use Tables XLV-XLVIII, II, 205, 206; how to use Table XLIX, II, 209, 210; inverted arch, size of skew-back, I, 89; inverted arch, strength of, I, 168; inverted arch, thrust on end pier, I, 169; iron, expansion, I, 67; iron rafter, II, 195-197; iron strut, II, 191-193; iron truss, II, 188-199; joint (pin) calculation of, II, 89-104; keyed girder, I, 213; lateral flexure, I, 28; loads on floors, girders, columns and walls, I, 79-82; moment of inertia of cross-section, arithmetically, I, 10; moment of inertia of cross-section, graphically, I, 268; neutral axis of cross-section, arithmetically, I, 8; neutral axis of cross-section, graphically, I, 268; pier, safe load on, I, 24; pier with abutting arches, I, 166; piles as short columns, I, 91; piles, safe load on, I, 93; pins, calculation of, II, 193-197; plate girder, II, 119, 120; reactions, amount of, arithmetically, I, 48, 55; reinforce plate, II, 196-198; reservoir wall, I, 115; riveted butt joint, double plate, II, 76-78; riveted butt joint, single plate, II, 73-76; riveted lap joint, II, 70-73; rivets in reinforce plates, II, 196-198; rivets in struts, II, 192, 193; rivets, shearing on, I, 32; rollers, II, 199; roof truss, II, 141-162; rupture, size of beam to resist, I, 55; safe loads on same timber with different manners of loading, I, 195; several loads (beam and cantilever), I, 48, 55, 255, 257; several loads (two on wooden girder), calculated graphically, I, 255; several

EXA

loads (five) on wooden girder, calculated graphically, I, 257; shearing on rivet, I, 32; shearing on wooden beam any point, I, 34; shearing on wooden beam at support, I, 33; shoe plate, II, 198; stirrup iron, I, 194; stone, expansion, I, 69; stone steeple, wind, II, 176, 177; strap at foot of truss, II, 186, 187; strength, comparative, of beams, I, 60; table of strains, II, 190; tension, I, 30; tie-beam, II, 183; tie-rod, II, 184, 185, 191; transverse strain on rafter, II, 195, 196; truss, II, 141–162; trussed beam of wood, calculation of, I, 231; trussed beam of wood, deflection of, I, 236; uniform and concentrated load (beam and cantilever), I, 237, 264; II, 119–135; uniform and concentrated load on iron beam, calculated graphically, I, 264; uniform load (beam and cantilever), I, 33, 34, 36, 63, 196–203, 262; uniform load on iron beam, calculated graphically, I, 262; wall, thrust against, of barrels, I, 147; wall to country house of brick, I, 143; wall to stage pit, I, 112; wall to warehouses of brick, I, 144; walls and piers of rubble masonry, I, 135; washers, II, 185; wind pressure, II, 189–191; wind pressure on chimney, I, 149; wooden rafter, II, 180, 182; wooden steeple, wind, II, 176, 177; wooden strut, II, 183, 184; wooden truss, II, 179–187; wrinkling, box girder flange, I, 27; wrought iron, compression, II, 209, 210.

Expansion, see Elongation, Examples; due to strain, I, 227–237; from strain, formula, I, 227; guarded against, if rollers used, II, 199; of materials, I, 67–69.

Extension, cast-iron under strain, II, 43–45; wrought-iron under strain, 43–45.

Extreme fibres, distance from neutral axis, for different sections, Table I, I, 12–21; strain on, with transverse strains, I, 49–51.

Eye-bar, arrangement on pin, II, 85–105; standard sizes, II, 53, 54; thickness of head, II, 89–91.

FACTOR-OF-SAFETY, for iron and steel, II, 1.

Fatigue, explained, I, 83; II, 46, 47.

Feet, expressed in metres, Table LIX, II, 260.

Fibres, see Extreme fibres, Transverse strength.

Flange, plate girder, average fibre stress, II, 134; plate girder, lateral flexure, II, 133, 134; plate girder, rivets in, II, 125, 126; plate girder, splicing of, II, 127; plate girder, strength of, Table XLIII, End of Vol. II; plate girder, thickness, II, 125; plate girder, value, II, 120; plate girder, where diminish thickness, II, 125; plate girder, wrinkling, II, 135; reduction of, *see Plate girder;* rivets, plate girder, II, 109; splicing of, plate girder, II, 113, 114; strain, plate girder, II, 108.

Flask, used in casting, II, 9, 10.

Flexure, see Lateral flexure.

Flitch-plate girder, see Examples; calculation of, I, 207–211.

Floor beams and girders, chapter on, I, 180–240.

Floors, weak floors how stiffened, I, 191, 192.

Flue, see Examples; chimney, area required, formula, I, 138.

Flux, used in smelting, II, 5.

Foot, see Shoe; see also, Table LXII, II, 262; decimal of each 1-64 inch, Table LVIII, II, 258, 259; of truss, designed, II, 186, 187.

Foot-bolt of truss, necessity for, II, 186.

Forging, see Welding.

Formula, see List of formulæ, I, ix–xiii and II, xiii, xiv; fundamental, I, 21; meaning of letters, I, 5, 6.

Foundations, see Examples, Cellar walls, Retaining walls, Walls, chapter on, I, 85–95; for machinery to avoid vibrations, I, 131, 132; sand, gravel and clay, I, 87, 91.

Fraction of inch, expressed in decimals, Table LVII, II, 257.

Freezing, to prevent, in columns, II, 202.

Friction, angle of, in soils, Table X, I, 99; of riveted plates, II, 64.

Frost, action of on foundations, I, 95; if wall frozen, I, 119; shape of wall to resist, I, 108.

Furlong, see Table LXII, II, 262.

Furring of walls, I, 120.

GALLERY beam, see Beams, Inclined beam, II, 211, 212.

Gallon, see Table LXII, II, 262.

Galvanizing, process of, II, 32.

Gill, see Table LXII, II, 262.

Girders, see Beams, Continuous girder, Examples, Flitch-plate girder, Floors, Inclined beam, Keyed girder, Plate girder, Trussed beams; and floor beams, chapter on, I, 180–240; continuous, I, 217–222; II, 211; deflection, iron, approximate formulæ, I, 223, 224; inclined, II, 212; iron beams, strength of, Table XV, I, 199; iron, length not to crack plaster, formulæ, I, 224; length of bearing, I, 216, 217; overhanging, II, 211, 212; strength of wooden, Table XIII, End of Vol. I; to ascertain amount of load on, I, 81,

GLA

[illegible] with [illegible] plate, I. 47–51. with keyed bolts, [illegible]

Glass, strength of, Table IV. II. 42; weight of, Table V. I. 45.

Gravel, see Table LXIII. II. 282.

Gravitation, the great basis of mechanical [illegible] centre of, I. 36; [illegible] of, I. 36; [illegible] I. 36; [illegible] I. 36; [illegible]

KEYED girder, see Examples; calculation of, I, 211-216.
Kilograms, reduced to cwts., Table LXI, II, 261; reduced to English weights, Table LXI, II, 261; reduced to pounds, Table LXI, II, 261; reduced to quarters, Table LXI, II, 261; reduced to tons, Table LXI, II, 261.
Kilometres, Table LIX, II, 260.
King post truss, II, 141-145.
Knot, see Table LXII, II, 262.

LAP-JOINT, see Examples, Joint, II, 62, 63.
Land mile, see Table LXII, II, 262.
Lateral flexure, see Examples; beams, formula, I, 204; formula, I, 28; in beams, I, 204-206; in iron beams, I, 239, 240; in top flanges of beams, girders, or trusses, I, 28, 29; plate girders, II, 133, 134; value of *y*, Table XVI, I, 205.
Lattice truss, II, 153.
Lead, effect of different kinds of strains, II, 48, 49.
Letters, in formulæ, meaning of, I, 5, 6.
Lever, law of, I, 48.
Limit of elasticity, see Elasticity, Elastic Limit.
Line of pressure, central in truss members, I, 74.
Lintels, wooden, in walls, I, 125, 126.
Liquid measure, see Table LXII, II, 262.
List of formulæ, I, ix-xii and II, xi, xii; of tables, I, vii, viii and II, vii-ix.
Litre, see Table LXII, II, 262.
Load, see Beams, Cantilever, Centre load (beam); Concentrated load, not central (beam and cantilever); End load (cantilever); Examples; Roof; Safe load; Several loads (beam and cantilever); Uniform and single load (beam and cantilever); Uniform load (beam and cantilever); and wind, one diagram, II, 170; effect of, if varied, II, 48, 49; effect on beam, I, 182-186; how assumed in trusses, II, 136; moving, *see Impact;* moving, I, 84; on truss chord, II, 137, 138; on truss parts, II, 136; rolling, effect of, II, 46-48; safe, on earth, clay, gravel, loam, sand, Table V, I, 45; to ascertain amount of, I, 80-83.
Loading, effect of manner of, Table XLI, End of Vol. II.
Loam, safe compression on, Table V, I, 45; weight of, Table V, I, 45.
Logarithms of numbers, Table LIII, II, 225-227.
Long measure, see Table LXII, II, 262.

MACHINE riveting, II, 59, 61.
Machinery, foundations for, to avoid vibrations, I, 131, 132.
Magnetite, iron ore, II, 2.
Malleable iron, see Cast-iron; classified, Table XXIX, End of Vol. II; description of, II, 18; forged cold, II, 18; resumé of qualities, II, 28; tests, Table XXXIV, End of Vol. II.
Manganese, in steel, tests, Table XXXIV, End of Vol. II.
Masonry, effect of different kinds of strains, II, 48, 49.
Materials, expansion and contraction of, I, 67-69; strength of, chapter on, I, 2-84; strength of, per square inch, Tables IV and V, I, 37-45; weight of, Tables IV, V and VI, I, 37-46; weight of, per cubic foot, Table VI, I, 46.
Measures and weights, English (U. S.) standard, Table LXII, II, 262; liquid, *see Table LXII*, II, 262; long, *see Table LXII*, II, 262; solid, *see Table LXII*, II, 262; square, *see Table LXII*, II, 262.
Member of truss with different strains, II, 139.
Metallic paint, description of, II, 31, 32; how covered, II, 32; to protect iron, II, 31, 32.
Metals, effect of different kinds of strains, II, 48, 49; recent tests, Table XXXIV, End of Vol. II; strength of, Table IV, I, 40-42; weight of, Table IV, I, 40-42.
Metres, expressed in feet and inches, Table LIX, II, 260.
Mile, land and sea, Table LXII, II, 262.
Millimetres, Table LIX, II, 260; reduced to English inch, Table LX, II, 261.
Mitis, classification of, II, 25-27.
Modulus of elasticity for plate girders, II, 117.
Modulus of rupture, see Rupture; metals, recent tests, Table XXXIV, End of Vol. II.
Mortar, see Cement; cement stains, I, 122; quality of, I, 118, 119; setting of, I, 119; strength of, Tables IV and V, I, 42, 45; thickness of joints, I, 117, 118; weight of, Tables IV and V, I, 42, 45; why necessary, I, 117.
Moulds, used in casting, II, 9, 10.
Moving load, see Impact, Load.
Moment, see Bending-moment; defined, I, 48.
Moment of inertia, see Examples; amount of, Table I, I, 12-21; explained, I, 180-182; for different sections, Table I, I, 12-21; of any section found graphically, I, 267-269.
Moment of resistance, see Centre load, Concentrated load, End load, Load, Several loads, Uniform and concentrated loads, Uniform load; amount of, Table I, I, 12-21; curve, II, 124;

MUC

explained, I, 182-186; for different sections, Table I, I, 12-21; graphical analysis of, I, 245; graphical method, formula, I, 245.
Muck bar, description of, II, 13; iron, Table XXIX, End of Vol. II.
Multiplication, rectangular sections, Table LV, 243-245.

N.

n, reasons for its values, II, 201, 202; value in compression formula, Table II, I, 23.
Natural sines, cosines, versines, coversines, tangents, cotangents, secants, cosecants, Table LI, II, 219, 220; sines, tangents and secants, Table LII, II, 221-224.
Neutral Axes, of arch, how found, I, 154, 155.
Neutral axis, see Examples; distance from extreme fibres, for different sections, Table I, I, 12-21; distance from extreme fibres proportioned to their respective resistance to compression and tension, I, 54; of any section, found graphically, I, 266, 269.
Notation, attention called to, II, 140; used in graphical analysis, I, 68-71; used in formulæ, meaning of letters, I, 5, 6.
Notched girder, explained, I, 216.
Numbers, logarithms of, Table LIII, II, 225-227.
Nuts, standard dimensions for, Table XXXIII, End of Vol. II; standard sizes, II, 53.

OPEN *hearth, see Table XXIX*, End of Vol. II; steel process, II, 20, 21.
Ounce, see Table LXII, II, 262.
Overhang to beam, II, 86-88, 211, 212.

PAINT, *see Metallic paint*.
Paper, weight of, Table VI, I, 46.
Parallelogram of forces, applied to graphical analysis of trusses, I, 72; defined, I, 71, 72.
Parts, decimal, of foot, per 1-64 inch, Table LVIII, II, 258, 259.
Pennyweight, see Table LXII, II, 262.
Perch, long measure, Table LXII, II, 262; of stone work, Table LXII, II, 262.
Permanent set, II, 42.
Phœnix columns, Table L, explained, II, 210, 211; properties of, Table L, II, 218.
Phosphorus, in steel, tests, Table XXXIV, End of Vol. II.
Piers, see Examples; and walls, chapter on, I, 117-153; narrow piers in walls,

PLA

I, 148; strength of, I, 134-138; strength of, formulæ, I, 134.
Pig-iron, best combinations, II, 7, 8; brands of, II, 6; chemical process, II, 5; grading of, II, 6, 7; manufacture of, II, 2-6; softeners, II, 7.
Piles, see Examples; base-courses over, I, 94, 95; iron piles, I, 94; long piles, I, 92-95; long, safe load on, formula, I, 93; sand piles, I, 92; sheath-piling, I, 94; short piles as columns, I, 91, 92.
Piping, in rolling iron, II, 13, 14.
Pin, see Bending, Bending-moment, Examples, Rivets; chapter on, II, 56-105; arrangement of eye-bars, II, 85-105; bending-moment, II, 80; bending-moment, graphically, II, 102-104; bending-moment on, II, 81, 82, 94-104; calculation of, *see Examples*; calculation of, II, 83; calculation of, in iron truss, II, 193-195; compression on, II, 81, 82; denial of bending-moment, II, 104, 105; description of, II, 56; designing joint, II, 83-105; iron, compression on, Tables XXXV-XXXVII, End of Vol. II; iron, shearing, bending and tension of, Tables XXXVIII-XL, End of Vol. II; reducing strains, II, 84, 85; shearing on, II, 81, 83, 93, 94; standard sizes, II, 53, 54; steel, compression on, Tables XXXV-XXXVII, End of Vol. II; steel, shearing, bending and tension of, Tables XXXVIII-XL, End of Vol. II; stress for steel, II, 70; stress for wrought-iron, II, 70.
Pin-joint, see Examples; calculation of, example, II, 88-104.
Pint, see Table LXII, II, 262.
Pitch, greatest for rivets, II, 65; least for rivets, II, 65; of rivets, II, 57, 58.
Planing, of column ends, II, 202.
Plastering, plate girder, not to crack, Table XLI, End of Vol. II; deflection, not to crack, I, 56, 57.
Plate, see Reinforce plates, Plate girder; cover in riveting, II, 63, 64; crushed by rivet, II, 65-70; friction from riveting, II, 64; sheared by rivet, II, 65-70; tests, Table XXXIV, End of Vol. II; torn by rivet, II, 65-70; weakened by rivets, II, 62; wrought-iron not to wrinkle, I, 25-27.
Plate girder, see Beam, Elasticity, Examples, Flitch Plate Girder, Friction, Girder, Modulus of Elasticity, Plate; chapter on, II, 106-135; angle bars, strength of, Table XLII, End of Vol. II; angles one piece, II, 114, 115; approximate formula, I, 65; approximate thickness of flange, I, 65; bending-moment, II, 124, 125, 132, 133; box girder, description of, II, 107; calculation of, II, 108-119; cambering of, II, 107; deflection, Table XLI, End of Vol. II; deflection,

formulæ, I, 228, 229; deflection of, II, 117; deflection of, arithmetically, II, 132; deflection of, by table, II, 132; deflection of, graphically, II, 131, 132; description of, II, 106; detailing of, *see Examples;* flange, average fibre stress, II, 134; flange rivets, II, 125, 126; flange strain, II, 108; flange, strength of, Table XLIII, End of Vol. II; flange thickness, II, 125; flange thickness, graphical method, formula, I, 270; lateral flexure, II, 133, 134; length not to crack plaster, formula, I, 230; length of bearing, II, 131; manner of loading, Table XLI, End of Vol. II; moment of resistance II, 124; not to wrinkle, I, 25–27; pitch of rivets, II, 65; plastering, not to crack, Table XLI, End of Vol. II; plates not full length, II, 107; practicable sizes, II, 51, 52; reduction of flanges, graphical method, I, 269–271; rivets in flange, II, 109, 110; rivets in stiffners, II, 128, 129; rivets in web, II, 108, 109; rivets in web angle, II, 129–131; splicing flange, II, 113, 114, 127; splicing plates, II, 113; splicing web, II, 113; stiffners to web, II, 127, 128; strength of, Table XLI–XLIII, End of Vol. II; tendency to slide, II, 112, 113; uneven angles, II, 107; value of angle irons, formula, I, 270; value of angles, II, 120; value of flange, II, 120; web buckling, II, 115; web rivets, II, 128; web, splicing of, II, 129; web stiffeners, II, 115–117; web, strength of, Table XLI, End of Vol. II; web thickness, II, 115, 127; web, uneven thickness, II, 107; where diminish flange thickness, II, 125; with diminishing flange, deflection of, I, 229–231; wrinkling of flange, II, 135.

Pole, see Graphical analysis, see also Table LXII, II, 262.

Pounds, see Table LXII, II, 262; reduced to kilograms, Table LXI, II, 261.

Pressure, see Line of pressure, Reservoir walls, Retaining walls, Walls.

Puddling, of wrought-iron, II, 13.

Puddling-ball of iron, Table XXIX, End of Vol. II.

Punching for rivets, II, 58, 59; steel cold, II, 37.

Quart, see Table LXII, II, 262.

Quartering, see Wood.

Quarters, see Table LXII, II, 262; reduced to kilograms, Table LXI, II, 261.

Radius, see Square of radius of gyration.

Radius of gyration, Phœnix columns, Table L, II, 218.

Rafter, analysis of load on, II, 182, 183, 195, 196; compression on, II, 196; iron, *see Examples;* iron, calculation of, II, 195–198; transverse and longitudinal strains in, I, 74; transverse strain on, II, 137, 138, 182, 183, 195, 196; wooden, *see Examples;* wooden, calculation of, II, 180–183.

Reaction, see Centre load, Concentrated load, End load, Examples, Load, Several loads, Uniform and concentrated loads, Uniform load; amount of (at both supports), I, 47, 51; amount of left (single load) formula, I, 47; amount of left (two loads) formula, I, 47; amount of right (single load) formula, I, 47; amount of right (two loads) formula, I, 47; at truss joints, II, 137, 138; beam with overhang, II, 86–88; continuous girders, Table XVII, I, 218, 219; from overhanging beam, II, 211, 212; graphical analysis of, I, 245; of continuous girder, II, 211; of wind on roofs, II, 165–174; shearing at left, formula, I, 33; shearing at right, formula, I, 33; truss ends, II, 138; trussed beams, Table XVIII, I, 220, 221.

Reaming, for rivets, II, 59.

Reciprocals of numbers, Table LIV, II, 228–242.

Rectangular sections, multiplication, Table LV, II, 243–245.

Reduction of flanges, see Plate Girder; plate girder, II, 125.

Reinforce plate, see Examples, Friction, Plate; rivets in, II, 196–198; to rafter joints, II, 196–198.

Repeated strains, effect of, II, 46–48.

Reservoir, shape of walls, I, 97; strength of walls, I, 103.

Reservoir walls, see Examples; pressure against, formulæ, I, 103.

Resistance, see Moment of resistance.

Retaining walls and cellar walls, chapter on, I, 96–116; buttressed, I, 107, 108; economical section, I, 96, 97; line of pressure, arithmetical theory of, I, 98–104; line of pressure, graphical method, I, 104–107; pressure against, formulæ, I, 98, 100, 103–105; value of p, Table XI, I, 101.

Reversed strains, effect of, II, 46–48.

Rivets, see Examples, Bending, Bending-moment, Plate girder, Riveting; chapter on, II, 56–105; bending-moment, II, 80; bending-moment, increase modulus, II, 61; bending-moment on, II, 81, 82; breaking of, II, 65–70; butt-joint, II, 63; chain, II, 62; compression of, II, 65–70; compression on, II, 81, 82; denial of bending-moment, II, 104, 105; description of, II, 56, 57; drilling for, II, 59;

example butt-joint, double plate, II, 76-78; example butt-joint, single plate, II, 73-76; example lap-joint, II, 70-73; greatest pitch, II, 65; hand, II, 60, 61; head of, II, 57; how joint yields, II, 65-70; in reinforce plates, *see Examples;* in reinforce plates, II, 196-198; in strut ends, II, 192, 193; in struts, *see Examples;* iron or steel, shearing, bending and tension of, Table XXXVIII, End of Vol. II; iron, safe compression on, Table XXXV, End of Vol. II; lap-joints, II, 62, 63; least pitch, II, 65; machine, II, 59, 61; number in flange leg, plate girders, II, 109; number in web leg, plate girders, II, 108, 109; pitch of, II, 57, 58; plate girders, location in flange, II, 109-112; punching for, II, 58, 59; reaming, II, 59; shearing of, II, 65-70; shearing on, II, 81, 83; shearing out of plate, II, 65-70; size to use, II, 65; staggered, II, 62; steel, safe compression on, Table XXXV, End of Vol. II; steel undesirable, II, 61; stress for steel, II, 70; stress for wrought-iron, II, 70; tail of, II, 56, 57; weakening of plate, II, 62; zigzag, II, 62.

Riveted girder, see Plate girder.

Riveting, see Rivets; cover plates, II, 63, 64; friction of plates, II, 64; of boilers, II, 64, 65; to distinguish, II, 60, 61.

Robert converter, claims made for, II, 54, 55.

Rock, foundations of, I, 90, 91.

Rocker, see Rollers.

Rod, see Bending, Tie-rod, see also Table LXII, II, 262; tension on, II, 83.

Rolled iron, see Wrought-iron.

Rollers, see Examples; effect of wind, II, 169, 170; example of, II, 199; expansion or contraction guarded against, II, 199; formula for, II, 199; how used, II, 199; secure truss sideways, II, 199; slip-joint in roof, when necessary, II, 199; three diagrams needed, II, 139; to truss, II, 169, 170; under truss, when used, II, 199; used for rolling iron, II, 14, 15; when needed, II, 139.

Rolling load, effect of, II, 46-48.

Rood, see Table LXII, II, 262.

Roof, see Abutment, Truss, Wind; allowance for snow, II, 140; circular, wind effect, II, 177, 178; contraction of truss, II, 139; expansion of truss, II, 139; horizontal tension, II, 165; horizontal thrust, II, 165; load on, II, 179, 180, 189; overturned by wind, II, 173-177; snow on, general rule, II, 138; table of strains, II, 190; thrust on abutment, II, 164, 165; weight of, II, 179, 180, 189; when rollers needed, II, 139; wind load line, II, 165-174;

wind on broken, II, 172, 173; wind on, general rule, II, 138; wind pressure on, II, 178, 189-191; wind reactions, II, 165-174.

Roof truss, examples of, II, 141-162; strain reversed, II, 138, 139; superfluous members, II, 143, 144.

Ropes, strength of, Table IV, I, 42.

Rubble work, see Stonework; dependence on mortar, I, 119.

Rupture, see Examples; formula for, in testing, II, 38; rivets, modulus larger, II, 61; when to calculate, Table IX, I, 63.

Rust, protection from, II, 31, 33.

SAFE loads, see Examples.

Sand, safe compression on, Table V, I, 45; weight of, Table V, I, 45.

Scissors truss, II, 157.

Screw, see Thread; standard dimensions for, Table XXXIII, End of Vol. II.

Screw ends, see Upset.

Sea mile, see Table LXII, II, 262.

Seasoning of wood, I, 189.

Secants, natural, Table LI, II, 219, 220; natural, Table LII, II, 221-224.

Sections, see Cross-section; rectangular, multiplication, Table LV, II, 243-245.

Separators, used between iron beams, I, 206.

Set, increasing, II, 46; permanent, II, 42.

Several loads (beam and cantilever), see, II, 87, 88, 182, 195, 212; *see Examples,* I, 48, 55, 255, 257; arithmetically, reactions, beam, I, 47, 48, 51; arithmetically, reactions, cantilever (equals sum of loads); arithmetically, point of greatest bending-moment, beam, I, 51, 52; arithmetically, point of greatest bending-moment, cantilever, I, 51, 52; amount of greatest bending-moment, beam, I, 52, 53; amount of greatest bending-moment, cantilever, I, 54; required moment of resistance at point of greatest bending-moment, beam, I, 50, 54; required moment of resistance at point of greatest bending-moment, cantilever, I, 50, 54; arithmetically, bending-moment and required moment of resistance at any point, beam, I, 50, 54; arithmetically, bending-moment and required moment of resistance at any point, cantilever, I, 50, 54; arithmetically, greatest deflection, beam, I, 56, 67; arithmetically, greatest deflection, cantilever, I, 57, 67; arithmetically, shearing, beam, I, 33-35; arithmetically, shearing, cantilever, I, 35; graphically, beam (cantilever similar), reactions, I, 245;

graphically, beam, (cantilever similar), point of greatest bending-moment, I, 243, 245; graphically, beam (cantilever similar), amount of greatest bending-moment, I, 245; graphically, beam (cantilever similar), required moment of resistance at point of greatest bending-moment, I, 243-245; graphically, beam (cantilever similar), bending-moment and required moment of resistance at any point, I, 243-245; graphically, beam (cantilever similar), greatest deflection, I, 246-248; graphically, beam (cantilever similar), shearing, I, 246.

Shapes, cast-iron columns, II, 202, 203.

Shearing, see Beams, Cantilever, Centre load, Concentrated load, Cross shearing, End load, Examples, Load, Several loads, Uniform and concentrated loads, Uniform load; across grain, formula, I, 32; across grain, graphical method, formula, I, 246; along grain, formula, I, 32; at any point of beam, I, 34; at left reaction, formula, I, 33; at reactions, I, 33; beam, vertical, any point, formula, I, 34; cantilever, vertical, any point, formula, I, 35; formulæ, I, 32, 33; horizontal in beams, I, 35; longitudinal, formula, I, 35; metals, recent tests, Table XXXIV, End of Vol II; of rivet, II, 65-70; on beams and cantilevers, Table VII, I, 58, 59; on pin, II, 93, 94; on rivets and pins, II, 81, 83; on rivets, pins and rods, Tables XXXVIII-XL, End of Vol. II; theory of, I, 31, 32.

Sheath-piling, explained, I, 94.

Shoe, preferably wrought-iron, II, 188; to iron truss, II, 198, 199.

Shoe plate, see Examples.

Shrinkage, see Cast-iron; of timber, I, 122; of walls, I, 121; of wood, I, 190.

Silicon, in steel, tests, Table XXXIV, End of Vol. II.

Sills, bedded hollow, I, 123.

Sines, natural, Table LI. II, 219, 220; natural, Table LII, II, 221-224.

Sizes of eye-bars, II, 53, 54; of flats, II, 52; of iron plates, II, 51, 52; of pins, II, 53, 54; of rivets, II, 65; of rounds, II, 52; of squares, II, 52.

Skin of cast-iron, II, 10, 11.

Slanting, see Inclined beam; beam, II, 212; surface, wind effect, II, 177, 178.

Slate, effect of different kinds of strains, II, 48, 49.

Sleeve nuts, II, 83, 84.

Sliding, in plate girders, II, 112, 113.

Slip-joints, how built, I, 123, 124; in roof, if rollers used, II, 199; where necessary, I, 95, 121.

Snow, allowance for, on roof, II, 140; general rule, II, 138; weight of, Table VI, I, 46; when not considered, I, 83.

Soils, angle of friction, Table X, I, 99; drainage of, I, 87; how to test, I, 86, 87; nature of, I, 85.

Solid measure, see Table LXII, II, 262.

Spanish tile, floor arch, I, 176-179.

Splicing flange, plate girder, II, 113, 114, 127; plate girders, II, 113; web, plate girder, II, 113, 129.

Square measure, see Table LXII, II, 262.

Square of radius of gyration, Table XLIX, II, 208, 209, 217; amount of, Table I, I, 12-21; for different sections, Table I, I, 12-21; how found, II, 209; of columns, II, 200; when constant, II, 191, 209.

Square roots, Table LIV, II, 228-242.

Squares, Table LIV, II, 228-242; area of, *see Table LVI*, II, 246-256.

Staggered riveting, II, 62.

Steel, see Beams, Cast-steel, Plate girder, Pins, Rivets; analysis varies, II, 8; annealing of, II, 30, 37; avoid great lengths, II, 51; bars stronger than plates, II, 37; beams and girders, deflection, formulæ, I, 223, 224; bending test, II, 40, 41; Bessemer, II, 20-22; Bessemer, tests, Table XXXIV, End of Vol. II; best process, II, 23; can be cast, forged, hardened and tempered, II, 19; castings of, II, 12; cast, tests, Table XXXIV, End of Vol. II; characteristics of, II, 19; Clapp-Griffiths, II, 23; classification of, II, 25-27; classified, Table XXIX, End of Vol. II; cold punching, II, 37; cost of fitting, II, 51, 52; crucible, tests, Table XXXIV, End of Vol. II; direct from ore, II, 20; distinguished from iron by carbon, II, 1; distinguished from wrought-iron, II, 39; ductility of, II, 41; economical designing, II, 50-54; effect of blue heat, II, 36; effect of cold rolling, II, 36; effect of different kinds of strains, II, 48, 49; effect varies with carbon, II, 37; elasticity of, II, 41-46; factor-of-safety, II, 1; fracture, II, 39, 40; hard, description of, II, 19; hardness great, II, 19; ingot, *see Table XXIX*, End of Vol. II; ingot, II, 22, 23; kinds made by mills, II, 24, 25; limit for plates, II, 51, 52; list of beams, channels, angles, tees, deck beams, zees, Tables XX-XXV, End of Vol. I; manufacture of, II, 18-25; mild, description of, II, 19; nature and use of, chapter on, II, 1-55; new deep beam, II, 135; not desirable for rivets, II, 61; open hearth, II, 20, 21; open hearth, tests, Table, XXXIV, End of Vol. II; pins, compression on, Tables XXXV-XXXVII, End of Vol. II; pins, shearing, bending and tension of, Table XXXVIII-XL, End of Vol. II; principal processes, II, 20; pro-

STE

[illegible] from page [illegible] to [illegible]. [illegible] Table XXX. End of Vol. II; [illegible] Table XXX. End of Vol. II; [illegible] Table XXX. End of Vol. II; [illegible] [illegible] strength [illegible] of [illegible] [illegible] of [illegible] [illegible] of [illegible]

sions for, Table XXXIII, End of Vol. II; standard sizes, II, 53.
Thrust, of roof on abutment, II, 164, 165.
Tie, curved, II, 160-164; curved, example, II, 163, 164; vertical in steeple, II, 177.
Tie-beam, see Curved, Examples; calculation of, II, 183; load on, II, 137, 138.
Tie-rods, see Examples: calculations of, II, 184, 185, 191; iron or steel, tension of, Tables XXXVIII-XL, End of Vol. II; tests, Table XXXIV, End of Vol. II; to iron floor beams, I, 206, 207; upset ends, II, 185.
Tiles, use in floor arches, I, 176-179; weight of, Table VI, I, 46.
Timber, see Wood; effect of different kinds of strains, II, 48, 49; halving of, II, 185; how to tell if sound, I, 188, 189; shrinkage of, I, 122.
Time, effect of, II, 46.
Tons, see Table LXII, II, 262; reduced to kilograms, Table LXI, II, 261.
Tower, strength of, I, 134-138; walls of, I, 132; walls, strength, formulæ, I, 134.
Transverse strain, see Beams, Cantilever, Graphical analysis, Rafter; added to longitudinal strains in rafters, I, 74; comparison of beams and cantilevers with different loads (strength and stiffness) I, 57; compression on upper fibres, I, 50; graphical analysis of, chapter on, I, 241-271; inclined beam, II, 182, 183, 195, 196; method of rupture, I, 47; moment of resistance of any fibre, I, 51; on rafter, *see Examples;* on rafter, II, 182, 183, 195, 196; on truss members, II, 136-138; simple rules for calculating arithmetically, I, 51-54; strength uniform cross-section, I, 50; tension on lower fibres, I, 50; when to overlook deflection in wooden beams, Table VIII, I, 62; wooden beams, one inch thick, strength of (uniform load) Table IX, I, 63.
Transverse strength, beams, rectangular, formulæ, I, 194, 195; cantilevers, rectangular, formulæ, I, 195; lower fibres, formula, I, 50; uniform cross-section, formula, I, 50; upper fibres, formula, I, 50.
Trapezoid, to find centre of gravity, I, 101.
Troy, see Table LXII, II, 262.
Truss, see Abutment, Rafter, Roller, Roof, Wind; adjustment of, II, 83, 84; allowance for snow, II, 140; analyzed graphically, I, 72-74; arched, II, 154; bearing area, II, 186, 198, 199; chord loaded, II, 137, 138; contraction of, II, 139; deflection, formulæ, I, 228, 229; designing of, II, 136; design permissible, II, 188;

distances between, II, 179; end reactions, II, 138; examples of, II, 141-162; expansion of, II, 139; foot of, design, II, 186, 187; foot-bolt, II, 186; hammer-beam, II, 160; horizontal tension, II, 165; how general rules apply, II, 136; Howe, II, 152; iron, *see Examples;* iron, detailing members and joints, II, 188-199; iron rafter, II, 195-198; iron shoe, II, 198, 199; iron strut, II, 191, 192; joint reactions, II, 137, 138; joints, how assumed, II, 136; king post, II, 141-145; latticed, II, 153, length not to crack plaster, formula, I, 230; load, how assumed, II, 136; load on parts, II, 136; pins, calculation of, II, 193-195; pitch of rivets, II, 65; reversed strains, II, 138, 139; rivets in struts, II, 192, 193; rollers under end, II, 139; scissors, II, 157; snow, general rule, II, 138; steps to design, II, 136; strains in, analyzed, chapter on, II, 136-178 strains in chords, I, 225-227; strains in members, II, 136; strapped foot calculated, II, 186, 187; superfluous members, II, 143, 144; tie-beam, II, 183; tie-rod, II, 184, 185, 191; transverse strain on members, II, 136-138; Warren, II, 153; Whipple, II, 153; wind and load, one diagram, II, 170; wind diagram, II, 138, 139; wind, general rule, II, 138; with abutment, II, 164, 165; with different strains, II, 139; with rollers, II, 169, 170; without abutment, II, 165; wood and iron, chapter on, II, 179-199; wooden, *see Examples;* wooden, detailing members and joints, II, 180-187; wooden, rafter, II, 180-183; wooden, strut, II, 183, 184.
Trussed beams, see Examples; or girders, I, 220-223; or girders, calculation of, I, 231-237; or girders, deflection of, I, 225, 231-237; strains in, Table XVIII, I, 220, 221.
Tubular beam, not to wrinkle, I, 25-27.
Turning of column ends, II, 202.

Underpinning of walls, I, 129, 130; *uniform and concentrated load (beam and cantilever), see* II, 87, 88, 182, 195, 212; *see Examples*, I, 237, 264; II, 119-135; arithmetically, reactions, beam, I, 51; arithmetically, reactions, cantilever (equals sum of loads); arithmetically, point of greatest bending-moment, beam, I, 51, 52; II, 132; arithmetically, point of greatest bending-moment, cantilever, I, 51, 52; arithmetically, amount of greatest bending-moment, beam, I, 52, 53; II, 132; arithmetically, amount of greatest bending-moment, cantilever, I, 53, 54; arithmetically, required mo-

ment of resistance at point of greatest bending-moment, beam, I, 50, 54; II, 132; arithmetically, required moment of resistance at point of greatest bending-moment, cantilever, I, 50, 54; arithmetically, bending-moment and required moment of resistance at any point, beam, I, 50, 54; II, 133; arithmetically, bending-moment and required moment of resistance at any point, cantilever, I, 50, 54; arithmetically, greatest deflection, beam, I, 56, 66, 67; II, 117, 132; arithmetically, greatest deflection, cantilever, I, 57, 66, 67; II, 117; arithmetically, shearing, beam, I, 33-35; arithmetically, shearing, cantilever, I, 35; graphically, beam (cantilever similar) reactions, I, 245; graphically, beam (cantilever similar), point of greatest bending-moment, I, 243; II, 124; graphically, beam (cantilever similar), amount of greatest bending-moment, I, 245; II, 124; graphically, beam (cantilever similar), required moment of resistance at point of greatest bending-moment, I, 243, 244; II, 124; graphically, beam (cantilever similar), bending-moment and required moment of resistance at any point, I, 243, 244; II, 124; graphically, beam (cantilever similar), greatest deflection, I, 246-248; II, 117, 131; graphically, beam (cantilever similar), shearing, I, 246; II, 117.

Uniform load (beam and cantilever) see Tables VII, VIII, IX, XII, XIII, XIV, XV, I, 58, 59, 62, 63, 199, and End of Vol. I; *see also* II, 87, 88, 119-135, 182, 195, 212; *see Examples*, I, 33, 34, 36, 63, 196-203, 262; compared to other loads, I, 57, 195, 196; arithmetically, reactions, beam, I, 48, 51; arithmetically, reactions, cantilever (equals load), arithmetically, point of greatest bending-moment, beam, I, 51, 52, 59; arithmetically, point of greatest bending-moment, cantilever, I, 51, 52, 58; arithmetically, amount of greatest bending-moment, beam, I, 52, 59; arithmetically, amount of greatest bending-moment, cantilever, I, 53, 58; arithmetically, required moment of resistance at point of greatest bending-moment, beam, I, 50, 54; arithmetically, required moment of resistance at point of greatest bending-moment, cantilever, I, 50, 54; arithmetically, bending-moment and required moment of resistance at any point, beam, I, 50, 54, 59; arithmetically, bending-moment and required moment of resistance at any point, cantilever, I, 50, 54, 58; arithmetically, greatest deflection, beam, I, 56, 57, 59, 66, 223; arithmetically, greatest deflection, cantilever, I, 57, 58, 66; arithmetically, shearing, beam, I, 33-35, 59; arithmetically, shearing, cantilever, I, 35, 58; arithmetically, strength of beam, I, 194; arithmetically, strength of cantilever, I, 195; graphically, beam (cantilever similar), reactions, I, 245; graphically, beam (cantilever similar), point of greatest bending-moment, I, 243; graphically, beam (cantilever similar), amount of greatest bending-moment, I, 245; graphically, beam (cantilever similar), required moment of resistance at point of greatest bending-moment, I, 243, 244; graphically, beam (cantilever similar), bending-moment and required moment of resistance at any point, I, 243, 244; graphically, beam (cantilever similar), greatest deflection, I, 246-248; graphically, beam (cantilever similar), shearing, I, 246.

Upset ends, to tie-rods, II, 185; screw ends, explained, II, 52; screw ends, when economical, II, 52, 53.

Upward pressure, overhanging beam, II, 211, 212.

U. S. Standard, see English.

VARIABLE strains, effect of, II, 49.

Vaulted arch, how calculated, I, 179.

Versines, natural, Table LI, II, 219, 220.

Vertical tie, steeple, II, 177.

Vibrations, from machinery, to avoid, I, 131, 132.

WALLS, see Cellar walls, Examples, Retaining walls, Underpinning; and piers, chapter on, I, 117-153; buttressed, average weight, formula, I, 107; connections between old and new, I, 124; corbels in, I, 150-153; damp-proofing of, I, 87, 119; furring of, I, 120; hollow, I, 128, 129; how underpinned, I, 129, 130; of towers, I, 132; pressure from barrels, formula, I, 147; shape for reservoirs, I, 97; strength of, I, 133-146; thickness of, Rondelet's rule, I, 133; thickness required, formulæ, I, 141; thrust of barrels, I, 146-148; timber in, I, 124-126; to ascertain amount of load on, I, 81-83; to resist bulging, I, 141-146; tower and chimney, strength, formulæ, I, 134; wind pressure on, I, 148, 149, 178.

Warren truss, II, 153.

Washers, see Examples: bearing area, II, 184; size of, II, 185.

Water, weight of, Table VI, I, 46; weight of one gallon, *see Table LXII*, II, 262.

Web angle, plate girder, rivets in, II, 129-131; buckling, II, 115; plate girder, rivets, II, 128; plate girder,

splicing of, II, 129; plate girder, stiffeners to, II, 127, 128; plate girder, strength of, Table XLI, End of Vol. II; plate girder, thickness, II, 127; rivets, plate girder, II, 108, 109; splicing, in plate girders, II, 113; stiffeners to, II, 115–117; thickness in plate girder, II, 115.

Weight, Avoirdupois and Troy, *see Table LXII*, II, 262; of iron, per yard, II, 18; materials, Tables IV–VI, I, 37–46; per cubic foot of materials, Table VI, I, 46.

Weights, English, reduced to kilograms, Table LXI, II, 261.

Weights and measures, English (U. S.) standard, Table LXII, II, 262.

Welded steel and iron, classified, Table XXIX, End of Vol. II.

Welding, description of, II, 18; iron, II, 29, 30; joints in steel, II, 37; malleable cast-iron, II, 18.

Whipple truss, II, 153.

Wind, amount of pressure on roofs, II, 139, 140; and load, one diagram, II, 170; braces, II, 173, 174; broken roof, II, 172, 173; circular surface, II, 177, 178; effect different, if rollers used; II, 139; effect if rollers, II, 169, 170; effect on growing timber, I, 188; effect on steeple, II, 174–177; general rule, II, 138; load line on roof, II, 165–174; on roofs, approximate rule, II, 140; pressure, *see Examples;* pressure, depends on inclination, II, 139, 140; pressure, how assumed, I, 83; pressure, how it acts, II, 139, 140; pressure on chimneys, I, 148–150; pressure on octagonal roof, II, 178; pressure on walls, I, 148, 149; II, 178; reactions, unequal, II, 165–174; reverses strains, II, 138, 139; slanting surface, II, 177, 178; steeple, pressure on, diagonally, II, 178; strain diagram, II, 138, 139; to overturn roof, II, 173–177; wall, pressure on, I, 148, 149; II, 178.

Wire, steel, tests, Table XXXIV, End of Vol. II; tinned, tests, Table XXXIV, End of Vol. II.

Wood, *see Cracks, Timber;* compression of, per square inch, Table IV, I, 37–39; decay of, I, 190, 191; formation of, I, 186, 187; heart wood, I, 187, 188; knots in, I, 188; medullary rays, I, 187; methods of seasoning, I, 189; quartering explained, I, 190; seasoning cracks, II, 187–189; shrinkage of, I, 190; strength of, per square inch, Table IV, I, 37–39; weight of, Table IV, I, 37–39; wind shakes, I, 188.

Wooden, *see Truss.*

Wooden steeple, wind on, example, II, 174–176.

Wrinkling, *see Examples;* formula, I, 25; plate girder flange, II, 135; safe breadth of plates to resist, Table III, I, 26; strain, *see Table III*, I, 26;

strain, breadth of plate required, I, 25–27; strain, thickness of plate required, I, 25–27.

Wrought-iron, *see Column, Iron, Plate;* analysis of, II, 8; avoid great lengths, II, 51; bar iron strongest, II, 35; bases preferable, II, 207; bending test, II, 40, 41; case hardening, II, 30, 31; classification of, II, 25–27; classified, Table XXIX, End of Vol. II; columns, Table XLIX, explained, II, 208, 209; columns, safe loads on, Table XLIX, II, 217; columns, shapes, II, 206–209; compression, example, II, 209, 210; contraction from strain, II, 43–45; cost of fitting, II, 51, 52; distinguished from steel, II, 39; double refined, II, 16; ductility of, II, 41; economical designing, II, 50–54; effect of blue heat, II, 36; effect of cold rolling, II, 36; effect of different kinds of strains, II, 48, 49; effect of rolling, II, 17; elasticity of, II, 41–46; extension from strain, II, 43–45; extra refined, II, 15, 16; fracture, II, 39, 40; free from carbon, II, 1; heat before welding, II, 35; limit for plates, II, 51, 52; manufacture of, II, 12–18; method of rolling, II, 14, 15; muck bar, II, 13; nature and use of, chapter on, II, 1–55; nature of, II, 18; Phœnix columns, Table L, explained, II, 210, 211; Phœnix columns, properties of, Table L, II, 218; pins, compression on, Tables XXXV—XXXVII, End of Vol. II; pins, shearing, bending and tension of, Tables XXXVIII—XL, end of Vol. II; piling of, in rolling, II, 13, 14; produced from pig, II, 9; puddling of, II, 13; refining of, II, 12–16; re-rolling, defect of, II, 35; résumé of qualities, II, 28; result of re-rolling, II, 16, 17; rivets, safe compression on, Table XXXV, End of Vol. II; rivets, shearing, bending, and tension of, Table XXXVIII, End of Vol. II; safe testing strain, II, 41, 42; single refined, II, 15; sizes of flats, II, 52; sizes of plates, II, 51, 52; sizes of rounds, II, 52; sizes of squares, II, 52; stress for rivets and pins, II, 70; testing of, II, 39–42; tests, Table XXXIV, End of Vol. II; tie-rods, tension of, Tables XXXVIII—XL, End of Vol. II; use of old iron, II, 17; weight per yard, II, 18; zee bar columns, II, 207–209.

Wrought steel, tests, Table XXXIV, End of Vol. II.

YARD, *see Table LXII*, II, 262.

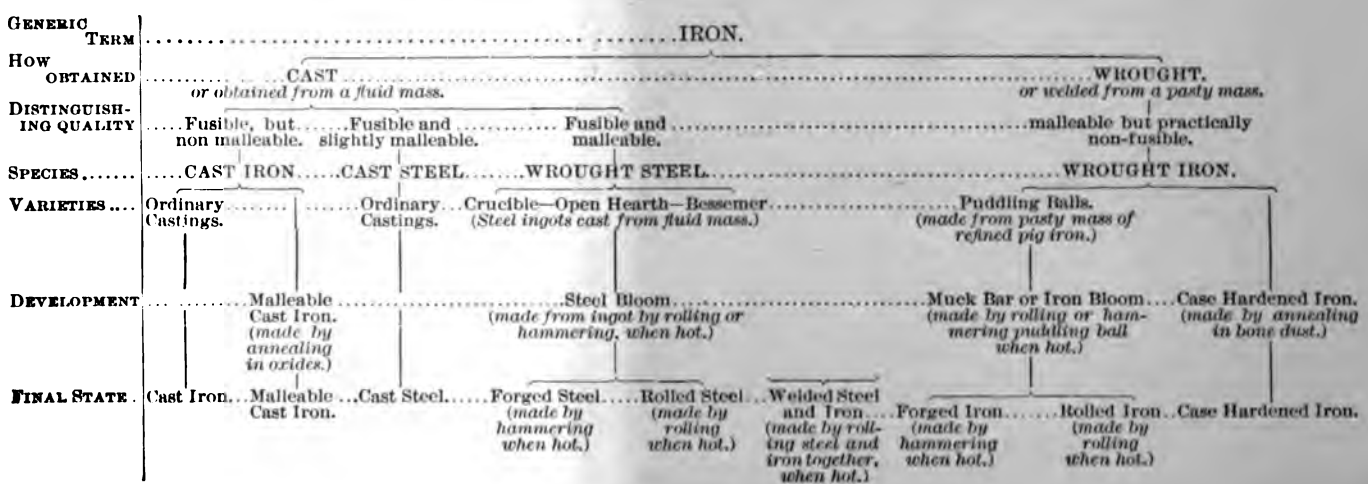
ZEE bars, columns of, II, 207–209.

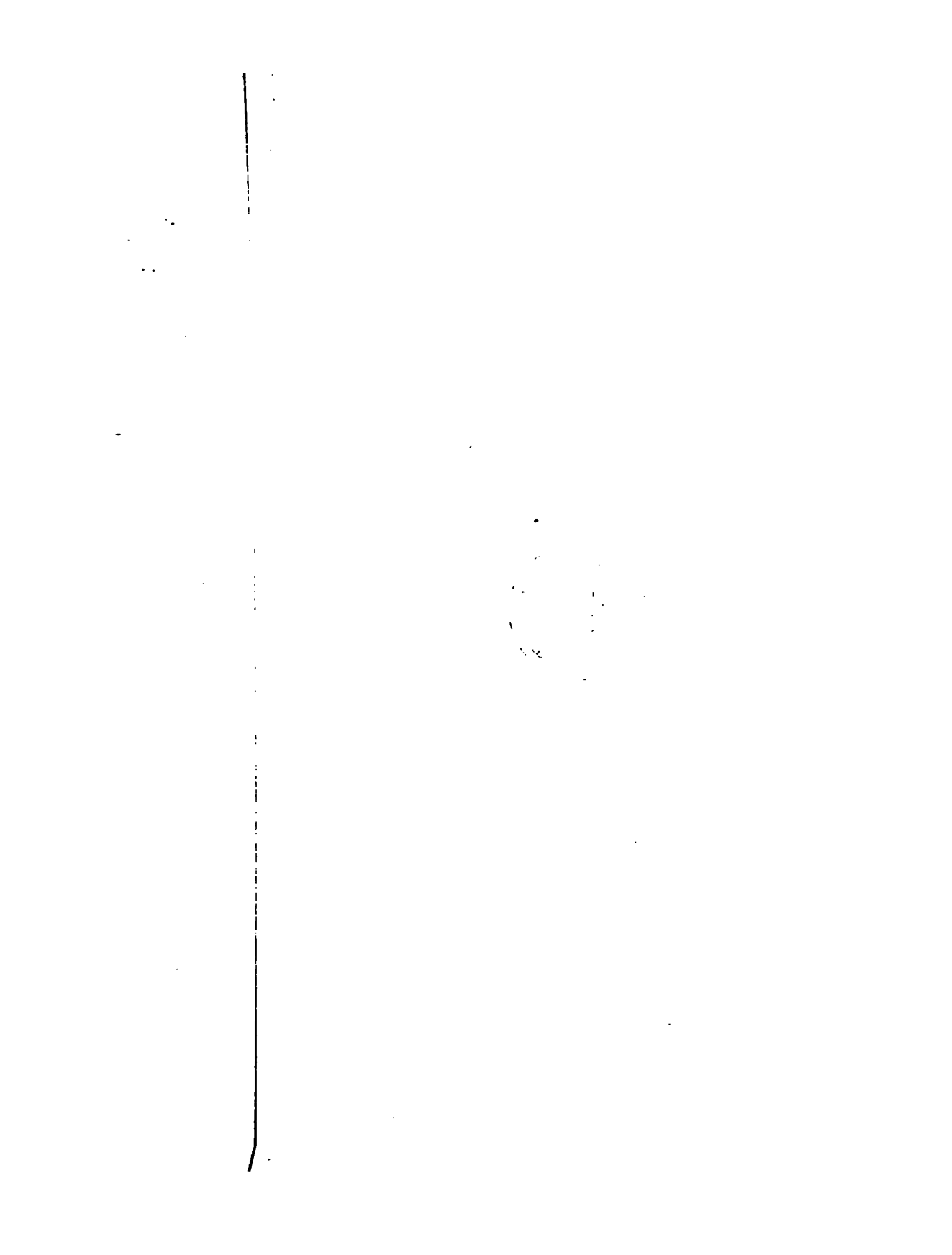
Zees, list of steel and iron, Table XXV, End of Vol. I.

Zigzag riveting, II, 62.

TABLE XXIX.

Classification of Structural Irons and Steels.





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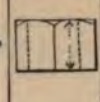
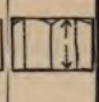
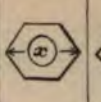
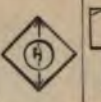
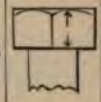
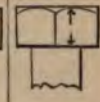
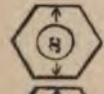
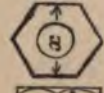
II. PROPORTIONS FOR

and BOLT HEADS.

Thickness Rough.	Thickness Finish.	Short Diameter Rough.	Short Diameter Finish.	Long Diameter Rough.	Long Diameter Rough.	Thickness Rough.	Thickness Finish.
							
¼	3/16	⅛	1/16	1/16	1/16	¼	1/16
5/16	¼	¼	1/16	⅛	⅛	5/16	⅛
⅜	5/16	⅜	⅛	⅛	⅛	⅜	⅛
7/16	⅜	⅜	⅛	1/16	1/16	7/16	⅛
½	7/16	½	⅛	1	1 1/16	½	⅛
9/16	½	½	⅛	1⅛	1⅛	9/16	⅛
⅝	9/16	⅝	1/16	1⅛	1⅛	⅝	1/16
¾	⅝	¾	1/16	1⅛	1⅛	¾	1/16
⅞	¾	⅞	1/16	1⅛	1⅛	⅞	1/16
1	15/16	1⅛	1/16	1⅛	1⅛	1	1/16
1⅛	1 1/16	1⅛	1/16	1⅛	1⅛	1⅛	1/16
1¼	1⅛	1⅛	1/16	1⅛	1⅛	1¼	1/16
1⅜	1⅛	1⅛	1/16	1⅛	1⅛	1⅜	1/16
1½	1⅛	1⅛	1/16	1⅛	1⅛	1½	1/16
1⅝	1⅛	1⅛	1/16	1⅛	1⅛	1⅝	1/16
1¾	1⅛	1⅛	1/16	1⅛	1⅛	1¾	1/16
1⅞	1⅛	1⅛	1/16	1⅛	1⅛	1⅞	1/16
2	1⅛	1⅛	1/16	1⅛	1⅛	2	1/16
2¼	1⅛	1⅛	1/16	1⅛	1⅛	2¼	1/16
2½	1⅛	1⅛	1/16	1⅛	1⅛	2½	1/16
2¾	1⅛	1⅛	1/16	1⅛	1⅛	2¾	1/16
3	1⅛	1⅛	1/16	1⅛	1⅛	3	1/16
3¼	1⅛	1⅛	1/16	1⅛	1⅛	3¼	1/16
3½	1⅛	1⅛	1/16	1⅛	1⅛	3½	1/16
3¾	1⅛	1⅛	1/16	1⅛	1⅛	3¾	1/16
4	1⅛	1⅛	1/16	1⅛	1⅛	4	1/16
4¼	1⅛	1⅛	1/16	1⅛	1⅛	4¼	1/16
4½	1⅛	1⅛	1/16	1⅛	1⅛	4½	1/16
4¾	1⅛	1⅛	1/16	1⅛	1⅛	4¾	1/16
5	1⅛	1⅛	1/16	1⅛	1⅛	5	1/16
5¼	1⅛	1⅛	1/16	1⅛	1⅛	5¼	1/16
5½	1⅛	1⅛	1/16	1⅛	1⅛	5½	1/16
5¾	1⅛	1⅛	1/16	1⅛	1⅛	5¾	1/16
6	1⅛	1⅛	1/16	1⅛	1⅛	6	1/16



Rough Head—one and one-half diameter of bolt + ⅛.



Finished Head—one and one-half diameter of bolt + 1/16.

Rough head—one-half distance between parallel sides of head.

Finished head—diameter of bolt—⅛.

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TA

VALUES FOR

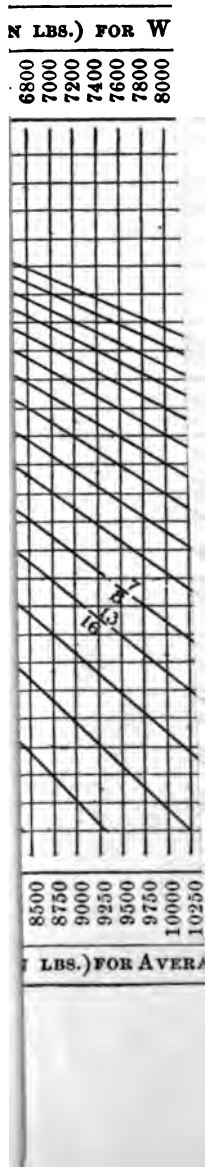

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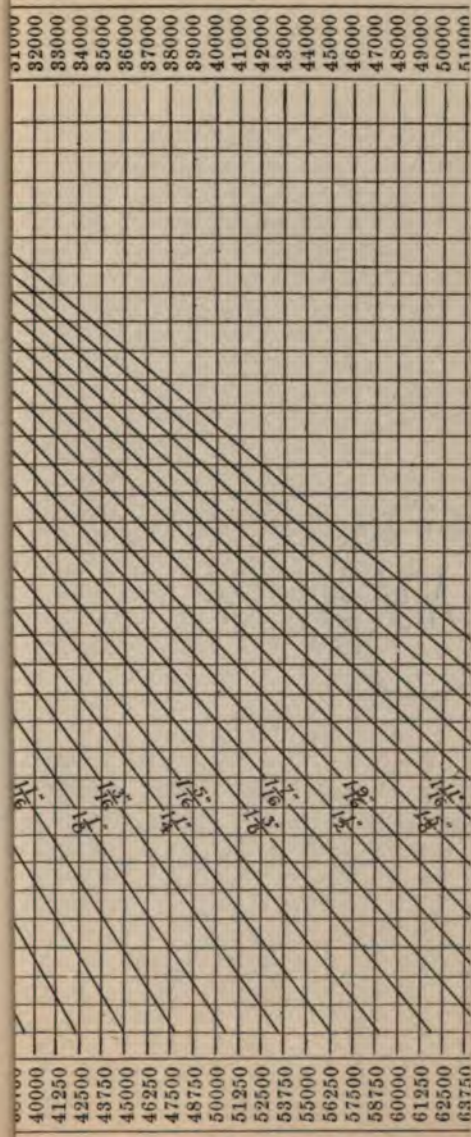
TABLE XXXVI.

FOR IRON AND STEEL PINS AND BOL

VROUGHT IRON TAKEN AT 12000 LBS. PER



RAGE STEEL TAKEN AT 15000 LBS. PER SQ

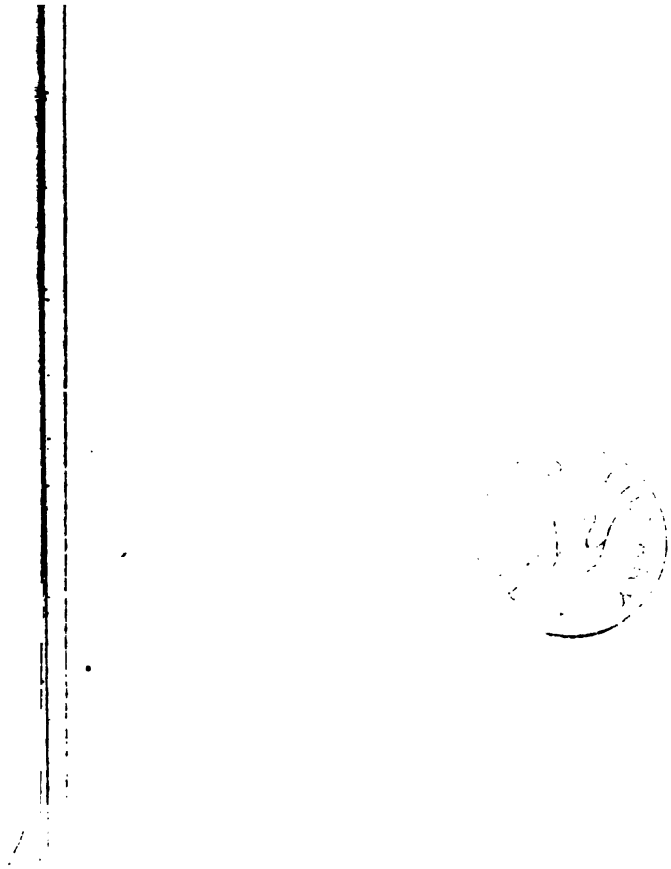
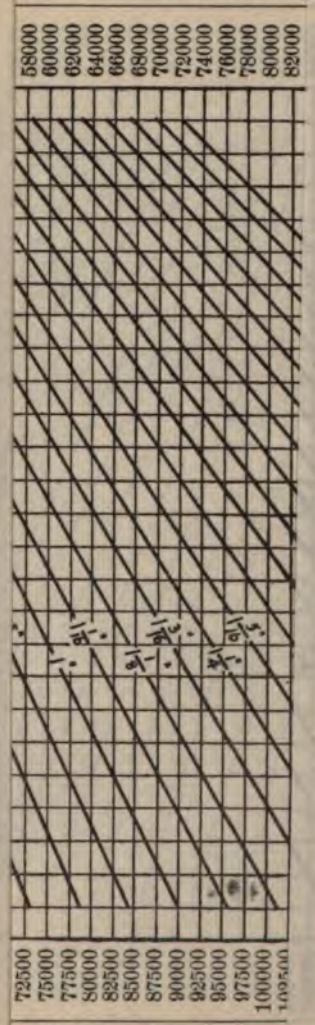


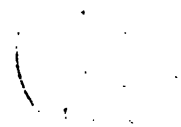
TABLE XXXVII.

UE FOR IRON AND STEEL PI

R WROUGHT IRON, TAKEN



R AVERAGE STEEL, TAKE



TABLE

.BING, BENDING AN

VETS, PINS AND RODS

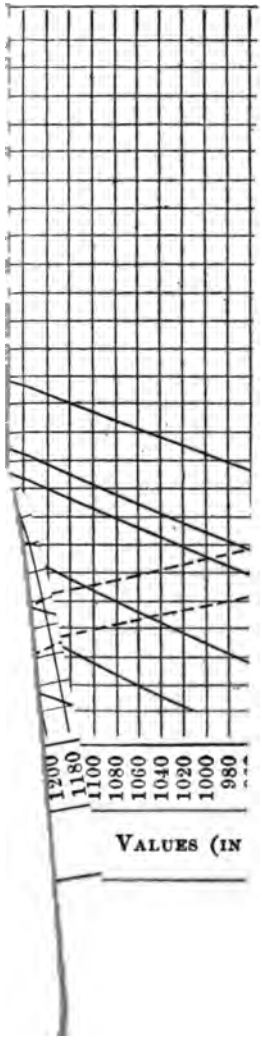
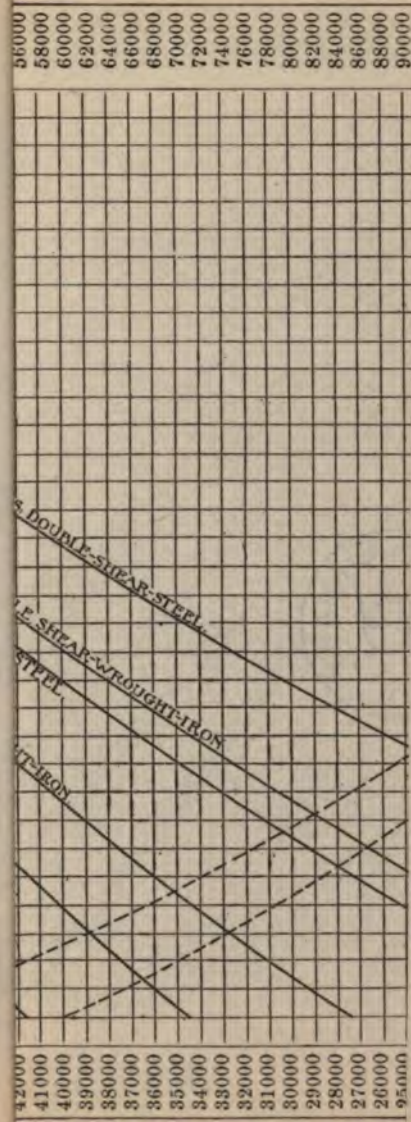


TABLE XXXIX.

R IRON AND STEEL PINS AND

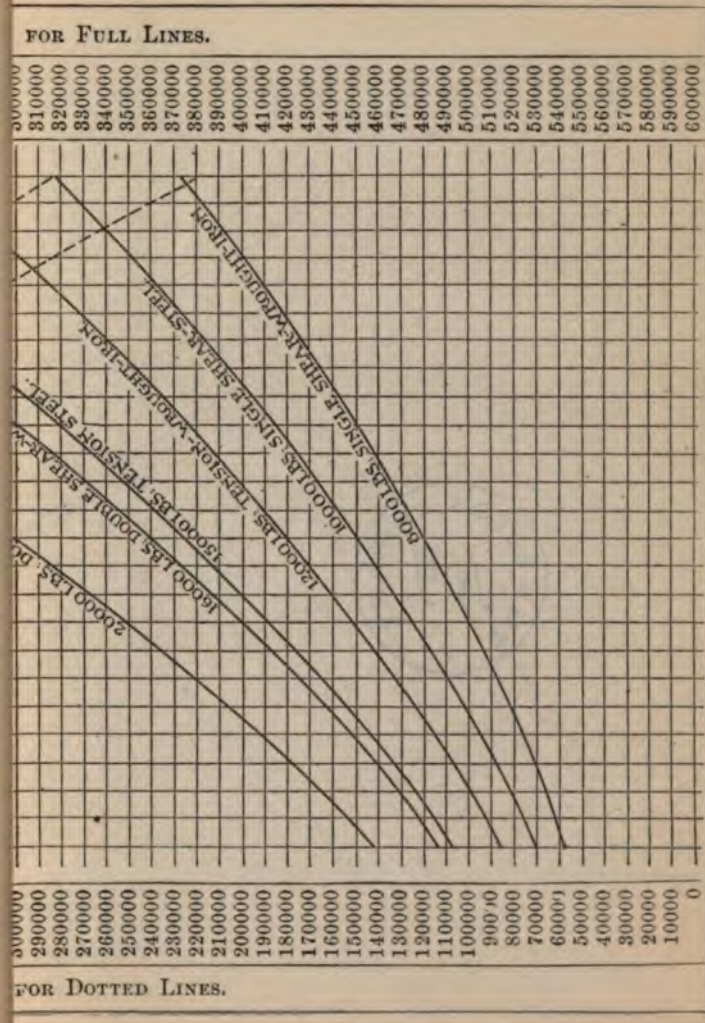
ALUES (IN LBS.) FOR FULL LINES.

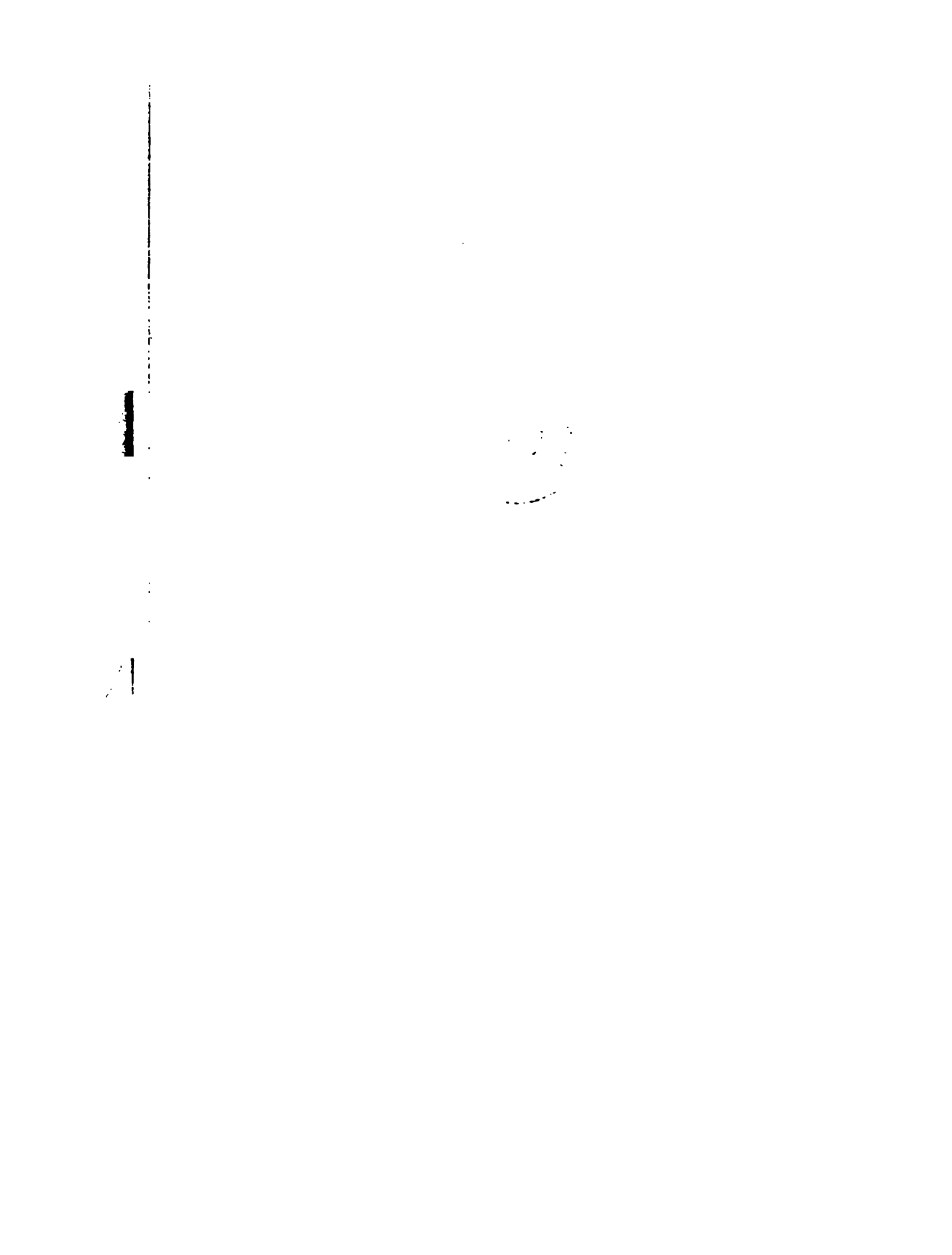


UES (IN LBS.) FOR DOTTED LINES.

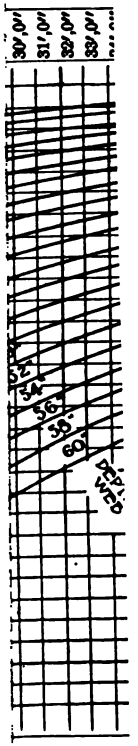
L.

Steel Pins and Rods 3 to 6 Inches in Diameter.





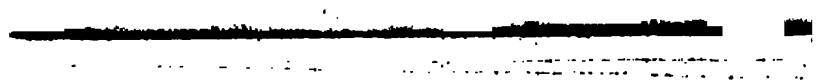
IRON RIV



OR DIFFERI

eflection	
r Steel.	
$= \frac{3.L^2}{200.d}$	C
$= \frac{3.L^2}{242.d}$	C

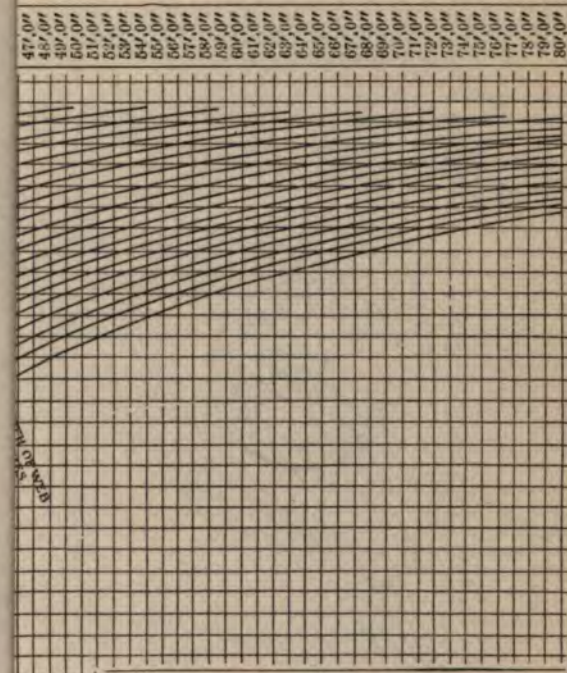
U = UNIFO
$= W_{uu}$ = CO



I.

RENGTH OF THE ANGLE BARS.

OF SPAN IN FEET.



IF ANGLE BARS ARE OF STEEL ALL RIVETS MUST BE OF STEEL.

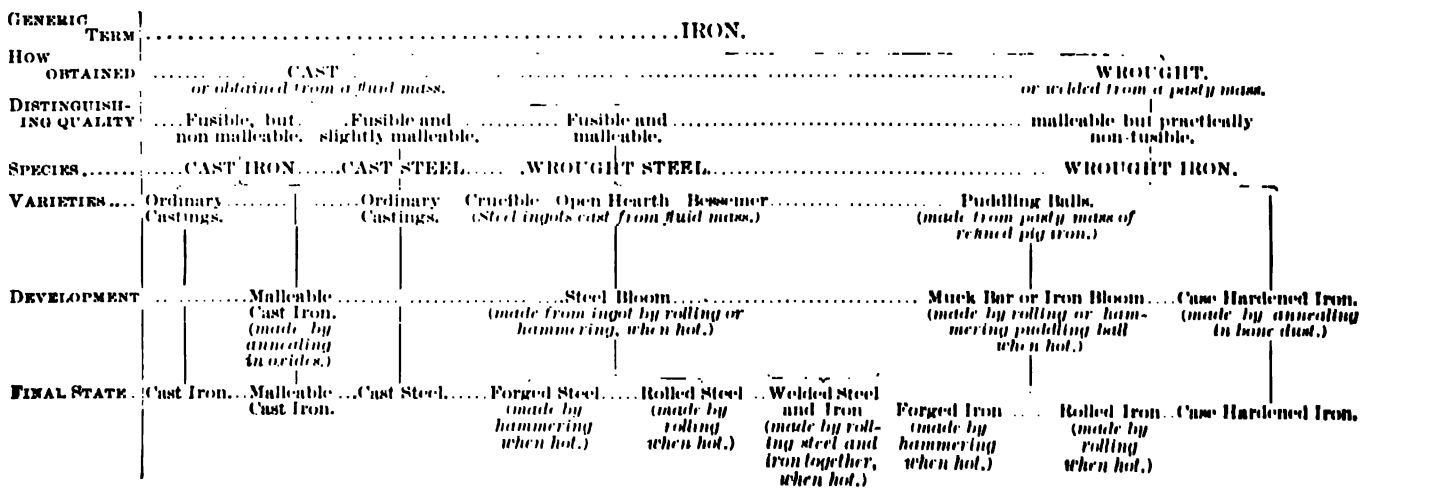
FOR STEEL ANGLE BARS

Add one quarter to safe load on iron bars, but length of span (in feet) must not exceed twice the depth of girder (in inches) or deflection will be too great for plastering.

LECTION, ETC., SEE TABLE XLI.

TABLE XXIX.

Classification of Structural Irons and Steels.



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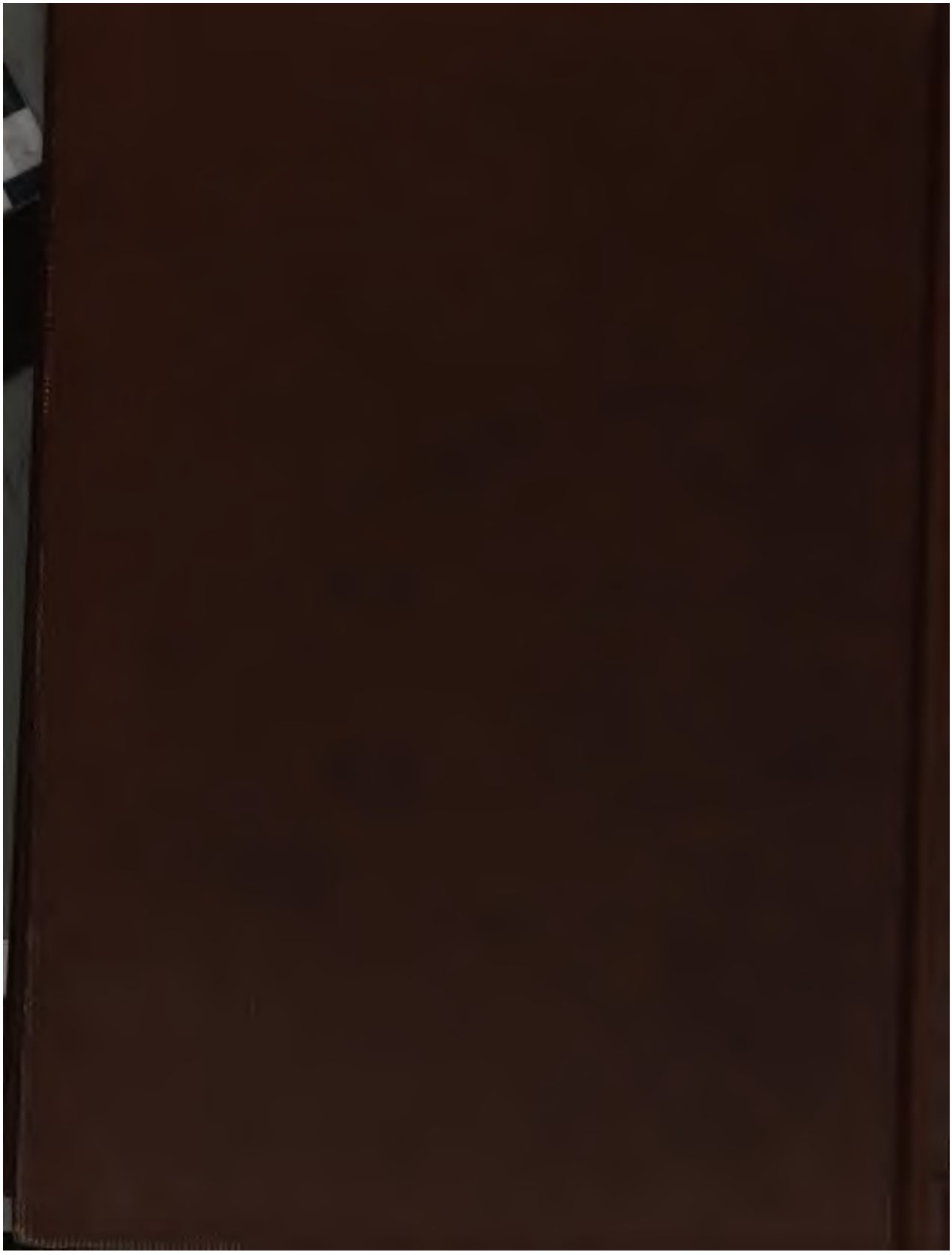
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